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AFCRL 507

LUNAR TRAJECTORY STUDIES

A. Petty
I. Jurkevich
M. Fabrize
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General Electric Company
Missile and Space Vehicle Department
Space Sciences Laboratory
Philadelphia, Pa.

Final Report

Contract No. AF 19(604)-5863

JUNE 1961

ELECTRONICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
Bedford, Mass.

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ABSTRACT

This report presents a description of several related IBM-7090 Fortran computer programs designed to provide a fast, accurate, and systematic procedure for determining initial conditions to the differential equations of motion from tracking data, lunar and interplanetary trajectories in n-body space, and satellite ephemeris compilations.

The theory and analytical formulation for each program is given in detail. Instruction in program usage is given with individual check problems provided to facilitate operational proficiency.

To verify the formulations the following examples were used:

1. Initial Condition Determinations - (Data for the asteroid Leuschnerina 1935 used in form of a check problem).
2. Trajectory Computation - Lunik III Data (included in interim report).
3. Ephemeris Computation - (Data for the asteroids Pallas and Vesta used in form of a check problem).

In addition to the above work, this report also contains a study of Lunar Trajectories. The types of trajectories considered are:

1. Error Analysis of Hyperbolic Impact Trajectories.
2. Lifetime of an Artificial Lunar Satellite.
3. Lunar Circumnavigation and Earth Return.

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I. INTRODUCTION

This report, which represents the final report on contract AF 19(604)-5863, is intended to cover only that work performed between July 1960 and June 1961. All work prior to this period has been given in an interim report.* All remarks made in the interim report concerning restrictions and assumptions pertaining to uncertainties in the physical constants employed and the resulting effects on the accuracy of the computations are applicable to this report.

A computer program designed to compute the trajectories of a body in the combined gravitational field of the Earth, Sun, and Moon, using true ephemerides of the positions of these bodies, has been operational for some time. It is described in the interim report.* This program has been extended to include the major planets and is known as the "n-body interplanetary trajectory program" (n = 9). The formulation of the equations of motion and the numerical integration techniques used are discussed in detail in the interim report. The main features and operational procedures are, however, included in Appendix A of this report.

Such a program is of great value in the computation of theoretical trajectories and orbits in which assumed sets of initial conditions are employed. However, employment of tracking data obtained from various radar and/or optical equipments located at a specific geographical location on the earth will require a great deal of additional hand computation before a "suitable set of initial conditions may be derived." This additional computation may assume several forms of which orbit determination, coordinate transformations,

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* AFCRL - TN - 60 - 1132 Scientific Report No. 1 (AF 19(604)-5863 Lunar Trajectory Studies and An Application to Lunik III Trajectory Prediction - Petty, A. F., Jurkevick, I.; July 1960.

equinox to equinox reductions, and compilation of ephemeris tracking data are prominent.

These hand computations are tedious and time consuming. If a sizable number are required the impracticality of manual computation becomes increasingly evident. To circumvent this problem, computer subroutines have been developed. These routines, used in conjunction with the "n-body interplanetary trajectory program", provide a high degree of flexibility in solving diverse orbital and trajectory problems.

An objective of this report is to present each program's analytical development and to illustrate its use in augmenting the inherent capability of the "n-body interplanetary trajectory program." Machine listings, together with programming instructions, are given for those with more than cursory interest. (See Appendix B)

Verification of the accuracy of these computer programs and routines was established by recomputing the orbits and trajectories of well known astronomical objects (asteroids). In no case were theoretical orbits or trajectories employed to test the programs.

In particular, extensive use has been made of the data for the asteroid Leuschnerina 1935 in developing the initial condition determination programs, both for the two position vector case and for the three angular position case. Also a recomputation of the orbits of the asteroids Pallas and Vesta were used to verify the ephemeris prediction accuracy of the "n-body interplanetary trajectory program." Residuals obtained by comparing computed values against tabular values as given by the American Ephemeris and Nautical Almanac is indicative of the accuracy achievable.

The computer program development mentioned above represents one of the two major aspects of this contract. The other is the employment of these programs in the study of several classes of Lunar Trajectories. The first of these, a circumlunar flight which passes at a distance of 7,000 km from the Moon (Lunik III) was described in detail in the interim report. Here, tracking data released through the Russian news agency TASS was employed to obtain a set of initial conditions to the differential equations of motion. A second circumlunar flight initiating from the Atlantic Missile Range (AMR) was also studied in some detail and the results are included in this report. Such constraints as range safety limits, launch time, and booster limitations were considered as well as the usual two point boundary value constraints. The effect of some of these constraints is indicated.

A major effort was given to the determination of initial condition error sensitivities for "Hyperbolic Lunar Impact Trajectories." Nominal impact trajectories were established on three separate dates and error tubes obtained. No attempt was made to artificially constrain the nominal trajectory to a normal impact. In all cases the center of the apparent lunar disc was taken as the nominal trajectories impact location.

The final study involving the life time of a lunar satellite over extended period of time (25 days) indicates the perturbative effects of the earth and sun on the orbital elements. The instantaneous perturbations of the elements as a function of time are presented for the first ten (10) days of the orbit. An interesting development arising from this study is the fact that for identical initial conditions the life time of the satellite in an Earth, Moon, Sun field exceeds 25 days (we did not ascertain the limit) while in an Earth-Moon field, the vehicle impacts the Moon after some 9 days.

This report contains the results of all of the above mentioned studies with the exception of the Lunik III trajectory. Also, a few additional special cases are included in Section VIII of this report to illustrate certain statements in the text.

II. INITIAL CONDITION DETERMINATIONS (GENERAL CONSIDERATIONS)

GENERAL COMMENTS

In the development of the following computer programs two branches of Astronomy have been used. These are spherical Astronomy and Celestial Mechanics. The former is concerned with the details of establishing precise and practical coordinate systems from which observational or tracking data can be readily employed in the theoretical equations developed in the latter. An attempt has been made in this report to follow the standard developments found in the Astronomical texts. Familiarity with these standard conventions is assumed. However, for each routine or program, all parameters are defined in terms of the units used and the direction in which they are measured. Appropriate references are given to the Astronomical literature throughout the discussions.

A. INTRODUCTION

Once a set of initial conditions becomes known, the differential equations of motion on the n-body problem may be numerically integrated. Such a set comprises the vehicle's position and velocity vector at an instant of time relative to a particular coordinate system. It may be determined at the end of thrust or at subsequent times.

This section's objective is to indicate methods for determining this set of initial conditions from observational or tracking data. Concern here is with single tracking stations whose tracking equipment cannot instantaneously measure or provide vehicle position and velocity vector. Tracking situations considered are:

(a) Vehicle position vector is measured or known at two instances of time.

(b) Vehicle angular position is measured or known at three instances of time. (no distance information available)

(c) Distance to the vehicle is measured or known at six instances of time. (no angular information is available)

It is to be noted that in all these tracking situations, six pieces of information are obtained. This is a necessary and sufficient condition to establish the required initial conditions.

Situation (a) corresponds to a radar recording the vehicle's range and angular coordinates. Situation (b) corresponds to an optical or infrared telescope recording of only the angular positions of the vehicle. Situation (c) corresponds to a radar recording of only the range to the vehicle.

In all cases it has been assumed that the time of each measurement is accurately known as well as the geographical position of the tracking station on the Earth's surface.

B. DETERMINATION OF THE INITIAL CONDITIONS FROM TWO POSITION VECTORS

Following Laplace, the position vector at any time t may be expressed in terms of the position and velocity vector at some time t_o by

$$\vec{r}(t) = f\vec{r}_o + g\dot{\vec{r}}_o \tag{1}$$

where $\vec{r}_o = \vec{r}(t_o)$, $\dot{\vec{r}}_o = \dot{\vec{r}}(t_o)$

and $$f = 1 - \frac{1}{2}\mu\tau^2 + \frac{1}{2}\mu\sigma\tau^3 + \frac{\mu}{24}(3\omega - 2\mu - 15\sigma^2)\tau^4 - \frac{\mu\sigma}{8}(3\omega - 2\mu - 7\sigma^2)\tau^5 + \ldots$$

$$g = \tau - \frac{1}{6}\mu\tau^3 + \frac{1}{4}\mu\sigma\tau^4 + \frac{\mu}{120}(9\omega - 8\mu - 45\sigma^2)\tau^5 + \ldots$$

$$\mu = \frac{MG}{|\vec{r}_o|^3}, \qquad \sigma = \frac{\vec{r}_o \cdot \dot{\vec{r}}_o}{|\vec{r}_o|^2}, \quad \omega = \frac{|\dot{\vec{r}}_o|^2}{|\vec{r}_o|^2}, \quad \tau = (t - t_o)$$

and where G is the gravitational constant and M is the mass of the Earth.

Note that f and g are scalar functions of the position and velocity at time t_o and the time interval between measurements τ. If the conditions at t_o are known, the entire trajectory is specified by equation (1).

The three scalar equations associated with (1) are:

$$\begin{aligned} X &= fX_o + g\dot{X}_o \\ Y &= fY_o + g\dot{Y}_o \\ Z &= fZ_o + g\dot{Z}_o \end{aligned} \tag{2}$$

Now if observations are made at time t and t_o, so that X, Y, Z, and X_o, Y_o, Z_o can be computed, the velocity components $\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$ may be expressed in terms of the scalar function f and g as

$$\begin{aligned} \dot{X}_o &= \frac{1}{g}(X - fX_o) \\ \dot{Y}_o &= \frac{1}{g}(Y - fY_o) \\ \dot{Z}_o &= \frac{1}{g}(Z - fZ_o) \end{aligned} \tag{3}$$

Since f and g are themselves functions of the velocity $\dot{\vec{r}}_o$, the velocity components are given only implicitly by (3). To determine the velocity components, f and g are initially approximated by the first and second terms in the series which defines them. Specifically, f and g are initially approximated by $1 - \frac{1}{2}\mu\tau^2$ and $\tau - \frac{1}{6}\mu\tau^3$ respectively.

Equation (3) is then solved, giving first approximations to $\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$. These values are used to recompute the f and g series, which in turn a used in (3) to recompute better approximations to $\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$. This procedure is repeated until successive iterations result in velocity components that differ by less than some prescribed tolerance.

C. DETERMINATION OF THE INITIAL CONDITIONS FROM THREE ANGULAR POSITIONS

(Distance Information Not Available)

Again following Laplace, but employing the entire development, computation of the initial conditions from angular observations alone is possible. Generally, the Laplacian method consist of writing two equations in two unknowns. One, the geometrical equation, can be expressed as

$$r^2 = \rho^2 + R^2 - 2\,(\vec{P} \cdot \vec{R})\,\rho \tag{4}$$

and the other, the dynamical equation, as

$$\rho \left[\vec{P} \times \dot{\vec{P}} \cdot \ddot{\vec{P}}\right] = \left[\vec{P} \times \dot{\vec{P}} \cdot \ddot{\vec{R}}\right] + \left[\vec{P} \times \dot{\vec{P}} \cdot \vec{R}\right] / r^3 \tag{5}$$

in which $\vec{\rho}$ is the radius vector between the observing station and vehicle, and $\vec{r}$, the geocentric distance of the object at time t. These are the unknown quantities. The quantity $\vec{R}$ is the geocentric radius vector of the observing station and is assumed to be known. The quantity $\vec{P}$ is a unit vector directed along the line from the observing station to the vehicle. Its components are the direction cosines of the observations and together with the components of $\dot{\vec{P}}_o$ and $\ddot{\vec{P}}_o$, enable evaluation of the triple scalar products in the dynamical equations.

For computational facility various schemes have been advocated. The methods differ only in the quantities taken as the independent variables. Thus, the straight or original Laplacian method directly uses the magnitudes of $\vec{\rho}$ and $\vec{r}$.

Leuschner's modification uses $\rho \cos \delta$ and $|\vec{r}|$ as the variables, and Stumpff employes $|\vec{r}|$ and one of the components of $\vec{r}$ as the principal variables. The quantity $\rho \cos \delta$ is known as the curtate distance, and δ represents the declination.

We have chosen to program Stumpff's method. This method derives its principal advantage from the use of the ratios of the direction cosines and the reduction of all the determinants from the third to second order. In the original Laplacian formulation, all the formulas were expressed in terms of third order determinants which correspond to the triple scalar product. The following formulation of Stumpff's Method has been employed. (See Herget - "Computation of Orbits")

Let

$$U = \frac{y+Y}{x+X} = \tan\alpha\,, \quad V = \frac{z+Z}{x+X} = \sec\alpha \tan\delta\,, \quad P = Y - UX,$$

$$Q = Z - VX \qquad (6)$$

where the small x, y, z are components of the vehicle and the large X, Y, Z are the components of the tracking site both referred to the same geocentric equatorial coordinate system. The latter are known quantities since we assume the geographic coordinates of the tracking station on the Earth's surface are known. The quantities α (the right ascension) and δ (the declination) are the measured observables. Three sets $\alpha_1 \delta_1$, $\alpha_2 \delta_2$, $\alpha_3 \delta_3$ of these observables are recorded and are the fundamental six pieces of information required for the trajectory determination.

Cross multiplying the equations for U and V, introducing P and Q, and differentiating twice, obtains

$$\begin{aligned} y &= Ux - P & z &= Vx - Q \\ \dot{y} &= \dot{U}x + U\dot{x} - \dot{P} & \dot{z} &= \dot{V}x - V\dot{x} - \dot{Q} \\ \ddot{y} &= \ddot{U}x + 2\dot{U}\dot{x} + U\ddot{x} - \ddot{P} & \ddot{z} &= \ddot{V}x - 2\dot{V}\dot{x} + V\ddot{x} - \ddot{Q} \end{aligned} \qquad (7)$$

Substituting the dynamical conditions for each component of $\ddot{\vec{r}} - \dfrac{\vec{r}}{r^3}$ into the two bottom equations of (7) obtains

$$\frac{1}{2}\ddot{U}x + \dot{U}\dot{x} = \frac{1}{2}\ddot{P} + \frac{P}{2r^3}$$

$$\frac{1}{2}\ddot{V}x + \dot{V}\dot{x} = \frac{1}{2}\ddot{Q} + \frac{Q}{2r^3} \tag{8}$$

Let

$$D = \frac{1}{2}\ddot{U}\dot{V} - \frac{1}{2}\ddot{V}\dot{U}$$

obtaining

$$Dx = \frac{1}{2}(\ddot{P}\dot{V} - \ddot{Q}\dot{U}) + (P\dot{V} - Q\dot{U}) / 2r^3$$

$$D\dot{x} = \frac{1}{4}(\ddot{Q}\ddot{U} - \ddot{P}\ddot{V}) + (\ddot{U}Q - \ddot{V}P)/4r^3 \tag{9}$$

$$r^2 = x^2 + y^2 + z^2 = (1 + U^2 + V^2)X^2 - 2(UP + VQ)X + (P^2 + Q^2)$$

In equation (9) the only unknowns are x, $\dot{x}$, and r which can be obtained by a simple iteration between the equations. The approximate numerical values of the coefficients at time t_o, the time of the middle observation, may be obtained from the observations by writing a Taylor's series for the first and third observations as

$$W_1 = W_o + \dot{W}_o T_1 + \frac{1}{2}\ddot{W}_o T_1^2 + \ldots.$$

$$W_3 = W_o + \dot{W}_o T_3 + \frac{1}{2}\ddot{W}_o T_3^2 + \ldots. \tag{10}$$

which may be written in the form

$$(W_1 - W_o)\, T_1 = \dot{W}_o + \frac{1}{2}\ddot{W}_o T_1 = (W, 1)$$

$$(W_3 - W_o)\, T_3 = \dot{W}_o + \frac{1}{2}\ddot{W}_o T_3 = (W, 3)$$

and then

$$\dot{W}_o (T_3 - T_1) = T_3 (W, 1) - T_1 (W, 3)$$

$$\frac{1}{2}\dot{W}_o (T_3 - T_1) = (W, 3) - (W, 1)$$

where W denotes U, V, P or Q and $T_1 = (t_1 - t_o)$, $T_3 = (t_3 - t_o)$. Thus, all coefficients can be determined from the observations, and equations in (9) solved. With the solutions from (9), (7) may be employed to obtain the other components of the vehicle's position and velocity vectors at time t_o. Initial conditions are thus obtained.

The value of D is the controlling factor of the entire solution. This corresponds to the coefficient of ρ in the left-hand member of (5). If extremely small, it indicates the time interval between measurements is too small to make the solution very determinate. Also, it is easy to see that difficulties will arise in employing this method when observations exist in the neighborhood of 6^h or 18^h right ascension, due to the large, or even meaningless, values obtained for the derivatives of U. Methods for circumventing this problem are currently being considered e.g., rotating the coordinate system by a fixed amount, though this will probably only shift the problem to the V's for observations near the celestial pole.

Employing more than three observations will aid in alleviating this problem. This, however, is an entirely different computation involving differential correction procedures and will not be discussed here.

D. DETERMINATION OF INITIAL CONDITIONS FROM SIX RADAR RANGE MEASUREMENTS

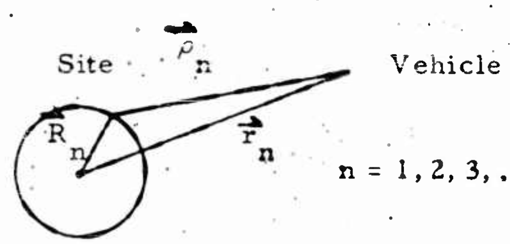
(Angular Information not available)

Initially, it appears that the initial conditions could be obtained from six radar measurements employing a line of reasoning similar to that used for two range vectors.

The observing site, the vehicle, and the dynamical center are connected by

$$\vec{\rho} = \vec{r} - \vec{R} \qquad (11)$$

as shown in the following sketch



$n = 1, 2, 3, \ldots 6.$

From the law of cosines

$$r_n^2 = R_n^2 + \rho_n^2 + 2\,\vec{R}_n \cdot \vec{\rho}_n \qquad (12)$$

Recall now that in Laplace's method

$$\vec{r}_n = f_n \vec{r}_o + g_n \dot{\vec{r}}_o \qquad (13)$$

From (13) it follows directly that

$$r_n^2 = f_n^2 r_o^2 + 2 f_n g_n \vec{r}_o \cdot \dot{\vec{r}}_o + g_n^2 \dot{r}_o^2 = (f_n^2 + 2 f_n g_n \sigma + g_n^2 \omega)\, r_o^2 , \qquad (14)$$

where σ and ω are defined as for the case of two range vectors. The same holds for the f_n and g_n series.

It is further convenient to express $2 \vec{\rho}_n \cdot \vec{R}_n$ as

$$2 (\vec{r}_n - \vec{R}_n) \cdot (\vec{R}_n) = 2 \vec{r}_n \cdot \vec{R}_n - 2 R_n^2 \tag{15}$$

Employing now (4) and (5), in (2) obtains

$$C_n = \rho_n^2 - R_n^2 = (f_n^2 + 2 f_n g_n \sigma + g_n^2 \omega) r_o^2 - 2f_n X_n x_o - 2 f_n Y_n y_o - 2f_n Z_n z_o - 2g_n X_n \dot{x}_o - 2g_n Y_n \dot{y}_o - 2g_n Z_n \dot{z}_o \tag{16}$$

In the above, X_n, Y_n, Z_n are the rectangular components of the vector $\vec{R}_n$ at the corresponding time τ_n at which the range ρ_n is measured.

Using obvious definitions, it is convenient to write equations (16) as follows:

$$a_{n1} r_o^2 + a_{n2} x_o + a_{n3} y_o + a_{n4} z_o + a_{n5} \dot{x}_o + a_{n6} \dot{y}_o + a_{n7} \dot{z}_o = C_n \tag{17}$$

$$n = 1, 2, 3, \ldots 6$$

A few remarks are pertinent with respect to this set. The unknown quantities are x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$, and r_o although the latter is given by $r_o^2 = x_o^2 + y_o^2 + z_o^2$. The coefficients a_{ni} are not really known, although, an approximation to these is available by taking first terms in the f and g series.

It is apparent that if r_o is included among the unknowns seven measurements instead of six are needed. On the other hand, a better approximation of f_n and g_n can be obtained by taking two terms in these series. This is, however, equivalent to estimating r_o. If this is the adopted procedure, only six measurements are required.

The computational scheme is then:

The input data are the six values of measured range; an estimate of r_o; X_o, Y_o, Z_o; sidereal time; six values of time at which the measurements are taken; and the rotation rate of the Earth.

From the above the coefficients a_{ni} are estimated and set (17) is solved. This results in quantities

$$r_o = (x_o^2 + y_o^2 + z_o^2)^{1/2}$$

$$v_o = (\dot{x}_o^2 + \dot{y}_o^2 + \dot{z}_o^2)^{1/2}$$

From these, better estimate of μ, σ and ω are obtained entering the f_n and g_n series. Using these, the whole procedure is repeated until the desired precision is obtained.

III. INITIAL CONDITION DETERMINATIONS (COMPUTER ROUTINES)

Section II presented the motivation behind the need for tracking subroutines. Summarizing, the purpose of tracking subroutines is to yield initial conditions required to initiate numerical integration of the differential equations of the n-body problem.

Basic ideas and computational formulas involved in the three chosen routines were outlined in Section II. In the present section, detailed step-by-step procedures, as programmed for the computer are given. In all three cases, however, discussion is more extensive than that prepared for computer use, reflecting the programs evolution. The entire development will be included since many procedures, though not employed in final computational programs, can be of interest to the user willing to effect personal modifications.

It should be noted that one of the three tracking schemes considered in this report proves unsuitable for numerical computations. Reasons for failure of the routine employing six ranges, measured from a single tracking station, will be discussed later.

The first tracking scheme to be described employs the vehicle's three measured angular positions.

A. DETERMINATION OF THE INITIAL CONDITIONS FROM THREE ANGULAR POSITIONS

This tracking method's purpose is to compute the components of the vehicle's position and velocity at some time t from measurements of its right ascension and declination at three instants of time. It is assumed that if the measured quantities are azimuth and elevation, they are transformed into the corresponding right ascension and declination by methods outlined in Section IV. It is further assumed that these methods are employed to prepare all measured data and auxiliary quantities for computations.

It must be emphasized that all tracking methods described in this report yield preliminary values of initial conditions. The significance is that a minimum of data is used to effect the computation. Consequently, no provisions are made to accept redundant data to obtain a better estimate of the desired quantities. Standard differential correction procedures can be used, eg. maximum likelihood estimation, or the conventional least squares method once the initial preliminary trajectory is obtained. Their inclusion in this report is omitted as they were not considered a part of the study.

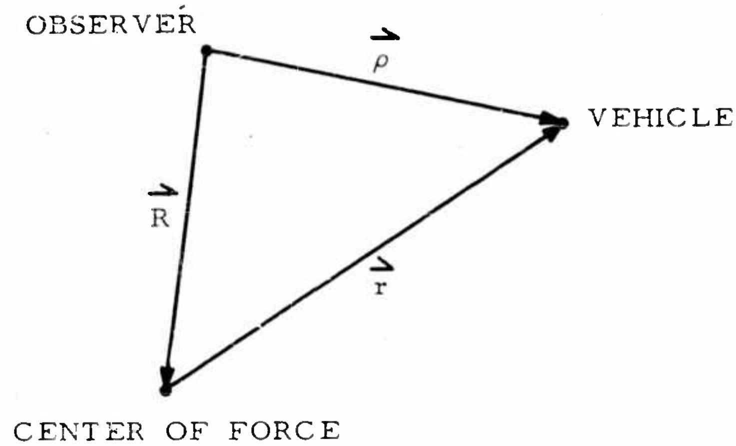
Initial estimates are based on a two body problem where the vehicle moves in the immediate neighborhood of some dominant mass. The latter will, in most cases, be the Earth.

It can be seen from Figure 1 that an astronomer usually considers the distances Observer → Vehicle and Observer → Center of Force positive. For purposes of this report it was found more convenient to measure distance Center of Force → Observer as positive. Distance $\vec{r}$ is in both cases measured positively from the center of the force.

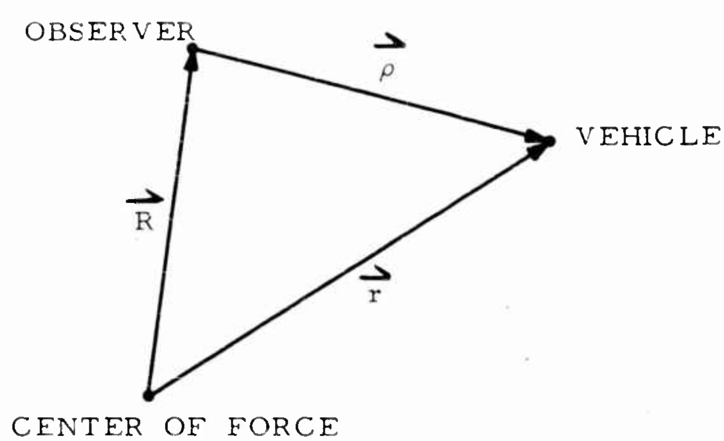
The new definition of the sign of $\vec{R}$ will result in expressions slightly different from those in the previous section. For this reason, the development of Stumpff's method is repeated. The choice of this particular method was discussed in Section II.

1. Determination of the Initial Conditions from Three Angular Positions

From Figure 1b, it can be seen that the observer's position,



(a) Astronomical convention



(b) Convention employed in the report

Figure 1. Relation Between the Observer, Vehicle and Center of Force

the vehicle's geocentric position, and position of the center of force are related by

$$\vec{r} = \vec{\rho} + \vec{R}$$
$$\therefore \vec{\rho} = \vec{r} - \vec{R} \tag{1}$$

In appropriate rectangular components this is:

$$\xi = \rho \cos \delta \cos \alpha = x - X$$
$$\eta = \rho \cos \delta \sin \alpha = y - Y \tag{2}$$
$$\zeta = \rho \sin \delta = z - Z$$

Employing the latter expressions define

$$U \equiv \mathrm{TAN}\, \alpha = \frac{y - Y}{x - X}$$
$$V \equiv \mathrm{SEC}\, \alpha\ \mathrm{TAN}\, \delta = \frac{z - Z}{x - X} \tag{3}$$

These equations yield in an obvious manner

$$Ux - UX = y - Y$$
$$Vx - VX = z - Z \tag{4}$$

or

$$Y - UX = y - Ux \equiv P$$
$$Z - VX = z - Vx \equiv Q \tag{5}$$

The last equations can be rewritten as

$$y = P + Ux$$
$$z = Q + Vx \qquad (6)$$

In the above, it must be noted, α and δ are measured in the coordinate system fixed at the observer's site. The components of $\overline{R}$ can always be found as soon as the location of the tracking station and the time of measurement are known. Both α, δ and X, Y, Z must be referred to the same coordinate system. It is recommended that some standard coordinate system be adopted, e.g., that referenced to the mean equinox of 1950.0.

Differentiating equations (6) obtains

$$\dot{y} = \dot{U}x + U\dot{x} + \dot{P} \qquad \dot{z} = \dot{V}x + V\dot{x} + \dot{Q}$$
$$\ddot{y} = \ddot{U}x + 2\dot{U}\dot{x} + U\ddot{x} + \ddot{P} \qquad \ddot{z} = \ddot{V}x + 2\dot{V}\dot{x} + V\ddot{x} + \ddot{Q} \qquad (7)$$

Note the equations of motion of the vehicle in the gravitational field of the dominant mass are given by

$$\ddot{x} = -\frac{x}{r^3}, \quad \ddot{y} = -\frac{y}{r^3}, \quad \ddot{z} = -\frac{z}{r^3} \qquad (8)$$

In these equations, units are chosen in such a way that the constant k appearing normally in (8) is equal to unity.

Employing equations (8) in (7) it follows that

$$\frac{1}{2}\ddot{U}x + \dot{U}\dot{x} = -\frac{1}{2}\ddot{P} - \frac{P}{2r^3}$$
$$\frac{1}{2}\ddot{V}x + \dot{V}\dot{x} = -\frac{1}{2}\ddot{Q} - \frac{Q}{2r^3} \qquad (9)$$

Solution of system (9) for x and $\dot{x}$ yields

$$x = \frac{\begin{vmatrix} -\frac{1}{2}\ddot{P} - \frac{P}{2r^3} & \dot{U} \\ -\frac{1}{2}\ddot{Q} - \frac{Q}{2r^3} & \dot{V} \end{vmatrix}}{\begin{vmatrix} \frac{1}{2}\ddot{U} & \dot{U} \\ \frac{1}{2}\ddot{V} & \dot{V} \end{vmatrix}} = \frac{-\left(\frac{1}{2}\ddot{P}\dot{V} - \frac{1}{2}\ddot{Q}\dot{U}\right) - \left(\frac{P\dot{V} - Q\dot{U}}{2r^3}\right)}{\frac{1}{2}\ddot{U}\dot{V} - \frac{1}{2}\ddot{V}\dot{U}} \tag{10}$$

Define the following quantities

$$\begin{aligned} D &\equiv \frac{1}{2}\ddot{U}\dot{V} - \frac{1}{2}\ddot{V}\dot{U} \\ A &\equiv -\left(\frac{1}{2}\ddot{P}\dot{V} - \frac{1}{2}\ddot{Q}\dot{U}\right) \\ B &\equiv -\left(P\dot{V} - Q\dot{U}\right) \end{aligned} \tag{11}$$

By virtue of equation (11), x can be rewritten as

$$x = \frac{A}{D} + \frac{B}{2D} \cdot \frac{1}{r^3}$$

Similarly

$$D\dot{x} = \begin{vmatrix} \frac{1}{2}\ddot{U} & -\frac{1}{2}\ddot{P} - \frac{P}{2r^3} \\ \frac{1}{2}\ddot{V} & -\frac{1}{2}\ddot{Q} - \frac{Q}{2r^3} \end{vmatrix} \tag{12}$$

$$D\dot{x} = -\left(\frac{1}{2}\ddot{U} . \frac{1}{2}\ddot{Q} - \frac{1}{2}\ddot{P} \cdot \frac{1}{2}\ddot{V}\right) - \frac{\left(\frac{1}{2}Q\ddot{U} - \frac{1}{2}P\ddot{V}\right)}{2r^3}$$

Defining quantities

$$G \equiv -\left(\frac{1}{4} \ddot{U} \ddot{Q} - \frac{1}{4} \ddot{P} \ddot{V} \right)$$

$$H \equiv -\left(\frac{1}{2} Q \ddot{U} - \frac{1}{2} P \ddot{V} \right)$$

$\dot{x}$ can be written as

$$\dot{x} = \frac{G}{D} + \frac{H}{2D} \cdot \frac{1}{r^3}$$

Finally r can be expressed in terms of x using equations (4).

Thus

$$r^2 = x^2 + y^2 + z^2 = x^2 + (P + Ux)^2 + (Q + Vx)^2$$

$$r^2 = \left(1 + U^2 + V^2 \right) x^2 + 2\,(PU + QV)\, x + \left(P^2 + Q^2 \right) \tag{13}$$

Defining

$$C = 1 + U^2 + V^2$$

$$E = 2\,(PU + QV)$$

$$F = P^2 + Q^2$$

r can be expressed as

$$r^2 = Cx^2 + Ex + F$$

Equations (10), (12), and (13) represent a system of simultaneous equations in x, $\dot{x}$, and r. Now, x and r are solved by (10) and (12), employing any convenient numerical procedure. Once r is available, $\dot{x}$ follows from (12). Other components of the position and velocity follow from equations (6) and (7).

The development of Stumpff's method will be concluded by giving, without proof, two infinite series frequently occuring in orbit determinations of the type considered in this report.

These are the f and g series of the Laplacian method of orbit computation. The simple idea behind these series is as follows. Suppose that position $\vec{r}_o$ and velocity $\dot{\vec{r}}_o$ of the vehicle are known at some time t_o. The motion of the vehicle in the neighborhood of this point can then be expressed as

$$\vec{r} = \vec{r}_o + \tau \dot{\vec{r}}_o + \tau^2/2! \ddot{\vec{r}}_o + \ldots.. \qquad (14)$$

and

$$\ddot{\vec{r}} = - \frac{\vec{r}}{r^3}$$

Successive differentiation of the last equation permits elimination of higher order derivatives in (14). It can be shown that as a result of this procedure equation (14) assumes the following form

$$\vec{r} = f \vec{r}_o + g \dot{\vec{r}}_o \qquad (15)$$

where

$$f = 1 - \frac{1}{2}\mu\tau^2 + \frac{1}{2}\mu\sigma\tau^3 + \frac{\mu\left(3\omega - 2\mu - 15\sigma^2\right)}{24}\tau^4$$

$$- \frac{\mu\sigma\left(3\omega - 2\mu - 7\sigma^2\right)}{8}\tau^5 + \frac{\mu}{720}\left[\left(630\omega - 420\mu - 945\sigma^2\right)\sigma^2\right.$$

$$\left. - \left(22\mu^2 - 66\mu\omega + 45\omega^2\right)\right]\tau^6 + \dots \qquad (16)$$

$$g = \tau - \frac{1}{6}\mu\tau^3 + \frac{\mu\sigma}{4}\tau^4 + \frac{\mu\left(9\omega - 8\mu - 45\sigma^2\right)}{120}\tau^5$$

$$- \frac{\mu\sigma\left(6\omega - 5\mu - 14\sigma^2\right)}{24}\tau^6 + \dots \qquad (17)$$

$$\mu = \frac{\bar{r}_o \cdot \bar{r}_o}{r_o^5} = \frac{1}{r_o^3} \qquad \sigma = \frac{\bar{r}_o \cdot \dot{\bar{r}}_o}{r_o^2} \qquad \omega = \frac{\dot{\bar{r}}_o \cdot \dot{\bar{r}}_o}{r_o^2}$$

Terms higher than τ^6 become too complicated for practical purposes. Note that $\tau = t - t_o$ is expressed in the appropriate units of time.

Use of f and g series is subject to the usual limitations of convergence. If any doubt exists concerning the latter, it is better to use the closed form of the series in question. The appropriate expressions can be found in Reference (1) pp. 48 or 75.

It is unlikely that in problems considered in this report the need for closed forms will ever appear.

The procedure written for the computer is based on the above development and it is summarized in a step by step form below:

Measured Data and Site Coordinates

Time and Date	Right Ascension	Declination	Rectangular Components of Site		
			X	Y	Z
t_1	α_1	δ_1	X_1	Y_1	Z_1
t_o	α_o	δ_o	X_o	Y_o	Z_o
t_3	α_3	δ_3	X_3	Y_3	Z_3

In the above, units of distance and time should be taken in accordance with the rules outlined in Section IV. The procedures of this chapter also determine the site coordinates.

2. Computer Program

The computation then proceeds as follows:

I. Compute

$$\mathrm{TAN}\ \alpha_i \equiv U_i$$

$$\mathrm{SEC}\ \alpha_i \qquad\qquad P_i = Y_i - U_i X_i$$

$$\mathrm{TAN}\ \delta_i \qquad\qquad Q_i = Z_i - V_i X_i$$

$$\mathrm{SEC}\ \alpha_i\ \mathrm{TAN}\ \delta_i \equiv V_i$$

$$i = 1,\ 0,\ 3$$

II. Compute

$$\tau_1 = k\left(t_1 - t_o\right)$$

$$\tau_3 = k\left(t_3 - t_o\right)$$

$$\tau_3 - \tau_1$$

The value of k is the reciprocal of the time unit employed for the particular problem.

In any modification of the Laplacian method of preliminary orbit determination, it is desirable that intervals $t_1 - t_o$ and $t_3 - t_o$ be nearly equal. It can be shown that, under these conditions, errors in numerical derivatives and accelerations of U, P, V, Q are of the second order.

III. Compute the following quantities

$$(U, 1) = \frac{U_1 - U_o}{\tau_1} \qquad (U, 3) = \frac{U_3 - U_o}{\tau_3}$$

$$(V, 1) = \frac{V_1 - V_o}{\tau_1} \qquad (V, 3) = \frac{V_3 - V_o}{\tau_3}$$

$$(P, 1) = \frac{P_1 - P_o}{\tau_1} \qquad (P, 3) = \frac{P_3 - P_o}{\tau_3}$$

$$(Q, 1) = \frac{Q_1 - Q_o}{\tau_1} \qquad (Q, 3) = \frac{Q_3 - Q_o}{\tau_3}$$

IV. Compute

$$\dot{U}_o = \frac{\tau_3 (U,1) - \tau_1 (U,3)}{\tau_3 - \tau_1}, \quad \dot{P}_o = \frac{\tau_3 (P,1) - \tau_1 (P,3)}{\tau_3 - \tau_1}$$

$$\dot{V}_o = \frac{\tau_3 (V,1) - \tau_1 (V,3)}{\tau_3 - \tau_1}, \quad \dot{Q}_o = \frac{\tau_3 (Q,1) - \tau_1 (Q,3)}{\tau_3 - \tau_1}$$

V. Compute quantities

$$\frac{1}{2} \ddot{U}_o = \frac{(U,3) - (U,1)}{\tau_3 - \tau_1}, \quad \frac{1}{2} \ddot{P}_o = \frac{(P,3) - (P,1)}{\tau_3 - \tau_1}$$

$$\frac{1}{2} \ddot{V}_o = \frac{(V,3) - (V,1)}{\tau_3 - \tau_1}, \quad \frac{1}{2} \ddot{Q}_o = \frac{(Q,3) - (Q,1)}{\tau_3 - \tau_1}$$

VI. Compute the quantities

(a) $D = \frac{1}{2} \ddot{U}_o \dot{V}_o - \frac{1}{2} \ddot{V}_o \dot{U}_o$

(b) $A = -\left(\frac{1}{2} \ddot{P}_o \dot{V}_o - \frac{1}{2} \ddot{Q}_o \dot{U}_o \right)$

(c) $B = -\left(P_o \dot{V}_o - Q_o \dot{U}_o \right)$

(d) $C = 1 + U_o^2 + V_o^2$

(e) $E = 2 \left(P_o U_o + Q_o V_o \right)$

(f) $F = P_o^2 + Q_o^2$

(g) $G = -\left(\frac{1}{2} \ddot{Q}_o \frac{1}{2} \ddot{U}_o - \frac{1}{2} \ddot{P}_o \frac{1}{2} \ddot{V}_o \right)$

(h) $H = -\left(\frac{1}{2} Q_o \ddot{U}_o - \frac{1}{2} P_o \ddot{V}_o \right)$

VII. Form the following equations

(a) $$x = \frac{A}{D} + \frac{B}{2D} \cdot \frac{1}{r^3}$$

(b) $$r^2 = C x^2 + E x + F$$

(c) $$\dot{x} = \frac{G}{D} + \frac{H}{2D} \cdot \frac{1}{r^3}$$

VIII. Solve (a) and (b) of step VII simultaneously for x and r. Designate desired solutions by x_o and r_o.

For the case of vehicles moving in the Earth's vicinity $r_o > 1$ in units of the Earth equatorial radius. Generally, equations (a) and (b) may result in three values of r. One is the desired one, the second represents the position of the center of force, and the third is entirely spurious.

IX. Using the value of r_o obtained in step VIII compute $\dot{x}_o$ from equation VII. (c)

$$\dot{x}_o = \frac{G}{D} + \frac{H}{2D} \cdot \frac{1}{r_o^3}$$

X. Compute

$$y_o = P_o + U_o x_o$$

$$\dot{y}_o = \dot{U}_o x_o + U_o \dot{x}_o + \dot{P}_o$$

$$z_o = Q_o + V_o x_o$$

$$\dot{z}_o = \dot{V}_o x_o + V_o \dot{x}_o + \dot{Q}_o$$

If the intervals of time between measurements are equal, the computed values of

$$x_o, y_o, z_o$$

$$\dot{x}_o, \dot{y}_o, \dot{z}_o$$

refer to the time of middle measurement.

These values, expressed in suitable units, are then employed as the initial conditions in the n-body Trajectory Program. If redundant data are available, preliminary values are used to initiate a differential correction program.

The following computation has not been programmed. It is included so that, if desired, it can be employed as a check on the quality of computation.

For times t_1 and t_3 compute values of f and g. In employing these values

$$x_1 = f_1 x_o + g_1 \dot{x}_o$$

$$y_1 = f_1 y_o + g_1 \dot{y}_o$$

$$z_1 = f_1 z_o + g_1 \dot{z}_o$$

and

$$x_3 = f_3 x_o + g_3 \dot{x}_o$$

$$y_3 = f_3 y_o + g_3 \dot{y}_o$$

$$z_3 = f_3 z_o + g_3 \dot{z}_o$$

Since positions of the observer at times t_1 and t_3 are known, it is possible to compute

$$x_1 - X_1$$

$$y_1 - Y_1$$

$$z_1 - Z_1$$

$$x_3 - X_3$$

$$y_3 - Y_3$$

$$z_3 - Z_3$$

Using these compute α_1, δ_1 and α_3, δ_3 from

$$\text{TAN } \alpha_1 = \frac{y_1 - Y_1}{x_1 - X_1} \text{ ; SEC } \alpha_1 \text{ TAN } \delta_1 = \frac{z_1 - Z_1}{x_1 - X_1}$$

and similar expressions at $t = t_3$.

The computed values of α_1, δ_1, α_3, δ_3 should be in reasonable agreement with the observed ones. Failure to achieve agreement may be due to:

(a) Error in computation

(b) Too long or too short a time interval

(c) Original observations too inaccurate

etc.

This concludes the discussion of the first tracking method.

B. DETERMINATION OF INITIAL CONDITIONS FROM TWO POSITION VECTORS

This section describes a procedure for determining velocity components of vehicle movement in the Earth's gravitational field from measurements of the distance and two associated angles at two different times.

In analysis, the technique offers a fast and accurate computer solution to establishing the complete initial conditions at some time t.

Laplace's method shows that position $\vec{r}$, at any time t, of an object traveling in a Keplerian orbit, can be expressed as a function of the position $\vec{r}_o$ and velocity $\dot{\vec{r}}_o$ at some time t_o, according to

$$\vec{r} = f\,\vec{r}_o + g\,\dot{\vec{r}}_o \tag{A}$$

where f and g are scalar functions of the position and velocity at time t_o and the time interval $t-t_o$. Since $\vec{r}$ and $\vec{r}_o$ are specified if two range and two angular coordinates are observed at time t and t_o, the velocity $\dot{\vec{r}}_o$ may be determined from (A), if f and g are known. Fortunately, for time intervals between observations of practical interest, functions f and g are represented by rapidly converging infinite series whose first dominant terms are functions of position $\vec{r}_o$ and the time interval $t-t_o$, but not of the velocity $\dot{\vec{r}}_o$. Thus, good first approximations of f and g are known, though the initial velocity is unknown.

The method described in this section is based on the above facts. The initial velocity $\dot{\vec{r}}_o$ is evaluated from Equation (A) after f and g have been approximated. This first approximation in velocity is used to recompute f and g, giving a better estimate of velocity. The procedure is repeated until desired precision is obtained. It is apparent the computation is iterative in which good initial estimates of the initial velocity components are not required. In fact, because of the very nature of the f and g series, the first approximations are quite satisfactory.

1. Determination of Equatorial Position Coordinates From Range, Azimuth and Elevation

Figure 2 shows the position of the observed object in the coordinate system fixed at the radar site. Let $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$ be the unit vectors along the respective axes, A-azimuth and E-elevation. Then the vector $\vec{\rho}$ from the observing station to the object is given by

$$\vec{\rho} = \rho \left[\cos E \cos A \, \hat{\zeta} + \cos E \sin A \, \hat{\eta} + \sin E \, \hat{\xi} \right] \tag{18}$$

Figure 3 shows that the position vector of the observing station is given by

$$\vec{R} = R \left[\cos \phi' \cos \Theta \, \hat{i}_x + \cos \phi' \sin \Theta \, \hat{i}_y + \sin \phi' \, \hat{i}_z \right] \tag{19}$$

where $\hat{i}_x$, $\hat{i}_y$, $\hat{i}_z$ are the unit vectors in the inertial geocentric cartesian coordinate system and ϕ' and Θ are the geocentric latitude and local sideral time (expressed in angular measure) of the site, respectively. The latter quantitites are computed according to the rules given in the chapter on conversion routines.

Rotation of the x, y, z coordinate system through an angle Θ about the z axis, and through an angle ϕ' about the y' axis, results in the following expression.

$$\begin{pmatrix} \hat{\xi} \\ \hat{\eta} \\ \hat{\zeta} \end{pmatrix} = \begin{pmatrix} \cos\phi' & 0 & \sin\phi' \\ 0 & 1 & 0 \\ -\sin\phi' & 0 & \cos\phi' \end{pmatrix} \begin{pmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{i}_x \\ \hat{i}_y \\ \hat{i}_z \end{pmatrix}$$

Let

$$\cos\phi' \cos\Theta = L$$

$$\cos\phi' \sin\Theta = M$$

$$\sin\phi' = N$$

then the unit vectors are related by:

$$\hat{\xi} = L\hat{i}_x + M\hat{i}_y + N\hat{i}_z$$

$$\hat{\eta} = -\frac{M}{\sqrt{L^2 + M^2}}\hat{i}_x + \frac{L}{\sqrt{L^2 + M^2}}\hat{i}_y \qquad (20)$$

$$\hat{\zeta} = -\frac{NL}{\sqrt{L^2+M^2}}\,\hat{i}_x - \frac{NM}{\sqrt{L^2+M^2}}\,\hat{i}_y + \sqrt{L^2+M^2}\,\hat{i}_z \qquad (20)$$

Using (20) in (18):

$$\vec{\rho} = \rho\left[\left(L\sin E - \frac{M}{\sqrt{L^2+M^2}}\cos E\sin A - \frac{NL}{\sqrt{L^2+M^2}}\cos E\cos A\right)\hat{i}_x\right.$$

$$+\left(M\sin E + \frac{L}{\sqrt{L^2+M^2}}\cos E\sin A - \frac{NM}{\sqrt{L^2+M^2}}\cos E\cos A\right)\hat{i}_y$$

$$\left.+\left(N\sin E + \sqrt{L^2+M^2}\cos E\cos A\right)\hat{i}_z\right]$$

From Figure 1b.

$$\vec{r} = \vec{\rho} + \vec{R} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_3$$

Thus, previous expressions yield

$$x = RL + \rho\left(L\sin E - \frac{M}{\sqrt{L^2+M^2}}\cos E\sin A - \frac{NL}{\sqrt{L^2+M^2}}\cos E\cos A\right)$$

$$y = RM + \rho\left(M\sin E + \frac{L}{\sqrt{L^2+M^2}}\cos E\sin A - \frac{NM}{\sqrt{L^2+M^2}}\cos E\cos A\right)$$

$$z = RN + \rho\left(N\sin E + \sqrt{L^2+M^2}\cos E\cos A\right)$$

To obtain the geocentric rectangular coordinates of the object referred to the equinox of date, range, azimuth, elevation, geocentric site latitude and sidereal time are needed.

If the computation is to proceed with respect to some other equinox, x, y, and z must be transformed according to the rules given elsewhere in this report.

This need, however, will be ignored for the present.

2. Determination of the Initial Conditions from Range, Azimuth and Elevation Measurements

Since motions in the Earth's immediate vicinity, over relatively short spans of time, are being considered, it is permissible to consider the object's path as a Keplerian Orbit. The latter is completely specified by six independent quantities, e. g., six orbital elements, six angular sightings of the position, three position and three velocity components at any time, etc.

In this problem, the tracking instrument supplies the range and angular data. Thus, two such observations at different times are sufficient to specify the orbit. However, since the n-body Trajectory Program requires position and velocity components as inputs, the above measurements must be made to yield velocity components at some time t as well.

From the previous section, two observations yield x, y, z at two different times.

Following Laplace, the position vector $\vec{r}$ at time t is expressed as:

$$\vec{r}\,(t) = f\,\vec{r}_o + g\,\dot{\vec{r}}_o \tag{21}$$

Development of this result was discussed in Scientific Report No. 1, and is found in references (1) and (2).

In expression (21): $\bar{r}_o = \bar{r}\left(t_o\right)$, $\dot{\bar{r}}_o = \dot{\bar{r}}\left(t_o\right)$

$$f = 1 - \frac{1}{2}\mu\tau^2 + \frac{1}{2}\mu\sigma\tau^3 + \frac{\mu\left(3\omega - 2\mu - 15\sigma^2\right)}{24}\tau^4 - \frac{\mu\sigma\left(3\omega - 2\mu - 7\sigma^2\right)}{8}\tau^5$$

$$+ \frac{\mu}{720}\left[\left(630\omega - 420\mu - 945\sigma^2\right)\sigma^2 - \left(22\mu^2 - 66\mu\omega + 45\omega^2\right)\right]\tau^6 + \ldots \tag{22}$$

$$g = \tau - \frac{1}{6}\mu\tau^3 + \frac{\mu\sigma}{4}\tau^4 + \frac{\mu\left(9\omega - 8\mu - 45\sigma^2\right)}{120}\tau^5 - \frac{\mu\sigma\left(6\omega - 5\mu - 14\sigma^2\right)}{24}\tau^6 + \ldots \tag{23}$$

where

$$\mu = \frac{1}{r_o^3}\ ; \quad \sigma = \frac{\vec{r}_o \cdot \dot{\vec{r}}_o}{r_o^2}\ ; \quad \omega = \frac{\dot{\vec{r}}_o \cdot \dot{\vec{r}}_o}{r_o^2}$$

and

$$\tau = k\left(t - t_o\right)$$

Depending on the object, it is convenient to express distances in astronomical units and time in 58.13244 days, or distance in Earth radii and time in units of 806.9275 seconds.

The procedure used was outlined in Section II and repetition is unnecessary.

3. Computer Program

A step-by-step outline of the computer solution is given below. Note that, in the final computer program, azimuth and elevation are not directly used but are first transformed to right ascension and declination referred to a suitable equinox.

I. Input data:

$$\rho_o \quad X_o, Y_o, Z_o \quad A_o E_o \quad t_o$$

$$\rho_1 \quad X_1, Y_1, Z_1 \quad A_1 E_1 \quad t_1$$

$\alpha_o, \delta_o, \alpha_1, \delta_1$, are obtained from $A_o E_o, A_1, E_1$

II. Compute

$$x_o = X_o + \rho_o \cos \delta_o \cos \alpha_o \qquad x_1 = X_1 + \rho_1 \cos \delta_1, \cos \alpha_1$$

$$y_o = Y_o + \rho_o \cos \delta_o \sin \alpha_o \qquad y_1 = Y_1 + \rho_1 \cos \delta_1, \sin \alpha_1$$

$$z_o = Z_o + \rho_o \sin \delta_o \qquad z_1 = Z_1 + \rho_1 \sin \delta_1$$

III. Compute

$$r_o = \left(x_o^2 + y_o^2 + z_o^2\right)^{1/2}$$

$$\mu = \frac{1}{r_o^3}$$

IV. Compute

$$f = 1 - \frac{1}{2} \mu \tau^2$$

$$g = \tau - \frac{1}{6} \mu \tau^3$$

V. Compute

$$\dot{x}_o = \frac{1}{g} (x - fx_o)$$

$$\dot{y}_o = \frac{1}{g} (y - fy_o)$$

$$\dot{z}_o = \frac{1}{g} (z - fz_o)$$

VI. Compute

$$\dot{\vec{r}}_o \cdot \dot{\vec{r}}_o = \dot{x}_o^2 + \dot{y}_o^2 + \dot{z}_o^2$$

$$\vec{r}_o \cdot \dot{\vec{r}}_o = x_o \dot{x}_o + y_o \dot{y}_o + z_o \dot{z}_o$$

$$\sigma = \frac{\dot{\vec{r}}_o \cdot \vec{r}_o}{r_o^2} \; ; \quad \omega = \frac{\dot{\vec{r}}_o \cdot \dot{\vec{r}}_o}{r_o^2}$$

VII. Compute f and g using complete equations (22) and (23).

VIII. Return to Step V. and compute new $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$. Repeat the entire procedure.

IX. Compare velocity components computed at each step with the corresponding velocity components obtained in the previous step.

When differences reach a value of less than $\epsilon = 4 \times 10^{-8}$ in appropriate units, the iteration procedure is terminated.

4. Evaluation of the Method

The evaluation of the computational method is summarized in Figure 4 and 5.

A time interval between observations of about one minute, it appears, results in the minimum error. When smaller intervals are used, round off errors become significant. For larger time intervals the f and g series become inaccurate. Figure 4 indicates that for a time interval between observations of five minutes the percentage velocity error for the lunar trajectory is .05 per cent corresponding to 12 mph error. Increasing the time interval to ten minutes, yields a velocity error of .5 per cent, or 120 mph. These results indicate the initial velocity may be precisely established, using a reasonable number of terms in the f and g series. Because of its large eccentricity, the lunar trajectory is a relatively worse case.

Figure 5 shows that, for a five minute interval, seven iterations are required to reach the solution for the lunar orbit. If the interval is increased to ten minutes, the velocity is established after eleven iterations.

Precision achievable by this technique is more than sufficient to establish preliminary initial conditions.

C. DETERMINATION OF THE INITIAL CONDITIONS FROM SIX RADAR RANGE MEASUREMENTS

This study's third tracking scheme attempted to utilize only range measurements. Underlying ideas, and the derivation of appropriate expressions, can be found in Section II. For present purposes it is sufficient to recall that the method involves a solution of six equations, linear in unknowns x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$ in which, at least initially, the coefficients are only approximations to the true ones. Furthermore, if the time intervals between observations are equal, the coefficients' numerical values are not far from unity. If the intervals are made deliberately unequal, some coefficients will be considerably larger or smaller than unity, but in a manner that they are basically multiples of each other.

The latter observations have the following unfortunate consequence.

There are many methods available for solving simultaneous linear equations. However, all methods, with one exception, involve many successive subtractions of quantities of nearly the same order of magnitude. This leads to the loss of significant figures which often makes the results meaningless.

Though the equations of the problem fell in the above category, an attempt was made to solve them by (a) inverting the matrix and (b) by successive elimination of the unknowns where division by the leading coefficient was employed. This was done for equally and unequally spaced time intervals.

In either case, a point in the solution was reached where the original seven significant figures were reduced to one or two. Use of double precision in the computer arithmetic was to no avail. Generally, after the first try the values of x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$ were larger by one or two orders of magnitude than those desired. No iteration was carried out because the structure of the equations indicated futility of further computation.

The one exceptional method hinted at above was the solution of simultaneous equations by the iteration method described in Reference (1).

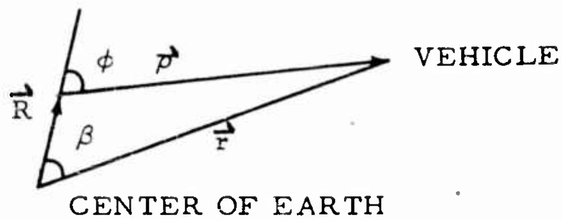
This method is free from the loss of significant figures due to the subtractions of nearly equal quantities. Unfortunately the iterative process converges only when each equation contains, compared to others, a large coefficient, and this coefficient must be associated with different unknowns in each equation. This implies that the dominant coefficients are arranged along the principal diagonal of the coefficient matrix. Generally, such an arrangement cannot be expected in the physical problem considered. A theorem given in Reference (3) states that, for the method to be applicable in each equation of the system, the absolute value of the largest coefficient must be greater than the sum of the absolute values of all the remaining coefficients in that equation.

Simulated problems constructed for testing the over-all method do not satisfy the above condition. Finally, it was found that the result of the first iteration is quite sensitive to the initial estimate of r_o. It appears that the problem is poorly conditioned. This difficulty cannot be attributed to unfavorable geometrical arrangement of the orbit and the observer, because in all other methods the selected example gave excellent agreement with the known values of x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$. The basic problem is apparently contained in the fact that the coefficients of the starting system of equations are not really known, and the approximations used for these are insufficient.

The conclusion drawn is that the determination of the vehicle's position and velocity components, from range measurements at a single station, is not practical as formulated in the previous section.

Before abandoning this investigation an attempt was made to reformulate the problem. In the previous scheme, the solution starts with a guess of r_o. The difficulty of finding a reasonably close initial value of geocentric distance is obvious. However, instead of starting with r_o, the angle between some reference (vertical), and the direction of the antenna to the body at some time t, could be estimated.

The geometry is then:



Using ϕ which is now supposed estimated, r is computed from

$$r^2 = R^2 + \rho^2 - 2\rho R \cos(\pi - \phi)$$

$$r^2 = R^2 + \rho^2 + 2\rho R \cos\phi$$

It is felt that ϕ can be estimated better than r. If this is allowed, it also follows that the right ascension and declination α, δ can be estimated. This in turn leads to values of topocentric coordinates ξ, η, ζ and then to geocentric position components x, y, z. The above follows from the well known relations

$$\xi = \rho \cos\delta \cos\alpha = x - X \tag{24}$$

$$\eta = \rho \cos\delta \sin\alpha = y - Y \tag{25}$$

$$\zeta = \rho \sin\delta = z - Z \tag{26}$$

Furthermore:

$$r = \sqrt{x^2 + y^2 + z^2} \tag{27}$$

Finally since ϕ is supposed known and thereby r, angle β can be computed from

$$r\,R\cos\beta = x\,X + y\,Y + z\,Z \tag{28}$$

Note in the above equations that the unknown quantities are α, δ, x, y, and z.

These equations can now be used to obtain initial estimates of these parameters.

Thus, from (27):

$$x = \left(r^2 - y^2 - z^2\right)^{1/2}$$

and using in (1), (2), (3) and (5):

$$\rho\cos\delta\cos\alpha = \sqrt{r^2 - y^2 - z^2} - X \tag{24'}$$

$$\rho\cos\delta\sin\alpha = y - Y \tag{25'}$$

$$\rho\sin\delta = z - Z \tag{26'}$$

$$r\,R\cos\beta = \sqrt{r^2 - y^2 \quad z^2}\,X + y\,Y + z\,Z \tag{28'}$$

Employing (25') and (26') in (24') and (28') one obtains

$$\rho\cos\alpha\cos\delta = \sqrt{r^2\;1\,(\rho\cos\delta\sin\alpha + Y)^2 - (\rho\sin\delta + Z)^2} \tag{24''}$$

$$\begin{aligned} r\,R\cos\beta = {} & X\sqrt{r^2 - (\rho\cos\delta\sin\alpha + Y)^2 - (\rho\sin\delta + Z)^2} \\ & + (\rho\cos\delta\sin\alpha + Y)\,Y + (\rho\sin\delta + Z)\,Z \end{aligned} \tag{28''}$$

Consider equation (24''). Following a lengthy algebraic reduction, this equation can be written as:

$$A_1 \cos^2 \delta + B_1 \cos \delta + C_1 = O$$

where

$$A_1 = \left[X \cos \alpha + Y \sin \alpha\right]^2 + Z^2$$

$$B_1 = 2 \left[X \cos \alpha + Y \sin \alpha\right] \left[\frac{R^2 + \rho^2 - r^2}{2 \rho}\right]$$

$$C_1 = \left[\frac{R^2 + \rho^2 - r^2}{2 \rho}\right] - Z^2$$

Similarly, equation (28'') can be reduced to

$$A \sin^2 \alpha + B \sin \alpha + C = O$$

where

$$A = \rho^2 \left(X^2 + Y^2\right) \cos^2 \delta$$

$$B = 2 \rho Y \cos \delta \left[\left(X^2 + Y^2\right) - r R \cos \beta + Z \left(\rho \sin \delta + Z\right)\right]$$

$$C = \rho \sin \delta + z \left[\left(\rho \sin \delta + Z\right)\left(X^2 + Y^2\right) - 2r RZ \cos \beta + 2Y^2 Z\right]$$

$$+ r^2 R^2 \cos^2 \beta + Y^2 \left(X^2 + Y^2\right) - 2 r RY^2 \cos \beta - X^2 r^2$$

Note that A_1, B_1, C_1 do not depend on δ, and A, B, C do not depend on α. Equations (24'') and (28'') are decoupled as far as α and δ are concerned. The two angular coordinates can be computed by successive approximations as follows. Assume, for instance, a value of α in (24'') and compute δ. Using this

δ in equation (28'') compute α. In principle, the assumed and computed values of α must be identical. Since the first try will not result in an identity, the value of assumed in (24'') must be modified until α assumed - α computed is reduced to a small quantity consistent with the desired precision.

The fact that equations (24'') and (28'') yield a multiplicity of roots will be ignored, and the proper values of α and δ will be assumed to have been found. This being the case, equations (24), (25), and (26) yield the corresponding values of ξ, η, ζ or x, y, z. Equations (24), (25), (26), and (28) can also serve as a partial control because the trigonometric functions involved must numerically be less than unity. If this is not the case, the value of r could be modified and the whole computation repeated. Assuming that reasonable values of x_o, y_o, z_o have been obtained, the procedure would be as follows. Using x_o, y_o, z_o obtained above in equation (17), Section II, namely,

$$a_{n1} r_o^2 + a_{n2} x_o + a_{n3} y_o + a_{n4} z_o + a_{n5} \dot{x}_o + a_{n6} \dot{y}_o + a_{n7} \dot{z}_o = C_n \qquad n = 1, 2, 3 \ldots 6 \tag{29}$$

a set of simultaneous equations in $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$ as unknowns is obtained. Thus, using two additional measurements of ρ, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$ can be computed. This procedure may result in better values of the velocity components since subtractive processes have been considerably reduced. Should this step result in reasonable values of $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$, equation (29) could be used in its entirety, and x_o, y_o, z_o could be considered as unknowns wherever they occur explicitly. The f and g series would be computed using x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$ obtained in the previous step.

The iteration, as described in Section II would be attempted only if the above scheme showed a reasonable success.

No computations based on this method have been attempted.

IV. AUXILIARY COMPUTATIONS AND CONVERSION SUB-ROUTINES

In the description of the n-body Trajectory Program, Interim Report #1, July 1960, it has been pointed out that the coordinate system used is referenced to the mean equinox of 1950.0. Thus, all basic information, i.e., planetary positions stored on magnetic tapes, is expressed in this frame of reference. Consequently, the components of the position and velocity vectors, which serve as the input to the trajectory program, must be consistent with the accepted coordinate system.

The initial conditions for the n-body Trajectory Program are usually derived from some tracking subroutine. Previously, it had been tacitly assumed that the observed quantities are given in the proper frame of reference, making the output usable in the trajectory program. In practice this assumption is not warranted. Observations will usually yield quantities referenced to the coordinate system, differing from 1950.0 by at least the precession effect. If these observations (measured with respect to the equinox of date) are used in a particular tracking subroutine, the resulting output will not be usable by the trajectory program. Thus, the output of the tracking computation, and the input to the n-body subroutine, must be matched by an appropriate transformation.

The purpose of this section is to describe such transformations and auxiliary computations provided with the n-body Trajectory Program.

Before discussing various detailed schemes, a number of general comments are necessary.

A. BASIC REQUIREMENTS

(a) Distances

Two systems of units are provided for all distance measurements. These are astronomical units (A. U.) and the equatorial Earth radii. The reason for this choice is that, in general, in all computations of the type considered in this report, it is convenient to keep magnitudes of distances near unity. Thus, for vehicles moving in the close vicinity of the Earth, the above condition obtains if the distances are expressed in terms of the Earth radius as a unit. Conversely the mean Earth-Sun distance is suitable in computations of the motion of probes to Venus, Mars, and other points in the solar system.

Both units have been provided to avoid limiting the program to lunar vehicles.

Thus, prior to computations, range measurements should first be converted to either astronomical units or Earth radii.

(b) Angles

All angular inputs employed by the conversion routines must be expressed in degrees and decimals of a degree. This particularly applies to right ascension α which is often expressed in time measure. Note the conversion factor is 15°/hour, $15'$/minute, and $15''$/second.

(c) Dates and Times

The dates and times of observation employed by the transformation subroutines occur in two forms.

(1) The date and time of observation shall be expressed in terms of universal time, in days and decimals of a day of

the same month, for all observations, even if this entails introducing a fictitious number of days in a month.

Example: Suppose that two measurements were made:

1935 Aug. 30.0006 UT and
1935 Sept. 2.9067 UT

They must then be expressed as

1935 Aug. 30.0006 UT and
1935 Aug. 33.9067 UT

(2) The date and time of observation to be also expressed in Julian Days (JD) and decimals of a Julian Day. Note that there are 365.25 days in a Julian year. An extensive table of Julian Day numbers is provided in Appendix A. Also, Julian Day numbers are tabulated in yearly issues of the Nautical Almanac and American Ephemeris.

Example: Days and times considered under (c) (1) expressed in JD are given by

JD 2428044.5006
JD 2428048.4067

(d) <u>Geographic Coordinates of the Observation Site</u>

In computations of the rectangular site coordinates, geographic coordinates of the tracking station are required. These shall be expressed as follows:

Geographic longitude of the site λ in degrees. The sign convention adopted is:

$\lambda > 0$ if East of Greenwich Meridian

$\lambda < 0$ if West of Greenwich Meridian

Geographic latitude of the site ϕ is taken as positive if North of the equator, and negative if South of the equator.

The altitude of the site above sea level $|h|$ is expressed in astronomical units or Earth equatorial radii.

Specific subroutines available in the program are considered below:

B. COMPUTATION OF RECTANGULAR GEOCENTRIC SITE COORDINATES

The geocentric components of the observer's position on the Earth's surface are required by all tracking subroutines. These components can be computed with respect to equinox of date, and then transformed into the coordinate system of 1950.0, or computed directly with respect to equinox of 1950.0. In both cases, the procedure programmed is as follows:

1. Reduction to Geocentric Latitude

For reasons which will not be discussed here (see Reference 4), it is necessary to reduce the observer's geodetic latitude to a geocentric one prior to computing the components of the observer's position vector. The appropriate expression utilized in the program is

$$\phi' = \phi + \frac{1}{3600}\left[-695''.6635 \sin 2\phi + 1''.1731 \sin 4\phi - 0''.0026 \sin 6\phi\right]$$

where ϕ' is the geocentric latitude.

2. Magnitude of the Geocentric Radius Vector of the Site

The geocentric radius vector R is determined from

$$R = h + a(.998320047 + .001683494 \cos 2\phi - .000003549 \cos 4\phi + .000000008 \cos 6\phi)$$

In the above, a = equatorial radius of the Earth and h is the altitude of the site above sea level. Note that R may be expressed either in astronomical units or equatorial Earth radii.

3. Computation of Sidereal Time

Since the Earth rotates about its axis, the components of R will be functions of time. The time involved is the "star time," generally called the sidereal time - a measure of the angle between the observer's meridian and the vernal equinox.

In the computer program, sidereal time is computed as follows:

(a) First Greenwich Mean Sidereal Time (GMST) at 0^h Universal Time as measured by the mean equinox of date is given by

$$GMST = \frac{1}{3600}\left[23925^s.836 + 8,640,184^s.542 \quad \frac{JD - 2415020.0}{36525} + 0^s.0929 \left(\frac{JD - 2415020.0}{36525}\right)^2\right]$$

Only that part of the resulting value is taken which is less than 24 hours. In the above, JD is the Julian date at midnight of the beginning of the day at which the GMST is desired, and 2415020.0 corresponds to the noon of January 0, 1900. Note that GMST is given in hours.

(b) The Local Sidereal Time is computed from

$$LST = \left\{GMST + 24(UT) + \frac{9^s.8565}{3600}\left[24(UT)\right] + \frac{\lambda}{15}\right\} 15 \quad \text{in degrees}$$

In the above, UT = universal time expressed in decimals of a day.

GMST computed in (a) differs from that tabulated in the Almanac by nutation terms.

4. Rectangular Components of the Observer

Once LST has been computed, the observer's position vector can be resolved in the coordinate system with respect to which LST is given.

Thus

$$X = R \cos \phi' \cos (LST)$$
$$Y = R \cos \phi' \sin (LST)$$
$$Z = R \sin \phi'$$

with respect to the mean equinox of date.

Note: If a close Earth satellite is used, X, Y, Z determine the observer's position with respect to the center of force. However, for the deep space probes, the center of force will most likely reside in the Sun. In this case, designate by $X_\odot$, $Y_\odot$, $Z_\odot$, the components of the Sun's position vector. These components are tabulated in the Almanacs. Quantities X, Y, Z, computed above are now small corrections for the parallax since observations are made from the surface of the Earth. The Sun's coordinates, with respect to the observation site, are given by

$$X'' = X_\odot + X$$
$$Y'' = Y_\odot + Y$$
$$Z'' = Z_\odot + Z$$

In astronomical practice, it must be mentioned, the Sun-to-Earth direction is taken positive. However, the Sun's coordinates tabulated in the

Almanacs are measured with respect to the Earth. This convention is not observed in this report. It is always assumed that the inertial origin rests at the center of force and all distances are measured positive from this center. Consequently, if tabulated values of the Sun's coordinates are used in formulas of this report, always reverse the algebraic sign of the tabulated value. If all computations are assumed to be carried out in the coordinate system of 1950. 0, X, Y, and Z of date must be converted to the standard reference system.

C. REDUCTION OF RECTANGULAR COORDINATES X, Y, Z OF DATE TO EQUINOX OF 1950. 0

Designate the rectangular components of a vector given with respect to equinox of date by X_D, Y_D, Z_D. The conversion of these to the coordinate system, fixed by the equinox of date, is effected by the following transformation:

$$\mathbf{X}_{1950} = \mathbf{A}\mathbf{X}_D$$

In this expression

$$\mathbf{X}_{1950} = \begin{pmatrix} X_{1950} \\ Y_{1950} \\ Z_{1950} \end{pmatrix} \qquad \mathbf{X}_D = \begin{pmatrix} X_D \\ Y_D \\ Z_D \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

where the elements of matrix a_{ij} are given by the following formulas

$$a_{11} = 1.00000000 - .00029696\,(\Delta t)^2 - .00000014\,(\Delta t)^3$$

$$a_{12} = -.02234941\,\Delta t - .00000676\,(\Delta t)^2 + .00000221\,(\Delta t)^3$$

$$a_{13} = -.00971691\,\Delta t + .00000206\,(\Delta t)^2 + .00000098\,(\Delta t)^3$$

$$a_{21} = .02234941\,\Delta t + .00000676\,(\Delta t)^2 - .00000221\,(\Delta t)^3$$

$$a_{22} = 1.00000000 - .00024975\,(\Delta t)^2 - .00000015\,(\Delta t)^3$$

$$a_{23} = .00010858\,(\Delta t)^2 - .00000003\,(\Delta t)^3$$

$$a_{31} = .00971691\,\Delta t - .00000206\,(\Delta t)^2 - .00000098\,(\Delta t)^3$$

$$a_{32} = -.00010858\,(\Delta t)^2 - .00000003\,(\Delta t)^3$$

$$a_{33} = 1.00000000 - .00004721\,(\Delta t)^2 + .00000002\,(\Delta t)^3$$

In the above expressions

$$\Delta t = \frac{2433281.5 - JD}{36525}$$

where JD 2433281.5 corresponds to the beginning of the Besselian year 1950 (namely 1950.0), and JD to the date and time at which X, Y, Z are given. Thus, Δt is in effect a measure of Julian centuries taken from 1950.0. Numerical constants in a_{ij} were taken from Reference 5.

D. REDUCTION OF (X, Y, Z) 1950.0 TO THOSE OF DATE

If matrix $\mathbf{A}_{D \to 1950}$ is known, this transformation is effected by

$$\mathbf{X}_D = \mathbf{A}^{-1}\,\mathbf{X}_{1950.0}$$

where $\mathbf{A}^{-1}$ is $\mathbf{A}$ reflected about the principal diagonal. However, it is simpler to recompute the transformation matrix. This is done simply by inverting the sign in Δt. Thus, the new Δt becomes

$$\Delta t = \frac{JD - 2433281.5}{36525}$$

Using this in the series expressions for a_{ij} and interchanging subscripts 1950.0 and D the desired transformation is obtained.

Note: In the American Ephemeris and Nautical Almanac it is now customary to tabulate solar coordinates with respect to both the mean equinox of the beginning of a given year and the equinox of 1950.0. In older Almanacs, solar coordinates were tabulated only with respect to the mean equinox f the beginning of the year. Under the latter conditions in t ... ming $X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$ between equinoxes, JD will correspond to the ginning of the Besselian year of interest.

The solar coordinates at other than tabular points are obtained from published ephemerides by interpolation. There is no reason why the above reduction formulas could not be used for this purpose, provided $X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$ are known for some date.

It is well known that the Sun is slightly ahead of the position given by theory from which the solar coordinates are computed. This discrepancy can be largely eliminated if the Sun's coordinates are interpolated for the time

$$t_{\text{Desired}} + 0^{d}.000282$$

It should be noted that the above transformations are valid for any vector and, consequently, equally applicable to transformations of the velocity components.

E. CONVERSION OF AZIMUTH AND ELEVATION TO RIGHT ASCENSION AND DECLINATION

Another important transformation is conversion of elevation-azimuth angular coordinates to right ascension and declination coordinates.

Conventional radars are not well adapted to equatorial mountings, and the angular output is of necessity given with respect to the local elevation-azimuth coordinate system.

Designate by

α - right ascension

δ - declination

E - elevation

A - azimuth

HA - hour angle

In the northern hemisphere, azimuth will always be measured from the north point clockwise through 360°.

The hour angle is measured West from the observer's meridian through 360°.

The right ascension is measured East from the first point of Aries through 360° The declination is taken positive North of celestial equator and negative South of the latter.

Since computations of preliminary orbits from angular data are more convenient when the latter are given in the equatorial system, elevation and azimuth are converted by the following expressions

1. Declination δ is computed from

$$\sin \delta = \sin \phi' \sin E + \cos \phi' \cos E \cos A$$

The algebraic sign of $\sin \delta$ gives the sign of δ .

2. The right ascension α is computed as follows:

First, the hour angle of the observed body is computed from

$$\sin HA = - \frac{\cos E \sin A}{\cos \delta}$$

$$\cos HA = \frac{\sin E \cos \phi' - \cos E \sin \phi' \cos A}{\cos \delta}$$

The above expressions determine both the hour angle and its quadrant.

The right ascension α of the observed object is determined from

$$\alpha = LST - HA$$

The result can be given in either time or angular measure. In this program α is always computed in degrees.

The Local Sidereal Time is taken from B, 3(b). Since the elevation and azimuth are measured with respect to the coordinate system as existing at the time of measurement, α and δ from the above expressions are referred to the equinox of date.

Often these quantities must be reduced to the standard equinox of 1950.0.

F. REDUCTION OF α AND δ FROM THE EQUINOX OF DATE TO THE REFERENCE EQUINOX

Let α and δ referred to the equinox of date be designated by α_D and δ_D. These are obtained by direct measurement or from computation D.

The reduction can be accomplished in two ways. One of these is described in Reference (4) p. 240. The other method is substantially simpler although it involves an iterative procedure.

Computation is arranged as follows:

Suppose that α_D and δ_D are given with respect to equinox of t_1 and values at t_2 are desired where $t_2 > t_1$.

1. Certain quantities m and n are computed for the middle of the interval involved. These are given by

$$m = \frac{1}{3600} \left[46.''09905 + 0.''0002790 \left(\frac{JD - 2433281.5}{365.25} \right) \right] \text{ degrees}$$

$$n = \frac{1}{3600} \left[20.''0426 - 0.''000085 \left(\frac{JD - 2433281.5}{365.25} \right) \right] \text{ degrees}$$

Note that (JD 2433281.5)/365.25 will correspond to $t = \frac{t_1 + t_2}{2}$

Using α_{t_1} and δ_{t_1} in expressions for the annual precession in right ascension and declination, results in

$$\frac{d\alpha}{dt} = m + n \sin \alpha_{t_1} \tan \delta_{t_1}$$

$$\frac{d\delta}{dt} = n \cos \alpha_{t_1}$$

The approximate values of α_{t_2} and δ_{t_2} are then given by

$$\alpha_{t_2} = \alpha_{t_1} + (t_2 - t_1) \frac{d\alpha}{dt}$$

$$\delta_{t_2} = \delta_{t_1} + (t_2 - t_1) \frac{d\delta}{dt}$$

The means between α_{t_2}, δ_{t_2} and α_{t_1}, δ_{t_1} are denoted by α' and δ'

2. Employing α' and δ' in

$$\left(\frac{d\alpha}{dt}\right)' = m + n \sin \alpha' \tan \delta'$$

$$\left(\frac{d\delta}{dt}\right)' = n \cos \alpha'$$

a better approximation is obtained for the effects of annua precession. Thus, more nearly correct values of α_{t_2} and δ_{t_2} are given by

$$\alpha'_{t_2} = \alpha_{t_1} + (t_2 - t_1) \left(\frac{d\alpha}{dt}\right)'$$

$$\delta'_{t_2} = \delta_{t_1} + (t_2 - t_1) \left(\frac{d\delta}{dt}\right)'$$

This procedure can be repeated as many times as desired.

Generally, even when $(t_2 - t_1)$ is as large as 50 years, two iterations lead to an error of less than one second of time in α, and less than a second of arc in δ. This is entirely sufficient for preliminary computations.

If the epoch $t_2 < t_1$ the procedure is the same as above.

Note, however, that the approximations are given by

$$\alpha_{t_1} = \alpha_{t_2} - (t_1 - t_2) \frac{d\alpha}{dt}$$

$$\delta_{t_1} = \delta_{t_2} - (t_1 - t_2) \frac{d\delta}{dt}$$

Thus, depending on the interval $\left| t_2 - t_1 \right|$ the reduction can proceed from any equinox to any other one.

G. EPHEMERIS COMPUTATION

The purpose of any tracking and orbit computation program is, not only to determine the orbit, but to predict the future positions of the body. These data, the so-called ephemeris, are needed to enable the observer to re-acquire the body if the tracking is intermittent.

Ephemeris information can best be given as angular data, and possibly range data with respect to a coordinate system fixed at the observer's site. The predicted positions can be given with respect to the equinox of date or the equinox of 1950.0.

The computational procedure is as follows:

1. The n-body program yields values of the vehicle's position x, y, z at a time t with respect to equinox of 1950.0. The units of distance can be astronomical units, Earth equatorial radii, or kilometers. Denote these components by $X_{t_i\,1950}$, $Y_{t_i\,1950}$, $Z_{t_i\,1950}$

2. At assigned universal times t_i rectangular coordinates of site X_{t_i}, Y_{t_i}, Z_{t_i} are computed by methods of section B. If the Sun's coordinates are desired, use the appropriate Almanac.

3. Reduce these to $X_{t_i 1950}$, $Y_{t_i 1950}$, $Z_{t_i 1950}$

4. The topocentric range ρ expressed in appropriate units is computed from

$$\rho_{t_i} = \left[\left(x_{t_i 1950} - X_{t_i 1950} \right)^2 + \left(y_{t_i 1950} - Y_{t_i 1950} \right)^2 + \left(z_{t_i 1950} - Z_{t_i 1950} \right)^2 \right]^{1/2}$$

5. The declination $\delta_{t_i 1950}$ at t_i is computed from

$$\sin \delta_{t_i 1950} = \frac{z_{t_i 1950} - Z_{t_i 1950}}{\rho_{t_i}}$$

The algebraic sign of $\zeta_{t_i 1950} = z_{t_i 1950} - Z_{t_i 1950}$ determines the sign of δ.

The right ascension $\alpha_{t_i 1950}$ is determined from expressions

$$\cos \alpha_{t_i 1950} = \frac{x_{t_i 1950} - X_{t_i 1950}}{\rho_{t_i} \cos \delta_{t_i 1950}}$$

$$\sin \alpha_{t_i 1950} = \frac{y_{t_i 1950} - Y_{t_i 1950}}{\rho_{t_i} \cos \delta_{t_i 1950}}$$

6. If desired, $\alpha_{t_i 1950}$ and $\delta_{t_i 1950}$ are reduced to the equinox of date by method outlined in section F.

Thus, computations indicated in 1 through 6 result in

$$\rho_{t_i}, \quad \alpha_{t_i\,1950}, \quad \delta_{t_i\,1950} \quad \text{or} \quad \alpha_{t_i} \quad \text{and} \quad \delta_{t_i}$$

If the observing instrument is a radar, more appropriate angular quantities are azimuth and elevation. These are obtained as follows.

7. Compute the Local Sidereal Time (LST) as measured by the equinox of date or equinox of 1950.0 using methods of Section B. 3.

8. Using $\alpha_{t_i\,1950}$ or α_{t_i} from G. 6 the Local Hour Angle (LHA) of the body is computed from

$$(LHA)_{t_i} = (LST)_{t_i} - \alpha_{t_i}$$

It must be observed that LST and α used in the above must be referred to the same equinox.

9. Elevation at time t_i is obtained from

$$\sin E_{t_i} = \sin \phi' \sin \delta_{t_i} + \cos \phi' \cos \delta_{t_i} \cos (LHA)_{t_i}$$

where ϕ' has been defined in B. 1. The algebraic sign of $\sin E_{t_i}$ determines the sign of E_{t_i}.

10. Azimuth is determined uniquely from expressions

$$\sin A_{t_i} = \frac{-\sin (LHA)_{t_i} \cos \delta_{t_i}}{\cos E_{t_i}}$$

and

$$\cos A_{t_i} = \frac{\sin \delta_{t_i} - \sin \phi' \sin E_{t_i}}{\cos \phi' \cos E_{t_i}}$$

A careful distinction must be made between $A_{t_i\ 1950.0}$, $E_{t_i\ 1950.0}$ and A_{t_i}, E_{t_i}. Indirectly, these values differ by effects of precession.

As a result of computations 6 through 10 azimuth and elevation are obtained referred to the horizon plane and zenith of date or that of 1950.0.

All computations discussed above are provided as separate subroutines with the main n-body Trajectory Program. An attempt has been made to make these subroutines as independent as possible. In some cases, however, total separation is not practical. This is particularly true in converting α, δ to A, E. This subroutine utilizes the hour angle which in turn requires Local Sidereal Time. However, the computation of LST is tied up with the computation of the observer's coordinates on the Earth's surface. Thus, in cases where LST alone is required, some superfluous information may be produced.

Occasionally, in tracking routines, it is required to find the sidereal time measured by equinox at time t_2 if the sidereal time measured by equinox t_1 is known. This reduction can be accomplished by utilizing methods of B. 3(a), (b), but for a few dates the use of the 7090 subroutines may not be warranted. Under these conditions, a simpler procedure is available and, although it is not a part of this program, it will be described below.

H. REDUCTION OF SIDEREAL TIME REGULATED BY A MOVING EQUINOX TO SIDEREAL TIME REGULATED BY A STATIONARY EQUINOX

The relation between the sidereal time regulated by a stationary equinox and the mean solar time is given by

$$d\Theta_s = 1.002737810 \, dt$$

The constant of proportionality is the ratio between the mean solar day and sidereal day. Its exact value varies depending upon the author. However, for most purposes the value given above is sufficiently accurate.

Integrating the above obtains

$$\Theta_s = \Theta_o + 1.002737810 \, (t - t_o) \qquad (30)$$

where Θ_o is the constant of integration. The sidereal time Θ_s can be obtained from the Almanac or the formula given in Section B.

$$\text{GMST at } 0^h \text{ UT} = \frac{1}{3600}\left[23925.^s836 + 8640184.^s542 \, \Delta t + 0.^s0929 \, (\Delta t)^2\right]$$

where $$\Delta t = \frac{JD - 2415020.0}{36525}$$

and $$JD = t_o$$

The Greenwich MST at any other universal time is then given by equation (1). It must be carefully noted that Θ_s computed by (1) is measured by the stationary equinox of t_o.

To correct this time to GMST, regulated by the moving equinox, recall that the general precession of the first point of Aries amounts to $50''.2675$/year as of 1950.0. Since the cosine of obliquity of the ecliptic for 1950.0 is $\cos E_{1950} = .91743695$, it follows that along a stationary equator precession is given by

$$50''.2675 \cos \epsilon_{1950} = 46''117/\text{year} = 3^s.074/\text{year} =$$
$$= 0^s.008417/\text{day} = 0^d.00000009742/\text{day}.$$

In t years the equinox will precess along the equator by an amount

$$\Delta \Theta_s = 3^s.0744841 \ t.$$

Sidereal time measured by the moving equinox is then given by

$$\text{GMST} = \Theta_o + 1.002737810 \ (t - t_o) \pm \Delta \Theta_s.$$

Positive sign is taken if $t > t_o$, negative one when $t < t_o$. In general then

$$\text{GMST} = \Theta_o + 1.002737810(t - t_o) + 3^s.0744841(t - t_o). \qquad (31)$$

To make the method clearer a numerical example is given.

Example:

Suppose that GMST at 0^h UT on Sept. 3, 1935, is equal to $22^h 44^m 48^s.38$. Let us compute the sidereal time on Sept. 3, 1950.

From Equation (1) it follows that

$$\text{GMST}_{1950} = 22^h 44^m 48^s.38 + 1.002737810 \ (2433527.5 - 2428048.5)$$
$$= 5494^d.9482431$$

Its fractional part gives

$$\text{GMST}_{1950} = 22^h\,45^m\,28^s.20$$

This is the GMST on Sept. 3, 1950, as measured by the stationary equinox of Sept. 3, 1935. However, to obtain GMST as measured by the equinox of Sept. 3, 1950, a correction for precession must be introduced. This correction is given by

$$\Delta\,\Theta_s = 3^s\!.0744841 \times 15^y\!.0006845 = 46^s\!.12$$

Since the equinox of 1950 is in advance of equinox of 1935 by the above amount, there results

$$\text{GMST}_{\text{Sept. 3, 1950}} = 22^h 45^m 28^s\!.20 + 46^s\!.12 = 22^h 46^m 14^s\!.3$$

<u>Converse Problem</u>

Given equinox of Sept. 3, 1950, at which GMST at 0^hUT is $22^h\,46^m\,14^s.3$, compute GMST on Sept. 3, 1935, as regulated by the stationary equinox of Sept. 3, 1950, and the moving equinox of Sept. 3, 1935.

$$\text{GMST}_{\substack{\text{on Sept. 3, 1935}\\ \text{measured by } \gamma\ 1950}} = 22^h\,46^m\,14^s\!.3 - 5494^d\!.0004610$$

The fractional part of this number gives

$$\text{GMST} = {}^d.9483151 = 22^h\,45^m\,34^s.4$$

Since the equinox of Sept. 3, 1935, lags the equinox of Sept. 3, 1950 by an amount

$$\Delta\,\Theta_s = 46^s.12$$

it follows

$$GMST = 22^h 44^m 48^s.30$$

as measured by the moving equinox of Sept. 3, 1935. It is also useful to note that $\Delta \Theta_s$ is the angle between the two equinoxes in question.

Thus, in equation (2) there exists a simple method of evaluating sidereal times as measured by any desired equinox, provided sidereal time is known at some instant of time.

The discussion of auxiliary subroutines will be concluded by giving several block diagrams of possible tracking computations involving situations frequently encountered in practice.

I. SCHEMATICS OF TYPICAL COMPUTATIONS

Problem 1

Consider a tracking station, whose geographic coordinates are ϕ' and λ that is capable of measuring angular positions of a close Earth satellite with respect to the geocentric equatorial coordinate system. The absolute minimum of information necessary to estimate the satellite's motion consists of α and δ measured at three different times. Assume α and δ are measured with respect to the equinox of date

The resulting computational scheme and the flow of information is shown in Figure 6. Its main features are as follows:

The first part of the computation can be carried out in two ways. The measured right ascension and declination, both with respect to equinox of date (α_{MD}, δ_{MD}), can be used directly in the three angle tracking routine to obtain a preliminary value of the vehicle position and velocity (x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$) at some time t_o.

The n-body Trajectory Program cannot accept these values as initial conditions since they are referenced to the equinox of date. Thus x_o, y_o etc., must be converted to the epoch of 1950.0. Under these conditions the proper routing is obtained by placing S_1 at position 2, S_2 at 2.

Another way is to convert measured data to epoch of 1950.0. Under these conditions the output of the three-angle tracking routine is directly usable by the 7090 program. The proper connections are then $S_1(1)$, $S_2(1)$.

The output of the 7090 program is in the form of rectangular components of position and velocity of the vehicle at some future time t referenced to equinox of 1950.0.

Following this, two possibilities exist. One is to compute the predicted angular positions with respect to 1950.0. The second possibility is to compute all predicted positions with respect to the equinox of date. Figure 6 shows both alternatives.

The user can select the flow of computation as desired by rearranging the order of various subroutines.

However, it is strongly recommended that all computations be carried out in a coordinate system referenced to the equinox of 1950.0. Thus, all measured data as well as auxiliary quantities should be expressed in this frame of reference. This accomplishes two things. First, the method is in accord with the commonly accepted astronomical practice of referring data to a standard epoch. Secondly, comparison with computations of other investigators is possible without need for further transformations.

In some instances, of course, adherence to this recommendation may be awkward.

This is particularly true in the case of ephemerides prepared for radar stations in terms of elevation and azimuth.

Problem 2

Consider a situation which is in all respects identical with that of problem 1 with the exception that the measured angular data are expressed in terms of azimuth A and elevation E.

The block diagram of the computation is then arranged as in Figure 7.

From this figure it can be seen that problem 2 requires the addition of a subroutine converting measured azimuth and elevation to right ascension and declination of date.

The essential output of ephemeris computation in this problem consists of azimuth, elevation, and range ρ

Problem 3

A tracking situation is now considered in which an instrument, such as radar, measures range ρ and the associated angle which may be given as elevation and azimuth, or right ascension and declination.

Because a complete position vector is measured, it is merely necessary to compute velocity components at some time t which, in conjunction with the measured position components, can be used as initial conditions for the main program.

If the tracking instrument is equatorially mounted, the computation can be arranged as shown in Figure 8.

As in Problem 1, the first part of the computation can be carried out in the coordinate system of date, or that referenced to equinox 1950.0. In the first case, the measured A and E are first converted to α and δ of date. These are used to compute the components of the measured position vector at two different times from expressions

$$\xi = \rho \cos \delta \cos \alpha$$
$$\eta = \rho \cos \delta \sin \alpha$$
$$\zeta = \rho \sin \delta$$

These quantities used in

$$x = \xi + X$$
$$y = \eta + Y$$
$$z = \zeta + Z$$

yield the components of the geocentric position vector of the vehicle. These components are employed in the two-range vector iterative routine to produce the velocity components at one of the measured instants of time. Before using these in the n-body Trajectory Program, conversion to the equinox of 1950.0 must be effected. The computation then follows the same lines as in Problems 1 and 2.

Switches S_1 and S_2 are connected to positions 1 and 2 for the proper flow of information.

If the iteration routine is to produce $x_\odot$, $y_\odot$, $z_\odot$, $\dot{x}_\odot$, $\dot{y}_\odot$, $\dot{z}_\odot$ referenced to epoch 1950.0, S_1 is connected to position 2 and S_2 to position 1.

Problem 4

Though similar to Problem 3, this situation's tracking instrument is assumed to be mounted equatorially. The measured data therefore consists of range ρ and the associated right ascension α and declination δ

The flow of information is very similar to that found in Figure 6 and shown in Figure 9.

Positions of switches S_1 and S_2 for proper flow of information are either $S_1(1)$ and $S_2(1)$ or $S_1(2)$ and $S_2(2)$, depending upon the referencing of the angular data to the equinox of date or 1950.0.

Diagrams similar to those given in Figures 6, 7, 8, and 9 can be constructed for any tracking subroutine. From these diagrams it is apparent that a substantial number of blocks contained in the conversion subroutines disappear if computations are consistently carried out in coordinates referenced to some standard equinox, e.g., 1950.0.

No block diagram is given for the case of 6 ranges. Operation of this routine in its present formulation was found unreliable.

Appendix B of this report consist of Operational Directories and FORTRAN Listings of programs discussed above, as well sample check problems for each subroutine.

V. ERROR ANALYSIS OF HYPERBOLIC LUNAR IMPACT TRAJECTORIES

A. INTRODUCTION

As a part of demonstrating the capabilities of the Lunar Trajectory Computer Program described earlier in this report a study was performed of tolerances in initial conditions involved in lunar flight impacting the Moon's surface.

For obvious reasons it was found impractical to consider all possible impact trajectories covering the velocity range from elliptical to hyperbolic ones. Likewise, there was no attempt to arrange trajectories by firing locations and dates. As a consequence of these considerations, it was decided to consider only trajectories of classes IS^{a+} or IS^{a-} in the sense of Egorov [10] These classes of trajectories refer to impacts occurring on the ascending branches of trajectories. This in turn implies that the velocity with respect to the Earth is in the hyperbolic range. The actual value of the initial velocity is, in principle, immaterial for purposes of this demonstration. However, in general, higher velocity trajectories impose tighter tolerances on orientations of the initial position and velocity vectors.

The general character of this study can be summarized as follows:

(a) A nominal hyperbolic trajectory is chosen.

(b) It is a three dimensional trajectory. There are no restrictions commonly invoked in assuming that the vehicle moves in the Earth-Moon plane.

(c) The vehicle moves in the field of the Sun, the Earth, and the Moon.

(d) Three dates are considered. These correspond to the following phases of the Moon.

(1) First Quarter (Nov. 7, 1959)

(2) Full Moon (Nov. 15, 1959)

(3) Last Quarter (Nov. 23, 1959)

This arrangement of the study may at least qualitatively reveal effects of the changing geometrical configuration of the three main bodies.

B. NOMINAL TRAJECTORIES

Nominal trajectories for each phase of the moon at the above dates have been established empirically. An essential requirement that has been imposed on these trajectories is that the hit must occur at the center of the apparent disc of the Moon. Its angular coordinates as seen from the Earth have been taken from the Nautical Almanac.

The initial conditions for the differential equations of motion were determined experimentally in units of A. U. and A. U. /hour. Upon converting we find the burnout altitude $h \approx 388.5$ st. miles and the burnout velocity $V = 7.177$ st. miles/sec. We have succeeded in holding these values constant for the three dates indicated. Thus, the only difference in initial conditions between the three sets of runs occurs in the orientation of vectors $\overline{\rho}$ and $\overline{v}$. (See Figure 10).

The following procedure was followed in arriving at a nominal trajectory. As a first step, conditions have been taken which result in an impact for an idealized problem based on either two body approximations or previous computer runs. These initial conditions are used in the Lunar program to obtain a trajectory. This trajectory in most cases strikes the Moon, but not at the desired place. Following this the initial velocity is changed by a small increment, and the computation is repeated. After three runs are obtained, an estimate is made of the differential correction required to effect the desired impact. This correction is usually not sufficient; consequently, the procedure described above is repeated several times until the strike occurs at the desired place. It was found that in all three phases of the moon the center of the apparent disc of the Moon could be hit in about 9 tries. The resulting error between the computed and observed angular coordinates is in the neighborhood of 1 second of arc.

C. SENSITIVITY OF THE IMPACT POINT TO ERRORS IN INITIAL CONDITIONS

In the description of the Lunar Trajectory Program it was indicated that the program accepts the components of the position and velocity vector (x_o, y_o, z_o; $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$) at some time t_o as the basic input. There are several disadvantages in using these inputs in the error analysis considered here. First of all, most measuring instruments work in a spherical coordinate system which is more natural. Thus, the above components are connected with the spherical coordinates by the usual transformation,

$$
\begin{aligned}
x &= \rho \cos \theta \cos \phi & \dot{x} &= v \cos \alpha \cos \beta \\
y &= \rho \sin \theta \cos \phi & \dot{y} &= v \cos \alpha \sin \beta \\
z &= \rho \sin \phi & \dot{z} &= v \sin \alpha
\end{aligned}
\tag{32}
$$

Thus the original inputs are replaced by θ, ϕ, α, β, ρ, v at time t_o. In the equations above, ρ is the magnitude of the initial position vector, θ is the right ascension, and ϕ is the declination. Similarly, v is the magnitude of the initial velocity vector, and α, and β are the two corresponding angular coordinates. The coordinate system employed here is an equatorial one in the astronomical sense. The above relations underscore another difficulty. So far as the computation of the trajectory is concerned x, y, z; $\dot{x}$, $\dot{y}$, $\dot{z}$ are certainly independent quantities. However, as soon as the measuring instrument enters the scene we find that these quantities are related. Thus it is, for instance, impossible to change ρ without changing x, y, z simultaneously. A similar situation exists with regard to the angular coordinates.

In view of the above it has been decided to employ spherical coordinates in our study of impact point errors. From the practical point of view this implies that any uncertainty in one of the spherical components must be converted to uncertainty in the rectangular components before it can be used in the computer program.

It has been found that in dealing with high speed trajectories (e. g. hyperbolic), 8-place accuracy in input is sufficient to furnish accurate, perturbed trajectories. However, in elliptical trajectories, where the magnitude of the velocity is decreased, it has become apparent that the lunar program demands accuracy of input to 10 or more places in order to furnish output of sufficient numerical accuracy. This would be especially true when mid-course guidance parameters are to be generated.

Response of the impact point to errors in one of the independent variables is evaluated by holding other variables fixed and varying the remaining one over a range of values. The maximum allowable error is that which results in a "skimming" impact on the Moon. The amount of miss due to an error in the corresponding variable can be conveniently measured in terms of the distance S on the surface of the Moon between the nominal and the perturbed impact points. This distance is in effect computed from six measured quantities ρ, θ, ϕ, v, α, and β, which are assumed to be mutually independent. Thus,

$$S = S(\rho, \theta, \phi, v, \alpha, \beta). \tag{33}$$

Since the deviation from the nominal impact point is measured, S is equivalent to an incremental miss, which to the first order of approximation can be expressed as

$$S = \Delta M = \left(\frac{\partial S}{\partial \rho}\right)\Delta\rho + \left(\frac{\partial S}{\partial \theta}\right)\Delta\theta + \left(\frac{\partial S}{\partial \phi}\right)\Delta\phi + \left(\frac{\partial S}{\partial v}\right)\Delta v + \left(\frac{\partial S}{\partial \alpha}\right)\Delta\alpha + \left(\frac{\partial S}{\partial \beta}\right)\Delta\beta, \tag{34}$$

where $\Delta\rho$, $\Delta\theta$, etc. represent errors in the corresponding quantities.

It is then evident that a successive independent variation of one of the variables will result in relations $S = S(\rho)$, $S = S(\theta)$, etc., each of which can yield partial derivatives in equation (34). These derivatives will be termed "error or sensitivity" coefficients. It is obvious that relation (34) assumes that for small deviations the miss can be written as a linear combination of errors. Just where a particular error ceases to be small cannot be evaluated until S is obtained.

Prior to evaluation of S the range of errors in a particular variable should be decided on in order to result in a skimming impact. There must be a sufficient number of points within this range in order to obtain S. One appraoch is to repeat the same procedure as used in establishing the nominal trajectory. The final approach adopted in this work will be described later in the report.

For the present, however, the description of the computation as employed here will be continued.

The early approach was to hold all variables but one constant at values corresponding to those of the nominal trajectory and to vary the remaining one by small increments. Values of x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ corresponding to these increments were computed from equation (32) by employing an LGP-30 Computer subroutine. The resulting quantities are then fed to the Lunar Trajectory Program. The output of this computation are the following quantities:

x, y, z and r	of the vehicle from the Earth
$\dot{x}, \dot{y}, \dot{z}$, and v	of the vehicle with respect to the Earth
x, y, z, and r	of the vehicle from the Moon
x, y, z, and r	of the vehicle from the Sun
t -	Corresponding time

The units employed are astronomical units, astronomical units per hour, and hours. The nominal radius of the Moon (assumed spherical) was taken as $R_M = 1.1625090 \times 10^{-5}$ AU. The initial values of θ, ϕ, α, β are obtained from equation (32).

As indicated earlier the run is terminated by prescribing either a maximum running time or a minimum distance from the Moon's center. In neither case, however, are the coordinates of the impact point obtained directly. These points must be obtained by interpolation from the computed points that give the position of the vehicle from the Moon as a function of time. The first step is to obtain the time of impact. For purposes of this investigation, use is made of Everett's interpolation formula in the form

$$f + f_0 + p\, \delta_{1/2} + E_2\, \delta_0^2 + F_2\, \delta_1^2. \tag{35}$$

Various quantities in this relation are obtained according to the following scheme:

t	f	$\delta_{n/2}$	δ_n^2	$\delta_{n/2}^2$
t_{-1}	f_{-1}			
		$\delta_{-1/2} = f_0 - f_{-1}$		
t_0	f_0		$\delta_0^2 = \delta_{1/2} - \delta_{-1/2}$	
		$\delta_{1/2} = f_1 - f_0$		$\delta_{1/2}^3 = \delta_1^2 - \delta_0^2$
t_1	f_1		$\delta_1^2 = \delta_{3/2} - \delta_{1/2}$	
		$\delta_{3/2} = f_2 - f_1$		
t_2	f_2			

In the above t is the independent variable (time) and f can stand for r, x, y, z or any other dependent variable. Constants E_2 and F_2 are given by

$$E_2 = \frac{p(p-1)(p-2)}{6}, \quad F_2 = \frac{p(p-1)(p+1)}{6}$$

where p is the interpolation fraction such that $0 < p < 1$.

In determining the selenocentric coordinates of the impact point equation (4) was employed in the form

$$R_M^{2} = \sum_{i=x,y,z} \left(X_{i0} + p\,\delta_{i\,1/2} + E_2(p)\,\delta_{i0}^{2} + F_2(p)\,\delta_{i1}^{2}\right)^{2} \tag{36}$$

A subroutine was written for the LGP-30 computer to obtain p from equation (36) by successive approximations using Newton's Method.

The above computation simultaneously yields values of x, y, z of the impact point.

At the time this study was begun, and largely by coincidence, several runs were terminated with only one point recorded after impact occurred. As (35) indicates, Everett's interpolation formula required two values of the argument on either side of the derived value of the function. To compensate for the lack of a sufficient number of tabular points, two additional sub-programs were written for the LGP-30. A Newtonian formula was employed based on five points and differences on a horizontal line. Also a six point Lagrangian interpolation formula was tested. Both methods, however, were inadequate because of their failure to provide the necessary degree of precision. An accurate value, arrived at by using Everett's scheme could only be approached to four places with Lagrange and Newton methods.

Using this comparison as the basis, machine operators have been advised to terminate a run on a more comfortable maximum time, or on impact plus three or more points.

The value of miss distance follows then directly from

$$\cos\theta = \frac{\vec{R}_M \cdot \vec{R}_M}{R_M^{2}} = \frac{X_N X_I + Y_N Y_I + z_N z_I}{R_M^{2}} \tag{37}$$

and N, I denote nominal and impact respectively, Finally

$$S = R_M \theta \qquad (3^{\cdot\cdot}$$

It must be realized that S, as computed here and used in subsequent work, contains no direction information.

S can now be plotted versus the variable in question, as, for instance, in Figure 11. Note that the discontinuity at the origin is the consequence of definition of S and the manner in which it is measured. The slope of such a relation represents $\frac{\partial S}{\partial \sigma_i}$ where σ_i is any of the independent variables.

The plot of $\frac{\partial S}{\partial \sigma_i}$ can be obtained by numerical differentiation of the $S(\sigma_i)$ relation. Caution must be exercised in putting too much trust in this result because in any process where numerical differentiation is involved there is a significant loss of accuracy. This is particularly true with regard to the first attempts where the number of points defining S was insufficient as, for instance, in Figure 12. In other cases where the distribution of points is more favorable $\frac{\partial S}{\partial \sigma_i}$ is considerably smoother, as shown in Figure 13. In either case, however, the linear trend of S or $\frac{\partial S}{\partial \sigma_i}$ extends only over a limited part of the lunar surface. Thereafter relation (3) ceases to be a good approximation.

The procedure just described was found somewhat wasteful of computing time. It was also found that it resulted in too many complete misses.

It was noticed, however, that the plot of the distance of closest approach r_p to the Moon's center versus error $\Delta\sigma_i$ resulted in a curve which could be represented remarkably well by a hyperbola of the form

$$\frac{(y - y_0)^2}{b^2} - \frac{(x - x_0)^2}{a^2} = 1.$$

An example of this is shown in Figure 14.

This situation was found to hold for all runs made in this investigation. It must be stated emphatically that there is no analytical justification for this and the use of this fact is predicated strictly on convenience.

The choice of hyperbola is not essential. It is conceivable that other curves could serve the same purpose. If it is agreed to use the above fact, the intersection of $r_p\,(\Delta\sigma_i)$ with R_M gives the maximum possible errors allowed, beyond which the Moon is missed entirely.

In addition, the bounding errors will indicate how $\Delta\sigma_i$ should be distributed to obtain $S(\sigma_i)$ with maximum efficiency. Uniform spacing is very important especially in the determination of the partial derivatives.

An item from the preceeding discussion deserving some elaboration is the determination of the fictitious distance of closest approach r_p. As in the case of the impact point, r_p cannot be obtained directly from the computer runs. To obtain this quantity the numerical minimum of r can be found by using various interpolation formulae. This procedure is not very accurate because the trajectory near the perilune is rather flat for the high velocity employed here. Secondly, the procedure is rather tedious.

It is possible to obtain r_p in a somewhat different manner by using computed tabular points. First it should be noted that regardless of velocity the trajectory which does not result in a capture is a hyperbola as far as the observer on the Moon is concerned.

The following equation can then be written:

$$r_p = a(e - 1), \tag{39}$$

where a is the real axis of the hyperbola and e its eccentricity.

Now at any three tabular points in the neighborhood of r_p,

$$r_1 = \frac{p}{1 + e \cos \theta_1}$$

$$r_2 = \frac{p}{1 + e \cos(\theta_1 - \Delta\theta_1)} \qquad (40)$$

$$r_3 = \frac{p}{1 + e \cos(\theta_1 - \Delta\theta)}$$

where $p = a(e^2 - 1)$ and θ is the true anomaly. Also,

$$\theta_1 - \theta_2 = \Delta\theta_1$$

$$\theta_2 - \theta_3 = \Delta\theta_2$$

$$\Delta\theta = \Delta\theta_1 + \Delta\theta_2 .$$

The increments of the true anomaly are obtained from

$$\cos \Delta\theta_1 = \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2} \text{ and } \cos \Delta\theta = \frac{\vec{r}_1 \cdot \vec{r}_3}{r_1 r_3}$$

Thus in equation (40), p, e, and θ, are unknown. The simultaneous solution of (40) yields

$$p = r_1 \frac{\frac{\sin \Delta\theta_1}{\sin \Delta\theta}(\cos\Delta\theta - 1) + (1 - \cos\Delta\theta_1)}{\frac{r_1}{r_2} - \cos\Delta\theta_1 + \frac{\sin\Delta\theta_1}{\sin\Delta\theta}\left(\cos\Delta\theta - \frac{r_1}{r_3}\right)} . \qquad (41)$$

The quantity θ_1 is obtained from

$$\frac{p - r_2}{p - r_1} \left(\frac{r_1}{r_2}\right) = \cos \Delta \theta_1 + \tan \theta_1 \sin \Delta \theta_1 . \tag{42}$$

The eccentricity follows from

$$e \cos \theta_1 = \frac{p}{r_1} - 1. \tag{43}$$

Finally, a is obtained from

$$a = \frac{p}{e^2 - 1} \tag{44}$$

All quantities necessary to obtain r_p from equation (39) are now available.

This computation was performed on the LGP-30 computer.

In the above a description was given of all the supplementary computations required to obtain the desired information from the main computer runs. These subroutines could very profitably be included in the main program in order to prevent the interruption of computation.

D. DISCUSSION OF RESULTS

As an example a discussion shall be given of the results obtained for the trajectory of Nov. 7, 1959.

The general character of incremental miss S as a function of $\Delta \rho$, $\Delta \theta$, $\Delta \phi$, Δv, $\Delta \alpha$, $\Delta \beta$, or ρ, θ, ϕ, v, α, β is shown in Figures 11, 15, 16, 17, 18, 19.

It is evident from these curves that for small errors (impacts not too close to the limb) S is a reasonably linear function of incremental errors. As the impact point moves closer toward the Moon's limb the curves break away from a linear variation. Values of errors for which the linear trend ceases to be a good approximation depend on a particular variable. This situation is shown much more clearly in plots of error coefficients. Two examples are given in Figures 12 and 13. The deviation from linearity can be either gradual or rather sharply defined.

Since this value of the error coefficient is only of significance in those cases where the curves of miss versus error are linear, the cases which fail to meet this qualification are not presented graphically.

A feature which is common to all plots of S is their asymmetry with respect to the nominal impact point. This is to be expected because the Moon is a moving target. It should be sufficiently clear that the larger the arrival an the greater the asymmetry in S with respect to the desired impact point. It may be noted that in this investigation no attempt was made to achieve a normal impact. (Arrival angle is defined as the angle between the velocity vector and the local vertical.)

From the plots of S as they stand the total permissible spread of errors cannot be readily determined. By this we mean themaximum deviation in each variable from the nominal value that results in a skimming hit at each limit. This is accomplished best from the plot of the "distance of closest approach" as a function of a particular error. This also alleviates the problem of computing too many runs which fail to impact the moon. A typical example of such plot is shown in Figure 14. The range of errors for other variables has been estimated from similar plots given in Figures 20, 21, 22, 23. Similar estimates were made for the November 15 and November 23 trajectories.

The range of errors for the November 7 trajectory is as follows:

$$-14.5 < \Delta \rho < 21.2$$

$$-\ .21 < \Delta \theta < \ .64$$

$$-\ .15 < \Delta \phi < .87$$

$$-79.0 < \Delta V < 101.0$$

$$-\ .202 < \Delta \alpha < .075$$

Values available for the November 15 case are

$$-\ .18 < \Delta \rho < 22.0$$

$$.43 < \Delta \theta < \ .06$$

$$-\ .65 < \Delta \phi < \ .08$$

$$-10.0 < \Delta V < 82.0$$

$$-\ .03 < \Delta \alpha < \ .32$$

$$-\ .025 < \Delta \beta < .264$$

For the November trajectory, the available values are

$$-\ 1.9 < \rho < \ 55$$

$$-\ .067 < \theta < \ 1.56$$

$$-\ .153 < \phi < \ .274$$

$$-\ 9.9 < V < 124$$

$$-\ .28 < \alpha < \ .41$$

$$-\ .35 < \beta < \ .002$$

Here $\Delta\rho$ is given in statute miles

ΔV in feet per second and

$\Delta \alpha$, $\Delta \beta$, $\Delta \phi$, $\Delta \phi$ in degrees

The initial conditions used in the three cases are given in the following list.

	November 7	November 15	November 23
x_o	$-3.3000000 \times 10^{-5}$	$-3.3000000 \times 10^{-5}$	3.3000000×10^{-5}
y_o	$-3.3000000 \times 10^{-5}$	3.3000000×10^{-5}	3.3000000×10^{-5}
z_o	4.0500000×10^{-6}	4.0500000×10^{-6}	4.0500000×10^{-6}
r_o	4.6844449×10^{-5}	4.6844449×10^{-5}	4.6844449×10^{-5}
$\dot{x}_o$	7.9198756×10^{-5}	5.8280834×10^{-5}	$-1.2900149 \times 10^{-4}$
$\dot{y}_o$	$-2.6305573 \times 10^{-4}$	2.6288784×10^{-4}	2.4008342×10^{-4}
$\dot{z}_o$	$-4.3421102 \times 10^{-5}$	6.9638123×10^{-5}	5.5450141×10^{-5}
V	2.7812974×10^{-4}	2.7812971×10^{-4}	2.7812971×10^{-4}
θ	225°	135°	45°
ϕ	$4^{\circ}57' 35''$	$4^{\circ}57' 35''$	$4^{\circ}57' 35''$
α	$-8^{\circ}.982$	$14^{\circ}.5$	$11^{\circ}.5$
β	$286^{\circ}.76$	$77^{\circ}.5$	$118^{\circ}.25$

Comparison of these three cases indicates that the geometrical arrangement of the three main bodies has a significant effect not only on the error range itself, but also on the error asymmetry. For instance, the tolerance in $\Delta\theta$ and $\Delta\phi$ in the November 15 case is very tight in the positive direction. Also the range in velocity became significantly smaller.

In none of the cases, however, do the tolerances become so small as to make the hit impractical for conditions specified above.

It is of interest to consider the effect of errors on the flight time of the vehicle. A few representative plots are shown in Figures 25, 26, 27, 28 and 29. It is evident from these that the variation in flight time is nearly linear with the magnitude of $\vec{\rho}$ and $\vec{v}$. The variation with angles, however, is sharply non-linear. It may also be noted that so far as the November 7 trajectory is concerned, errors $\Delta\theta$, $\Delta\phi$, and $\Delta\alpha\Delta\beta$ affect the flight time in opposite directions. Thus negative $\Delta\theta$, $\Delta\phi$ tend to increase the time of flight while negative $\Delta\alpha$, $\Delta\beta$ tend to decrease it. As was to be expected V has the most serious effect on the flight time.

The entire previous discussion was concerned with errors in only one of the initial quantities: When errors in ρ, θ, ϕ, v, α, β are present simultaneously, the increment in miss is found by (34) provided that precision measures $\Delta\sigma_i$ are known. This expression will, however, result in an estimate that is generally too high. If the errors $\Delta\sigma_i$ are entirely independent, it is more reasonable to compute the miss from

$$\Delta M = \left\{ \left[\frac{\partial S}{\partial \rho} \Delta\rho \right]^2 + \left[\frac{\partial S}{\partial \theta} \Delta\theta \right]^2 + \left[\frac{\partial S}{\partial \phi} \Delta\phi \right]^2 + \left[\frac{\partial S}{\partial V} \Delta V \right]^2 + \left[\frac{\partial S}{\partial \alpha} \Delta\alpha \right]^2 + \left[\frac{\partial S}{\partial \beta} \Delta\beta \right]^2 \right\}^{1/2} \tag{45}$$

Conversely, if the miss is not to exceed some predetermined value, equation (45) can be used to specify the value of $\Delta\sigma_i$. In the converse problem a question arises whether the effects of various parameters are equal or not. This question can be settled by examining the plots of S. In general, however, the precision with which the parameters must be measured can be estimated from

$$\pm \frac{\Delta M^1}{6} = \frac{\partial S}{\partial \rho} \Delta\rho = n_1 \frac{\partial S}{\partial \theta} \Delta\theta = n_2 \frac{\partial S}{\partial \phi} \Delta\phi = n_3 \frac{\partial S}{\partial V} \Delta V = n_4 \frac{\partial S}{\partial \alpha} \Delta\alpha = n_5 \frac{\partial S}{\partial \beta} \Delta\beta, \tag{46}$$

where n_i are measures of the strength of the effect. Thus, as indicated above, $\Delta\theta$ affects S n_1 times as much as $\Delta\rho$, etc. The quantity ΔM^1 is the relative tolerance on ΔM.

Perhaps, if further study was to be carried out, another approach to the "total miss" portion of the error analysis could be tried. By varying the six spherical parameters separately, within the impact region, an error tube would be generated which would encompass the total allowable error. For these trajectories, statistically speaking, the first moment, or mean error trajectory, could be computed. Using the mean as a measure of location, the second moment can be taken about it to obtain the variance, the square root of this quantity being the standard deviation. The standard deviation is merely a number, in the same units as the particular parameter in question, which measures the relative extent of the data concentrated about the mean and becomes larger as the data becomes more dispersed. With a large sample, an interval of two standard deviations will include about 95% of the trajectories. With this knowledge, the confidence limits on the allowable error can be computed.

However, it can be said without reservation that when the first moment of the error cone is computed it would not agree with the computed standard trajectory unless the arrival angle of the trajectory were normal. Immediately a problem becomes evident. By looking at Figure 30, a plot of longitude and latitude of the impact points, the allowable error is seen to be almost no better than the nominal trajectory itself for changes in the velocity angle $\Delta \alpha$. The implications of this become clear when it is realized that if a mean trajectory were to be computed from the data taken about a non-normally arriving nominal, the allowable error at the extremes would be fictitious. Under the same conditions, the tolerances probably would become so small that arrival at a predetermined point on the moon's surface would be impossible.

In light of the above discussion, certain desirable procedures can be ascertained which would be of value in predicting the likelihood of impact and the accuracy of impact about a desired point of the Moon based on the perturbations on the initial input.

Computing the impact points in terms of their latitude and longitude on the apparent disc gives not only an indication of the value of an error tube and the resulting measures of standard deviation, but also provides directional information as well as a measure of the miss distance. This information would lead to optimization of computer runs on a factual rather than guess-work basis. It is also concluded that the accuracy of prediction within prescribed confidence limits is a direct function of the impact angle. In the November 7 trajectory, this angle was computed at 50°, for November 15, 66°, and for November 23, 65° These angles are calculated by finding the direction cosines of a straight line approximation to the tangent line to the hyperbola at the point of impact. The direction cosines of the normal to the tangent plane are then computed. The products of the direction cosines are summed and this gives the cosine of the angle of impact.

This portion of the report shall be concluded with a brief discussion concerning the disturbing effects of the Sun on a vehicle moving in the Earth-Moon space.

For two of the three dates used in this study the computation of the standard trajectory was repeated with the Sun taken out of the program.

For a crude estimate it is sufficient to compare the distance of the vehicle from the Moon for equal flight times.

For the November 7 case, the distances differ by 5.3 miles after 20.250 hours (just before impact) while for the November 23 case, the difference is 4.8 miles at 20.969 hours, which is also just prior to impact. Thus the Sun's perturbation on the distance of the vehicle from the Moon is not a very significant one for the trajectories in question.

VI. LIFETIME OF AN ARTIFICAL LUNAR SATELLITE

The principal objective of this phase of the Lunar Trajectory Study was to examine in a cursory way the life time of an artificial satellite placed in orbit around the moon. No attempt at an all inclusive analysis of this problem was intended or made.

Using the "n body computer program" in a restricted four body analysis, e. g., Earth, Moon, Sun and Vehicle; a near lunar orbit was run and some interesting results were obtained.

Fixing the altitude of the injection point at 135.827 st. miles above the moons surface a range of instantaneous injection velocities were introduced. The first orbit of each of the resulting orbits is shown in Figures 54 and 55. The range between escape and impact has been covered. Taking the case $V = 3.0 \times 10^{-5}$ A. U. /Hour the run was extended for 25 days. The projections of this orbit for the first revolution are given in Figures 56, 57, and 58. The osculating orbital elements for this revolution are

$$i = 34^{\circ}.36$$

$$\omega = 90^{\circ}.02$$

$$\Omega = 4^{\circ}.88$$

$$a = 1.458 \times 10^{-5} \text{ A.U.}$$

$$e = .1933$$

$$T = 14.^{h}157$$

$$P \approx 2.^{h}5$$

and a plot of the apolune, perilune distances as a function of orbital life time are given in Figures 59A and 59B. As may be seen in Figures 59A and 59B several interesting features appear. First we see that the lifetime of the orbit is a strong function of the number of major bodies carried in the computations.

In the Earth, Moon, Sun field the orbit exceeds 25 days (the limit has not been determined). But for the Earth, Moon field with the Solar effect removed, the lifetime is reduced to 220 hours, impact with the Moon terminating the run. A similar situation exists when the Earth's influence is removed with the lifetime reduced to only a few hours.

Secondly there exists a definite pattern showing both the effects of long and short period perturbations. The variation in the osculating orbital elements was derived from the rectangular components of the vehicles position and velocity at six hour intervals for the first 10 days of the orbit. This was done for both the Earth, Moon, Sun and Earth, Moon geometries with the results indicated in Figures 60, 61 and 62. These figures clearly show the rotation of the line of apsides as well as the long period effects in i and Ω. Also to be noted is the definite divergence between the two cases. The length of time of these runs precludes any obvious identification of secular terms except in T. Care should, however, be exercised here as a long period term can over a short interval of time look like a secular term. To show the true periodic variation in a, e, and ω requires that these elements be recomputed for time intervals of the order of one hour instead of the six hour interval employed in these figures.

If a lunar satellite is to be employed to determine the geometrical figure and internal density gradient of the Moon by perturbation analysis of the satellite's orbits, several factors are obvious. From Figure 59B we can see that the difference in perilunes during the mid portion of the orbit is 1.207×10^{-8} A. U. or some 1.12 statute miles. Similarly, for the apolunes we have a difference not exceeding 1.961×10^{-8} A. U. or 1.82 miles. Since this is as close an orbit as one cares to discuss, first perilune is 1.697×10^{-7} A. U. or 15.76 statute miles above the lunar surface, the perturbations shown are about the maximum that can be expected. Hence any tracking equipment must be able to resolve these perturbations with a high degree of accuracy. If this can be done then an appropriate set of orbital elements can be computed for a known interval of time. Then a variation of parameters scheme can be introduced. As an example consider the planetary equations due to Lagrange in which we have

$$\dot{\Omega} = \frac{1}{na^2 (1-e^2)^{1/2} \sin i} \frac{\partial R}{\partial i}$$

$$\dot{(i)} = - \frac{1}{na^2 (1-e^2)^{1/2} \sin i} \frac{\partial R}{\partial \Omega} - \frac{\tan \frac{i}{2}}{na^2 (1-e^2)^{1/2}} \left[\frac{\partial P}{\partial \pi} - \frac{\partial R}{\partial \epsilon} \right]$$

$$\dot{\pi} = \frac{\tan \frac{i}{2}}{na^2 (1-e^2)^{1/2}} \frac{\partial R}{\partial i} + \frac{(1-e^2)^{1/2}}{na^2} \frac{\partial R}{\partial e}$$

$$\dot{a} = \frac{2}{na} \frac{\partial R}{\partial \epsilon}$$

$$\dot{e} = - (1-e^2)^{1/2} \frac{1 - (1-e^2)^{1/2}}{na^2 e} \frac{\partial R}{\partial \epsilon} - \frac{(1-e^2)^{1/2}}{na^2 e} \frac{\partial R}{\partial \pi}$$

$$\dot{\epsilon} = \frac{\tan i/2}{na^2 (1-e^2)^{1/2}} \frac{\partial R}{\partial i} + (1-e)^{1/2} \frac{1 - (1-e^2)^{1/2}}{na^2 e} \frac{\partial R}{\partial e} - \frac{2}{na} \frac{\partial R}{\partial A}$$

where $\sigma = -nT$, $\pi = \omega + \Omega$ and $\epsilon = \pi + \sigma$ and the other elements have the conventional meaning. The perturbing forces are contained in the disturbing function R.

If the left hand side of the above functions are known from the tracking data it is possible to compare these with values obtained from computations based on an assumed theoretical model. Comparisons can be made and in theory at least an improved theoretical model obtained. While it may be possible to arrive at a better figure for the moon in this way the likelihood of determining the density gradient is somewhat more uncertain since the force field is not a uniquely determined function of the density gradient.

VII. LUNAR CIRCUMNAVIGATION AND EARTH RETURN

In addition to the Lunik III trajectory* another Lunar Circumnavigation and Earth Return trajectory was analyzed from an entirely different point of view. In the Lunik III case we are primarily interested in the ability to reproduce the trajectory of the vehicle in Earth-Moon space from crude tracking data. In the case under present consideration we are concerned with the generation of such a trajectory subject to a number of additional constraints. These constraints are manifest in two ways, those associated with the ascent and those associated with the extraterrestial portions of the flight. In the former such practical problems as booster capabilities, range safety limits, and the launch on time problem are eminent. These must be matched to the geometrical constraints imposed by the extraterrestial portions of the flight.

An attempt has been made in this section to indicate the effects of these constraints on a Lunar Circumnavigation and Earth Return trajectory with particular emphasis on matching the ascent to the geometrical constraints. Consider the following as an initial set of constraints;

A. ASCENT TRAJECTORY

1. Launch Site: Cape Canaveral
2. Range Safety Limits: 85° - 125° in azimuth
3. Firing Azimuth Limits (Azimuth of the velocity vector at injection or burnout point) identical with range safety limits.
4. Flight path angle at injection limited to $0^{\circ} \leq \theta \leq 3^{\circ}$ (a booster characteristic)

B. CONDITIONS AT THE MOON

1. Distance of closest approach to Moon's surface $2000 \leq r_M \leq 3000$ miles

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*Described in Scientific Report #1

2. Vehicle to pass in front of moon on outbound leg and slightly below Moon's orbital plane. (Necessary to achieve Earth return in Northern Hemisphere)

3. Transit Time ≈ $3^{d}.25$ for outbound leg (enabling vehicle to be in vicinity of Moon for longest possible time)

C. EARTH RETURN

1. Vehicle to return to the Earth in the Northern hemisphere and in a suitable recovery area.

2. Vehicle to return to the Earth in direction of the Earth's rotation (direct motion)

3. Vehicle to return at an altitude of from $200 \leq h \leq 300$ miles above the Earth's surface.

In arbitrarily specifying a set of constraints such as that listed above it is possible to overdefine the problem. A solution may not exist for a particular firing date. An indication of this exists in this case but an exhaustive study to definitely ascertain whether such a situation is the case or not has not been made.

To match the ascent and geometrical constraints it is first necessary to determine the orientation of the velocity vector at the time of injection. Thus, we must specify the time (t), the firing azimuth (A_z) and the flight path angle (θ) at the injection point. Since we have already placed a constraint on θ our free parameters become t and A_z. Initially then we wish to select values of t and A_z such that the orbital plane of our vehicle intersects the orbital plane of the moon some $3^{d}.25$ after injection.

To obtain preliminary values for A_z and t a method based on a two body approximation was tried. It was realized at the outset that the final values for A_z and t could only be obtained from the n-body program by trial and error since a solution to the two point boundary value problem in n-body space does not exist in explicit form. However, it was hoped that the preliminary values would be sufficiently close to the final values for A_z and t to greatly reduce the convergence time. It is appropriate to mention at this point that we have selected a direct assent trajectory, fully realizing that for extreme declinations of the Moon one may be forced to employ a coasting orbit.

The results of such an approximation give A_z as a function of t, launching site latitude (ϕ) and the coordinates of the Moon. For any specific case ϕ and the coordinates of the Moon enter as fixed parameters. For this problem we have taken the time as September 25, 1960, and the Moon's coordinates at the vehicles time of arrival ($3^{d}.25$ later) as

$$\alpha_{☾} = 18^{h}\ 37^{m}\ 27^{s}.8$$

$$\delta_{☾} = -18^{\circ}\ 19'\ 51''.5$$

where $\alpha_{☾}$ is the right ascension and $\delta_{☾}$ is the declination of the Moon.

The injection point was taken as 1500 miles downrange at an azimuth value of 110° and altitude of 750000 ft over a spherical Earth.

A convenient relation between the firing azimuth and the launching time, can be obtained as follows:

Let (i_x, i_y, i_z) be unit vectors defining a geocentric equatorial coordinate System and $(\bar{\xi}, \bar{\zeta}, \bar{\eta})$ be a horizon-altitude system connected with the injection point. Then a unit vector $\bar{\ell}$ along the firing azimuth is given by

$$\bar{\ell} = \bar{\zeta} \cos A_z + \bar{\eta} \sin A_z .$$

Rotation into the (i_x, i_y, i_z) System gives

$$\bar{\ell} = -(\cos A_z \sin\phi \cos\alpha_L + \sin A_z \sin\alpha_L) i_x$$
$$+ (\sin A_z \cos\alpha_L - \cos A_z \sin\phi \sin\alpha_L) i_y$$
$$+ (\cos\phi \cos A_z) i_z$$

Unit vector normal to the plane of the orbit is given by $\bar{W} = \bar{r} \times \bar{\ell}$ where $\bar{r}$ is the unit vector defining the launching site.

From a dot product of W and a unit vector defining the Moon's direction at the encounter we obtain

$$\tan A_z = \frac{\sin(\alpha_L - \alpha_{☾})}{\sin\phi \left[\cos(\alpha_L - \alpha_{☾}) - \tan\delta_{☾} \operatorname{ctg}\phi\right]}$$

where α_L = Lauching site right ascension

$\alpha_{☾}$ = right ascension of the Moon

$\delta_{☾}$ = declination of the Moon

ϕ = launch site latitude

Since the n-body program employed in this search requires initial velocity components to be expressed in the geocentric equatorial system it was convenient to employ the following expressions:

$$\dot{x} = V_T \left[\sin\theta \cos\phi \cos\alpha_L - \cos\theta (\cos A_z \sin\phi \cos\alpha_L + \sin A_z \sin\alpha_L)\right]$$

$$\dot{y} = V_T \left[\sin\theta \cos\phi \sin\alpha_L + \cos\theta (\sin A_z \cos\alpha_L - \cos A_z \sin\phi \sin\alpha_L)\right]$$

$$\dot{z} = V_T \left[\sin\theta \sin\phi + \cos\theta \cos\phi \cos A_z\right]$$

where θ is the flight path angle, V_T is the total injection velocity, and ϕ is the latitude of the injection place.

Choosing now the launching site, injection point, lunar coordinates, and the trip time as specified earlier, we can plot the firing azimuth as a function of time on September 25, 1960. This plot is shown in Figure 63. (Only Eastward firings have been considered.) Note that the range in firing azimuth is quite restricted. There are two reasons for this. First of all firing on steep branches is impractical because any launching delays will require excessive re-adjustment of azimuth. Secondly most of the diagram lies outside the range safety limits of 85° to 125°.

From this point the procedure was rather straight forward. Three variables remained open to us. These were launching right ascension α_L, total velocity V_T, and the flight path angle θ. Trip time can hardly be considered a bonafide parameter.

The total velocity was fixed at a value which was found reasonable from previous studies. Then for values of $\theta = 1^{\circ}, 2^{\circ}, 3^{\circ}$ launching time was varied over the upper allowable part of the $A_z - t_L$ diagram. This is the region where the firing azimuth varies relatively slowly with launching time.

The resulting initial conditions $x_o, y_o, z_o, \dot{x}_o, \dot{y}_o, \dot{z}_o$ were employed in the n-body program. The bodies used were the Sun, Earth, Moon, and the vehicle. Note carefully that the Earth's oblateness was retained in the n-body program. Also, the initial conditions as used are based on a two body, approximation (no oblateness considered).

The very first runs indicated two problem areas. The first of these is that for the small range in θ one cannot use an arbitrary combination of A_z, t_L. In fact it appears that A_z, t_L diagram must include a grid of constant θ lines to be really useful. The situation as it exists shows that we are firing almost at right angles to the initial radius vector. The line of apsides of the resulting orbit turns almost 90° away from the intended point of encounter. One can compensate for this effect by a large flight path angle. Yet in our case this variable is restricted to a narrow range. Thus the only way to correct this problem is to employ smaller values of α_L. In our case this forced us to employ a value for $A_z = 85^{\circ}$ which is practically the limit of the allowable range.

The second problem is the fact that first runs placed the vehicle far below the moon. Part of the rotation of the line of apsides described above, and the great dip of the trajectory below the Moon's orbital plane can be attributed to the effect of Earth oblateness.

To make the z component of the vehicle in the Moon's vicinity acceptable at all, it was found necessary to aim at a declination much different from that of the actual Moon. In our case the final value was in the neighborhood of -8° degrees as compared to -18° for the actual position of the target.

This experience indicates that the two body estimates of the launching conditions have an extremely limited value and should be modified. Since the declination of the target point must be watched so closely, this quantity became in our case an additional variable.

Our approach indicated finally that a reasonable circumnavigating trajectory results for the following conditions:

$$\alpha \, \mathbb{C}_T = 279^{\circ}\, 22'\, 12''$$

$$\delta \, \mathbb{C}_T = -7^{\circ} 40''$$

$$V_T = 2.6200445 \times 10^{-4} \text{ AU/hr}$$

$$\theta = 2^{\circ}$$

$$\delta_L = 19^{\circ}\, 13'\, 22''$$

$\alpha_L = 124^\circ.00$

Transit time to the Moon approximately 75 to 85 hours and the total trip time between 7.5 to 8 days.

A limited error analysis of this trajectory was made over the following range of input parameters:

$$123^\circ.4 \leq \alpha_L \leq 124^\circ.00$$

$$-8^\circ 19' 52'' \leq \delta_{☾_T} \leq -7^\circ 19' 52''$$

$$2.6200226 \times 10^{-4} \leq V_T \leq 2.6200666 \times 10^{-4}$$

$$1^\circ \leq \theta \leq 3^\circ$$

One of these trajectories is shown in Figures 64, 65 and 66.

The resulting variation of distances of closest approach to the Moon and on return to the Earth are shown in Figures 67, 68, 69 and 70.

It can be seen that r_M and r_E as functions of $\delta_{☾_T}$ reach a minimum in the vicinity of $-7^\circ 40'$. However, these minimum values do not satify the original specification. Although the closest approach to the Moon is satisfactory and meets requirements, the value of r_E is entirely too large. At any rate $\delta_{☾_T}$ can no longer be used to improve either distance.

The variation of these two distances with θ exhibits a similar trend. Thus r_E reaches a certain minimum which is contrary to specifications from the start. Not much improvement can be hoped for using this variable.

The next variable, the velocity V_T, causes changes in r_E and r_M in opposite directions. The point of intersection does not satisfy specifications. A limited hope of improving these two distances lies in the fact that slopes of the two functions are quite different. Thus at the cost of a slight degradation of one distance, one may obtain a very substantial improvement in the other. No attempt was made to investigate this possibility.

Sensitivity of r_E and r_M to changes in the right ascension of the launching point α_L are shown in Figure 70. Its nature is exactly the same as that of the previous plot.

It appears then that for this trajectory we have reached near optimum conditions. Despite this fact the distance of closest approach on return to the Earth is entirely unsatisfactory. It must be noted that the above discussion ignored the question of transfer of angular momentum during scattering of the vehicle by the Moon. As pointed out by Egorov and Sedov in their papers, this is a parameter which is quite important in determining characteristics of the return leg of the trajectory. In fact specification B. 2 is a direct consequence of this consideration.

Results of this study can be summarized as follows:

1. Initial conditions established on the basis of a two body problem were found highly unsatisfactory when used in a more realistic model of the Earth-Moon System. One of the causes appears to be the Earth oblateness and the restricted range available for flight path angles θ.

2. A "figure 8" trajectory was found which satisfies nearly all engineering restrictions at launch.

   The distance of closest approach at the Moon is satisfactory.

   The distance of closest approach to the Earth is unsatisfactory, being in the neighborhood of 10000 miles and occasionally even higher than this value. The return, however, does occur in the northern hemisphere in direct motion. The problem of proper recovery site was not studied for obvious reasons.

The trip time achieved is satisfactory.

3. The limited error analysis indicates that the above trajectory is nearly optimum as far as the distances of closest approach are concerned. Thus, little hope exists for any further improvement.

4. It must be remarked that the above was a direct ascent trajectory. It is conceivable that employment of a coasting arc before final injection will result in better circumnavigation as well as recovery distances. Without question, the coasting arc will largely eliminate constraints associated with the launch on time problem and facilitate solution of the geometrical problem.

With regard to trajectories of this type Figure 71 shows the differences between considering a two body plus oblateness, three body, a four body, and a four body plus oblatness effect on close approaches to the Moon. Since the latter three cannot be handled analytically the only hope of getting a reasonable approximation for A_z as a function of t rests in modifying the two body method as outlined by an oblatness term. No attempt was made to do this on this contract.

VIII. EPHEMERIS COMPUTATION - PALLAS AND VESTA

One of the tasks which the n-body interplanetary trajectory computer program is capable of performing is the compilation of ephemerides. To test this facet of the program, it was decided to reproduce part of the orbit for two of the better observed asteroids the coordinates of which are tabulated in the American Ephemeris and Nautical Almanac. Observational inaccuracies in the ephemerides of artificial earth satellites together with atmospheric drag effects precluded their use. Furthermore, they would only serve as a check on the near earth accuracy of the program, the perturbative forces of the planets being of little or no consequence.

The ephemerides for Pallas and Vesta, as given in the almanac, are tabulated for each day and represent smoothed values for which the integrations were adjusted along the entire orbit. A discussion of the methods employed, intervals selected, etc. can be found in Vol. XI, Part IV of the Astronomical Papers. The smoothing technique employed greatly improves the accuracy of the ephemeris. Individual errors in the order of 5 seconds of arc between the computed and observed values of right ascension and declination is indicative of the accuracy of the observational data for which the ephemerides are compiled. Hence, only six decimal places are printed out in the Almanac. One unit in the last decimal place corresponding to 1×10^{-6} A. U. throughout.

Our purpose here is to select a small segment of observed data and to predict the future positions of the body. This represents the case most useful to artificial satellite ephemeris compilations in which the future position of the body is of interest. This will be compared with runs obtained by selecting data over a larger segment of the orbit indicating the improvement to be expected. The latter will indicate the need for a time history (past history of the satellite's position) of some extent if future position type ephemerides are to be accurately obtained.

For our computations we have used Pallas and Vesta in the combined force field of the Sun, Venus, Earth, Mars, Jupiter and Saturn. Selected data on these two asteroids is given in Table 1.

TABLE I

	Radius (Km)	Volume (cm^3)	Mass (grams)	Sidereal Period (days)	a (A. U.)	e	i (degrees)
Pallas	240	6×10^{22}	20×10^{22}	1684	2.767	.235	34.8
Vesta	190	30×10^{21}	10×10^{22}	1325	2.361	.088	7.1

It should be noted that they represent both extremes with regard to orbital eccentricity and inclination that is found among the brighter asteroids.

Selecting ten tabular geocentric positions from the almanac, a numerical differentiation scheme was employed to obtain the velocity vector at one of the tabular values and hence a complete set of initial conditions for the n-body trajectory program was obtained. The data represented a period of time of ten days. A run was made using these conditions. Then a linear differential correction scheme for each component of the velocity vector was applied to improve the original estimates of the velocity components. This was accomplished by employing the tabular position data at three day intervals but now extended over a period of sixty days. A set of normal equations were obtained the solution of which yielded the desired corrections. This entire procedure was again repeated at three day intervals but with the period extended to one hundred and fifty days for Vesta, and with six day intervals over one hundred and fifty days for Pallas.

The results of these computations are shown in Figure 72 for Pallas and 73 for Vesta. As may be seen, the ephemerides computed from the short time history soon indicate large errors. While the residuals are small over the fitted portion of each orbit they soon build up to substantial errors further along in time. We can conclude that in compiling ephemerides of this type a long time history for the object under study is a prime requisite. This would allow the entire orbit to be fitted at one time a procedure used in astronomy. This does not necessarily mean that a great number of data points are required but rather that they be obtained at selected intervals over the entire orbit. Intervals in the order of 20 to 40 days are common in astronomical practice. In the case of Pallas and Vesta, whose periods are 4.6 and 3.6 years respectively, the 20 day interval necessitates the use of some 84 and 66 points respectively.

An additional effect that can be noticed in Figures 72 and 73 is a slight oscillation in the residuals. A satisfactory explanation of their cause has not as yet been determined. Generally speaking, the amplitudes of the oscillations are slightly less for Pallas than for Vesta.

References:

1. Paul Herget, "The Computation of Orbits," published privately by the author, 1948.

2. F.R. Moulton, "An Introduction to Celestial Mechanics," MacMillan Co., New York, 1914.

3. J.B. Scarborough, "Numerical Mathematical Analysis," The Johns Hopkins Press, 1955, p. 164.

4. W.M. Smart, "Textbook on Spherical Astronomy," Cambridge Univ. Press, 1956, p. 195.

5. "Planetary Coordinates for the Years 1960-1980," H.M. Nautical Almanac Office, London, 1958.

The reader interested in detailed aspects of practical astronomy can find a wealth of material in the following references:

6. Simon Newcomb, "A Compendium of Spherical Astronomy," Dover, 1960.

7. H.C. Plummer, "An Introductory Treatise on Dynamical Astronomy," Dover, 1960.

8. W. Chauvenet, "A Manual of Spherical and Practical Astronomy," Dover, 1960.

9. W.M. Smart, "Celestial Mechanics," Longmans, 1953.

10. Egorov, V.A., Certain Problems of Moon Flight Dynamics, USPEKHI FIZICHESKIKH NAUK, Vol. 63, 1957.

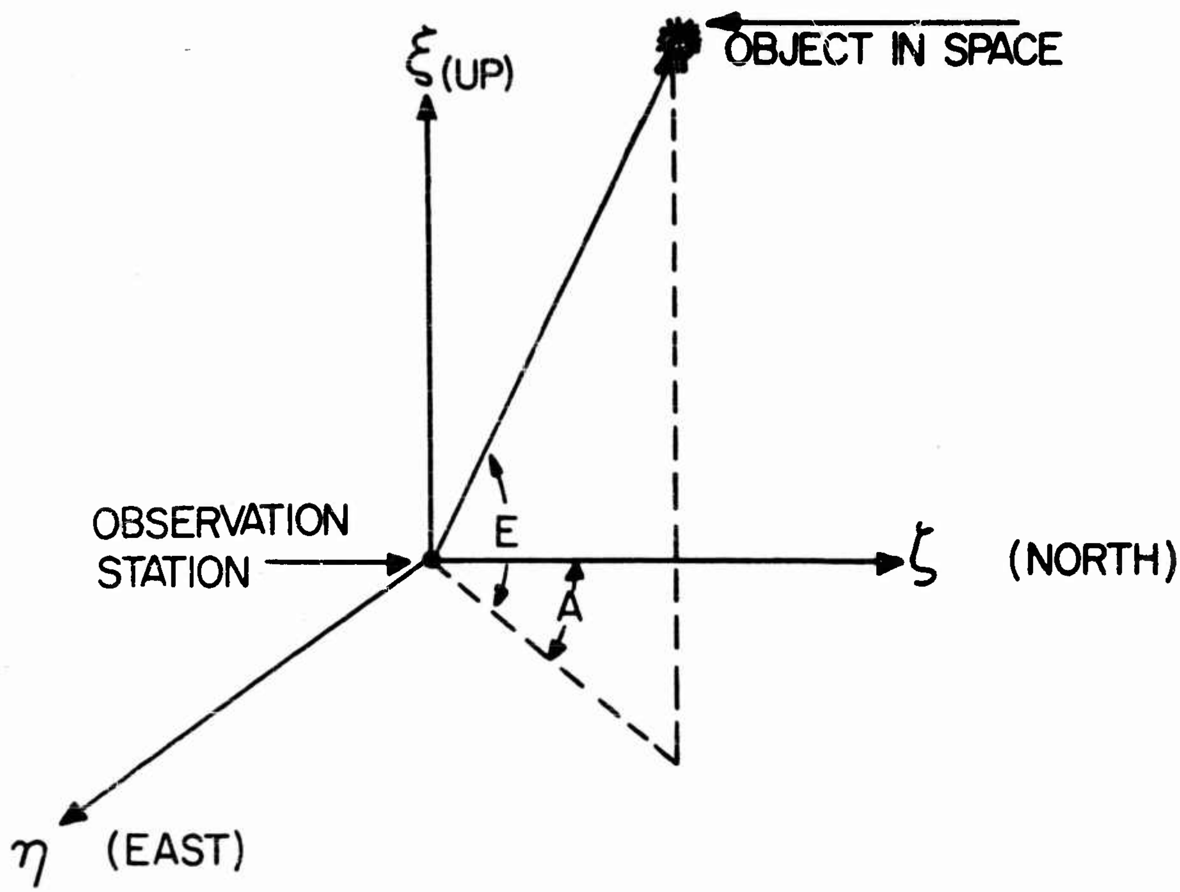


Figure 2. Topocentric Coordinate System

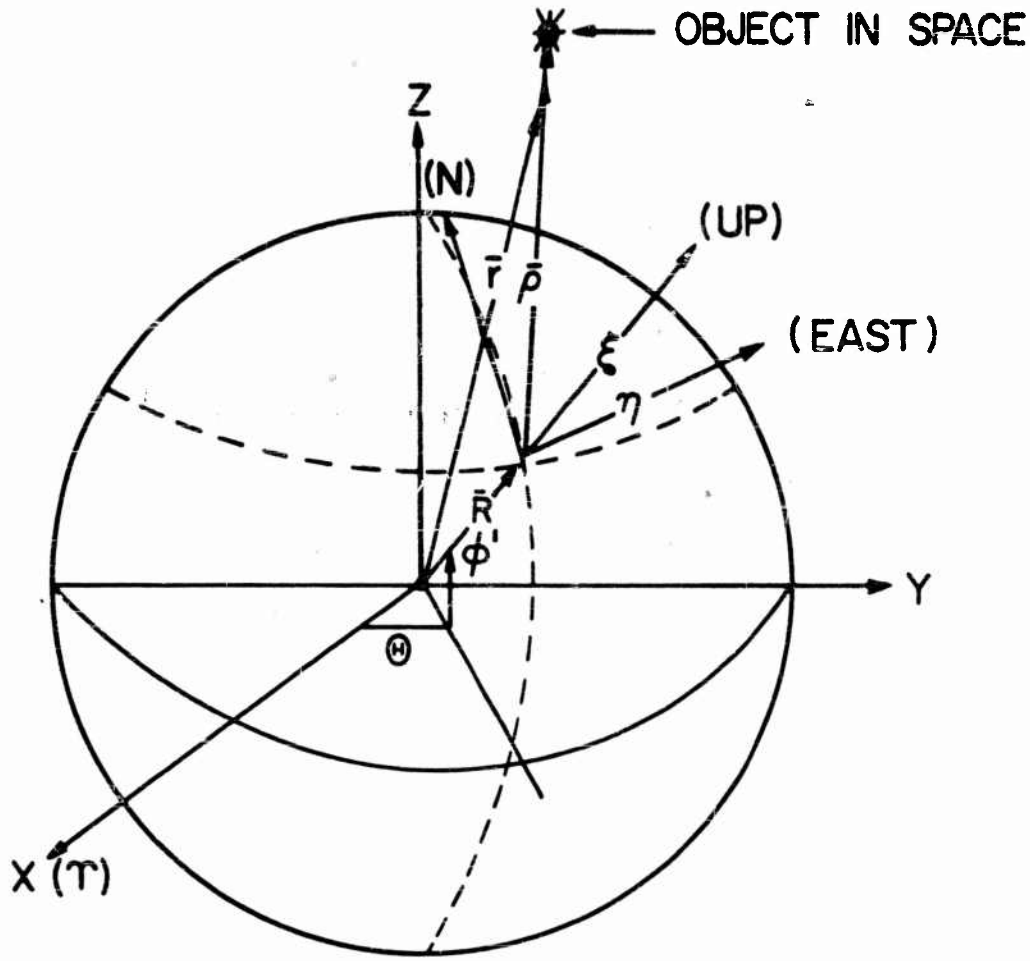


Figure 3. Geocentric and Topocentric Coordinate System

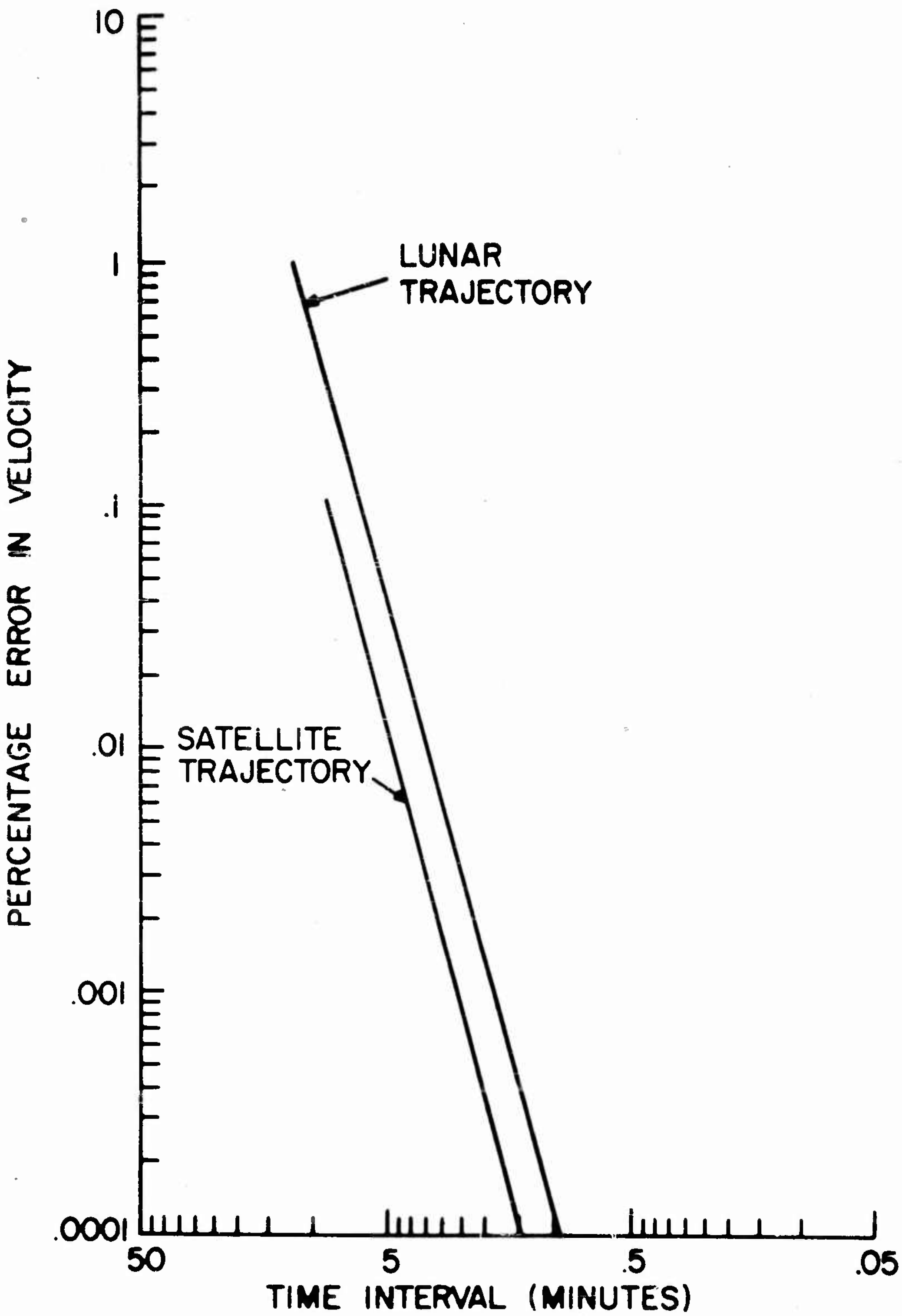


Figure 4. Percentage Error in Velocity as a Function of the Time Interval Between Positions

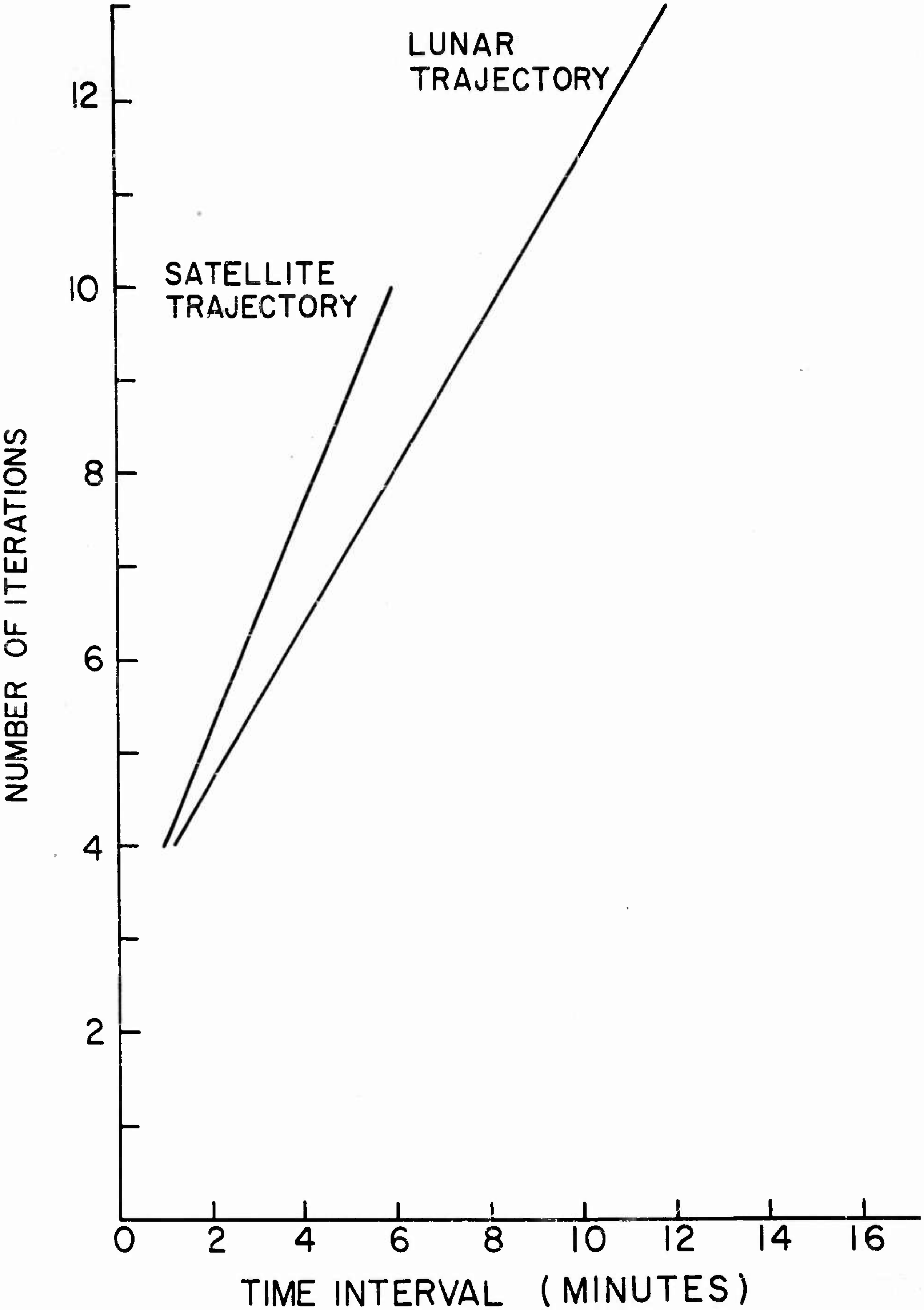


Figure 5. Number of Iterations Required to Reach a Solution as a Function of the Time Interval

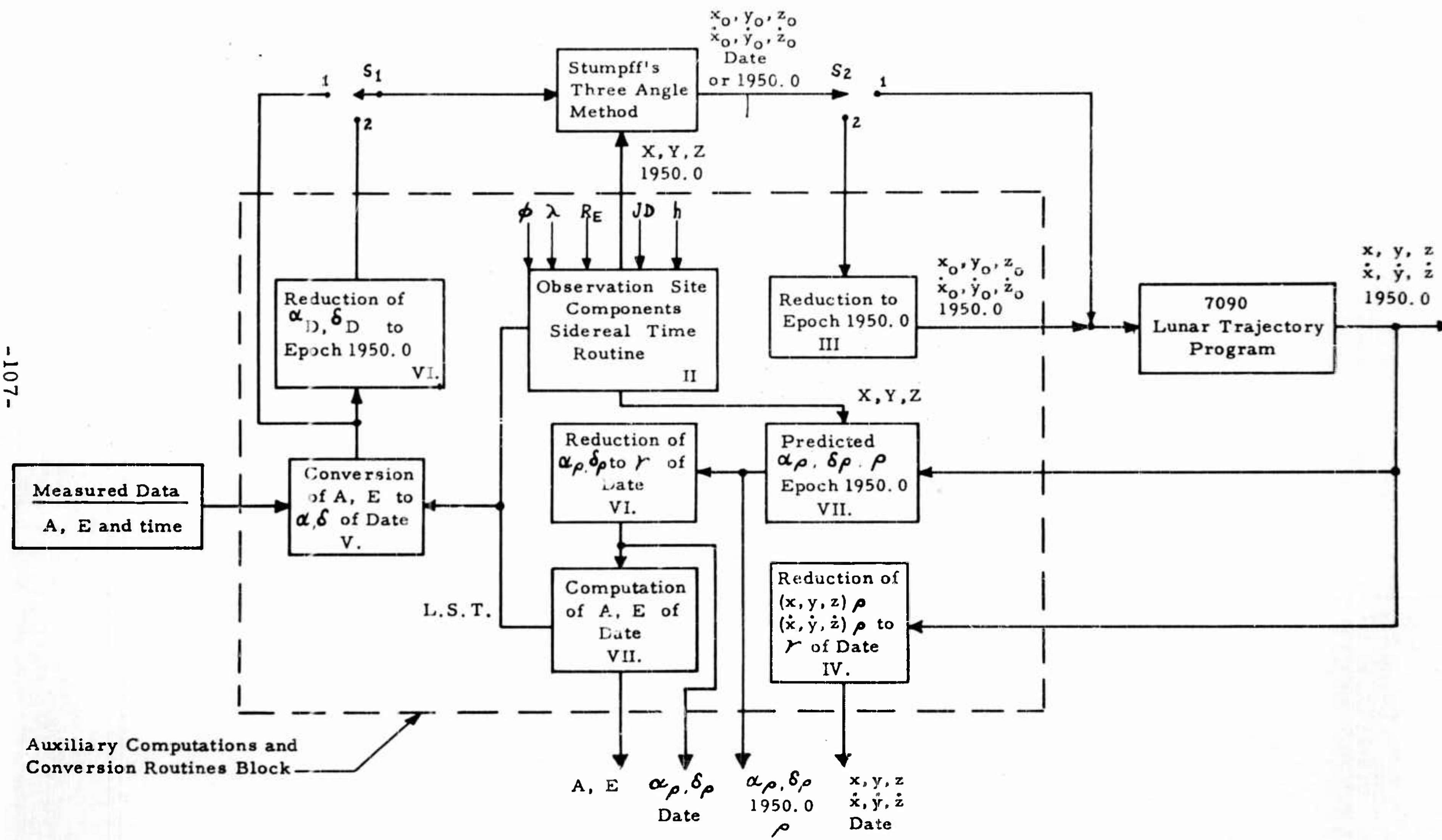


Figure 6. Schematic No. 1

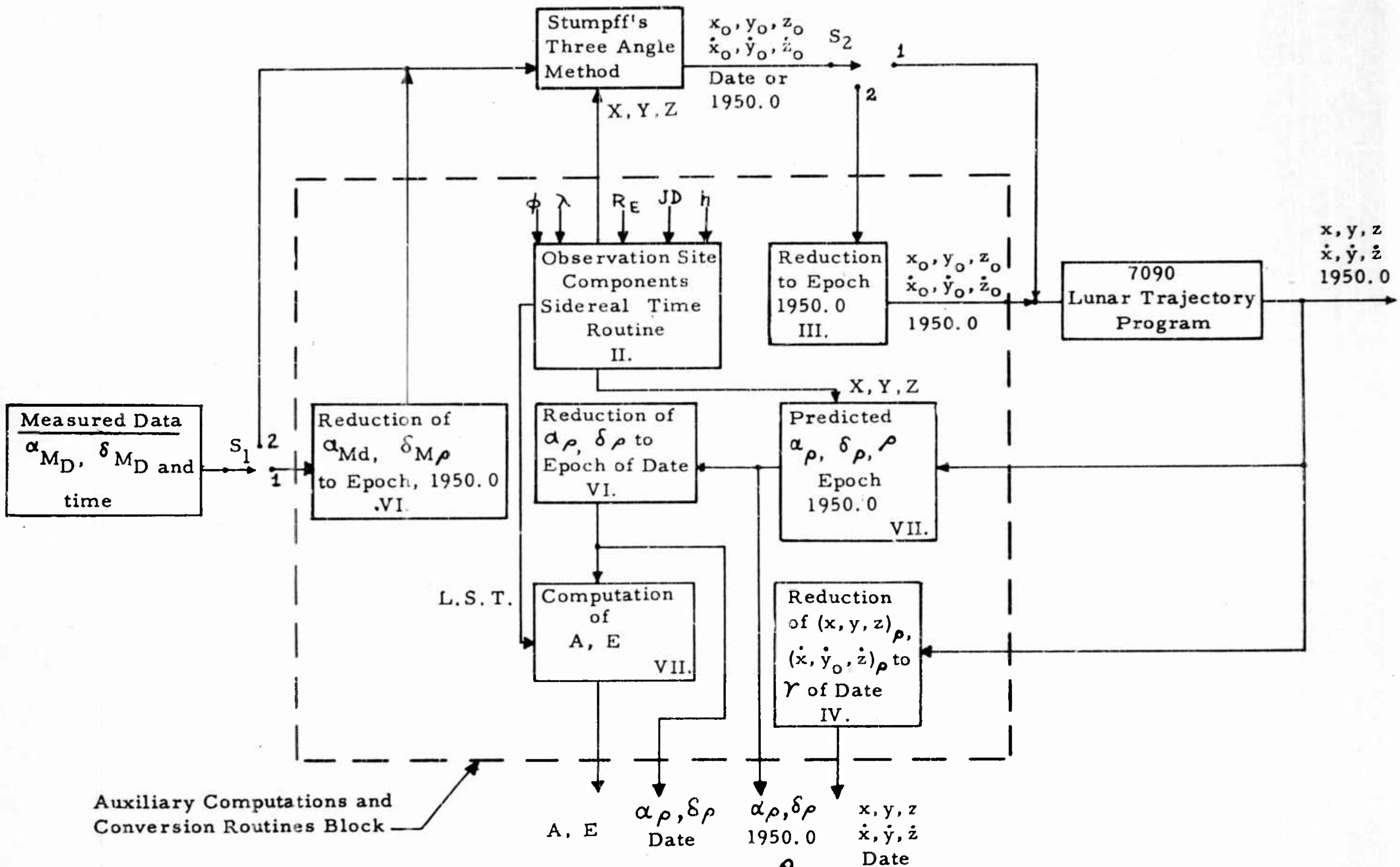


Figure 7. Schematic No. 2

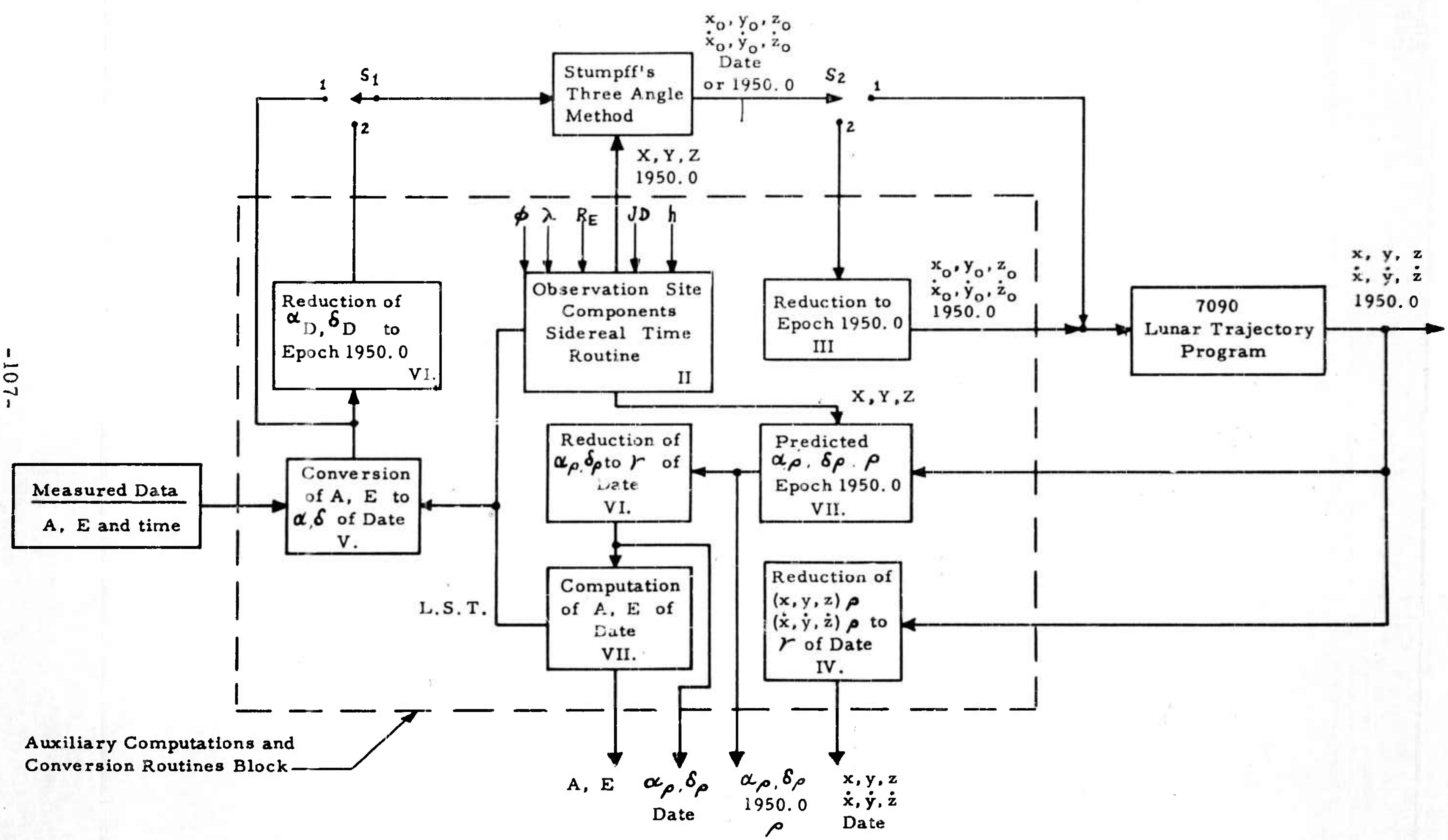


Figure 6. Schematic No. 1

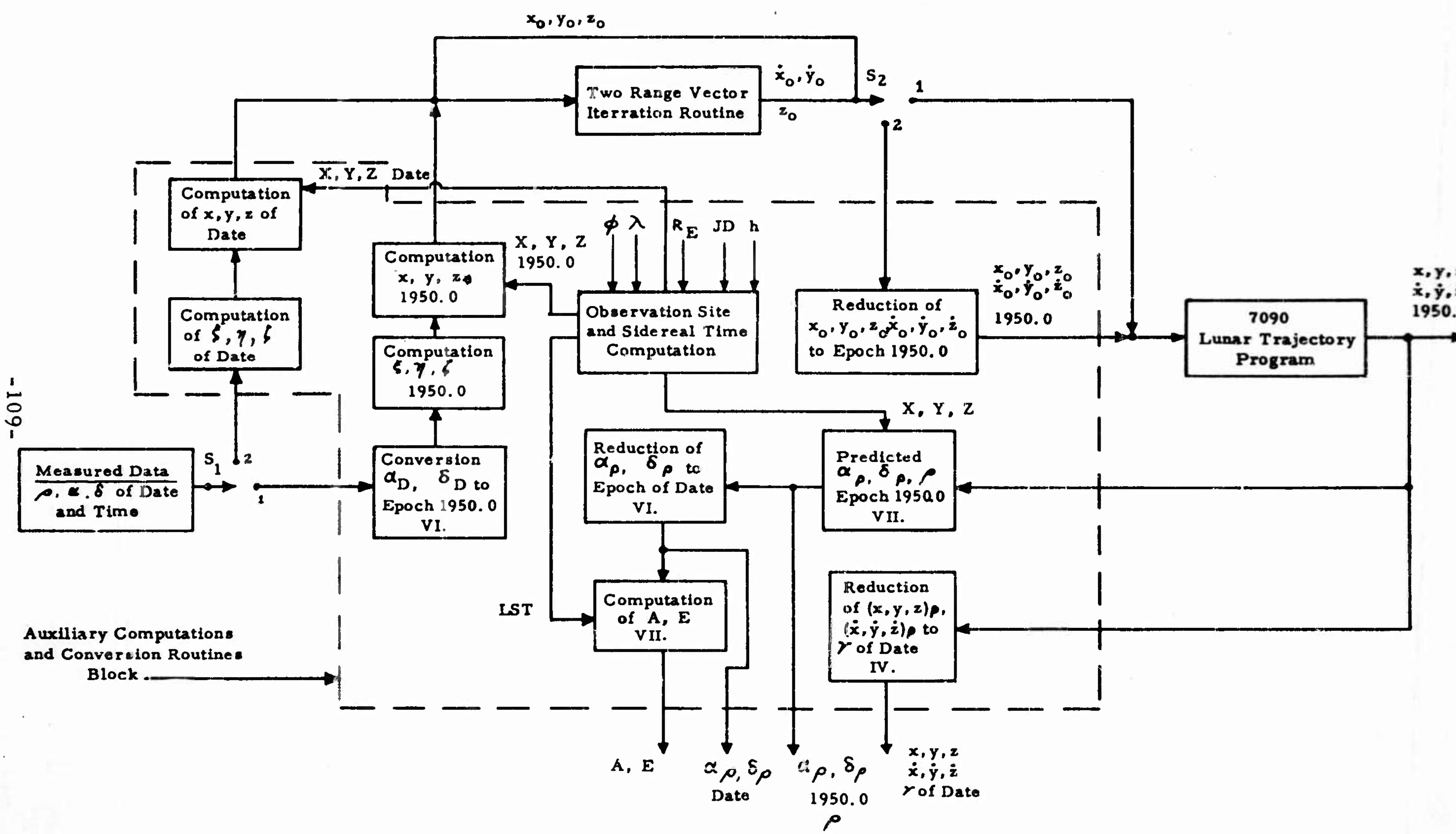


Figure 8. Schematic No. 3

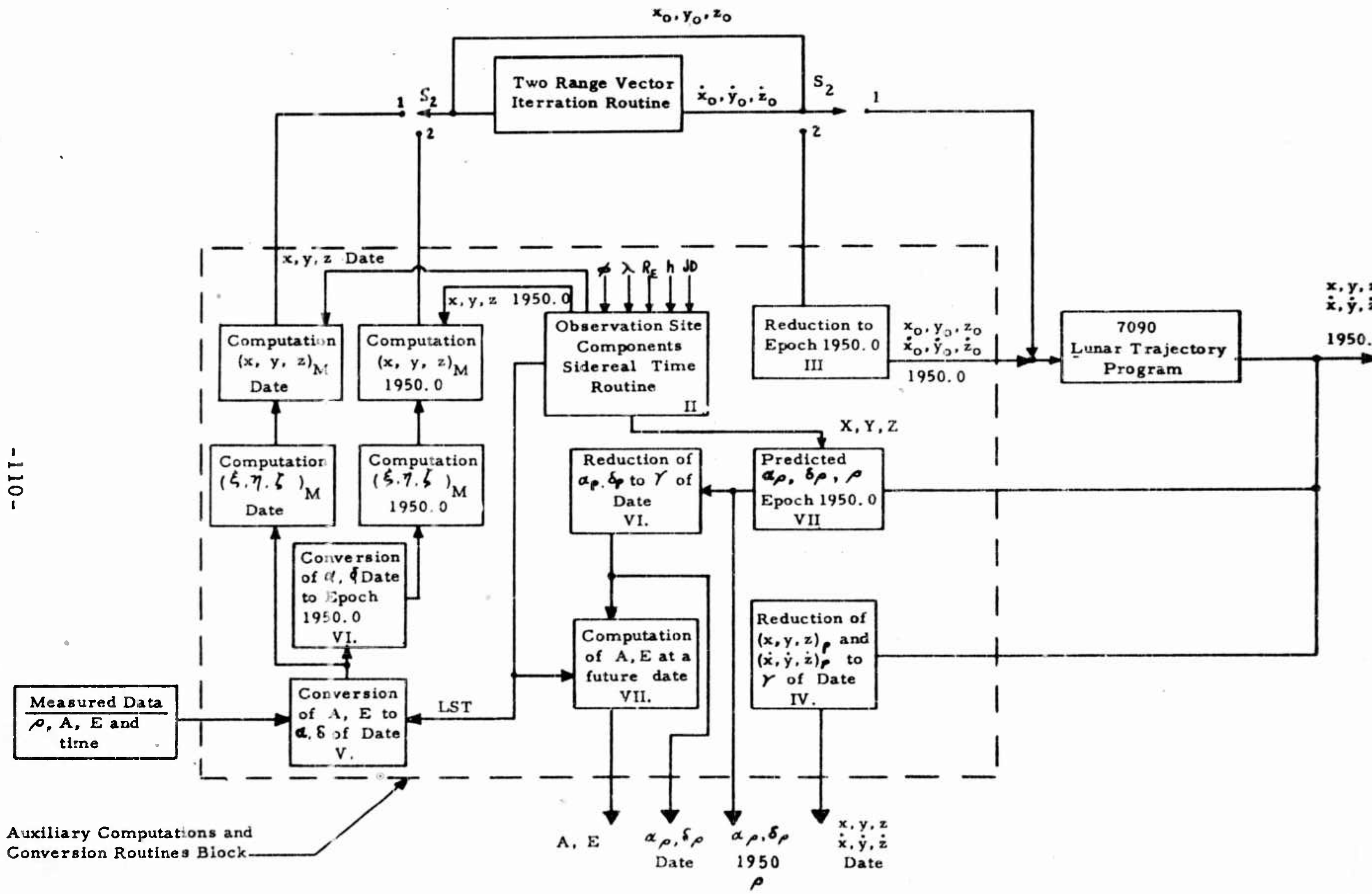
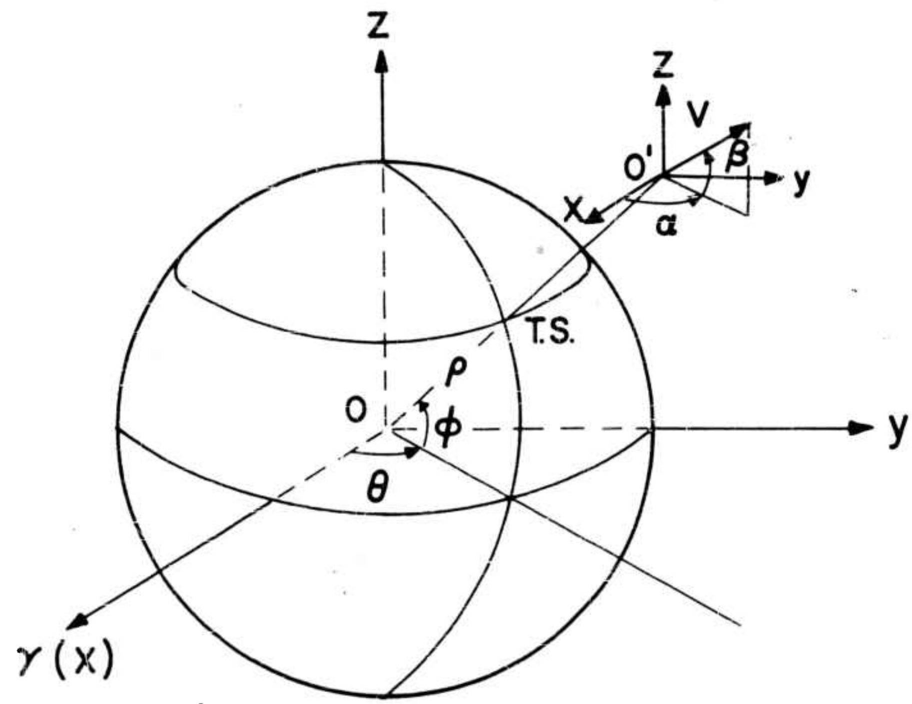


Figure 9. Schematic No. 4



$\rho = \overline{OO'}$

O - CENTER OF THE EARTH

T.S. - TRACKING SITE

T.S. - O' = h = BURNOUT ALTITUDE

Figure 10. Coordinate System Used in Error Analysis

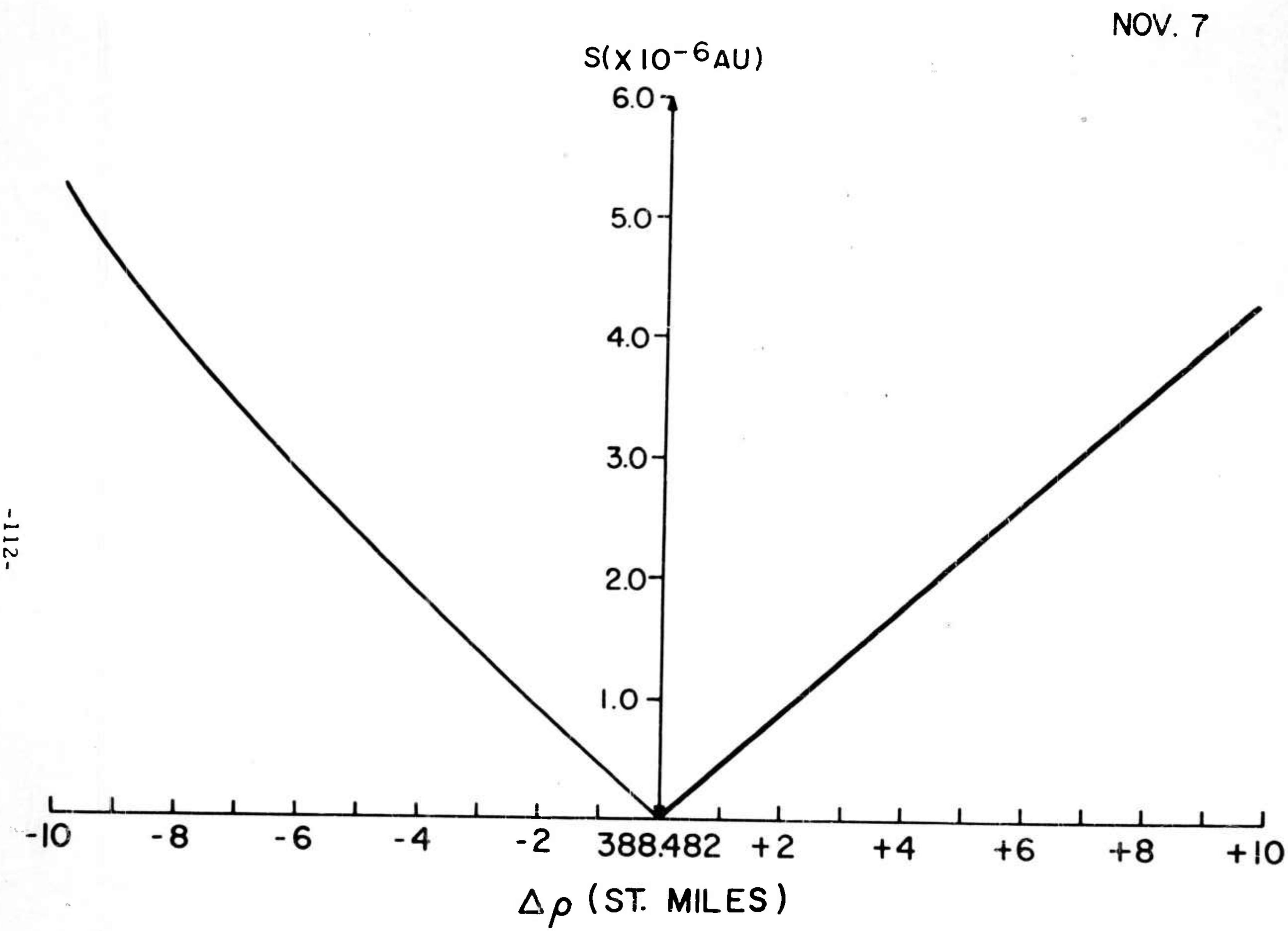


Figure 11. Miss Distance as a Function of Error in Initial Position

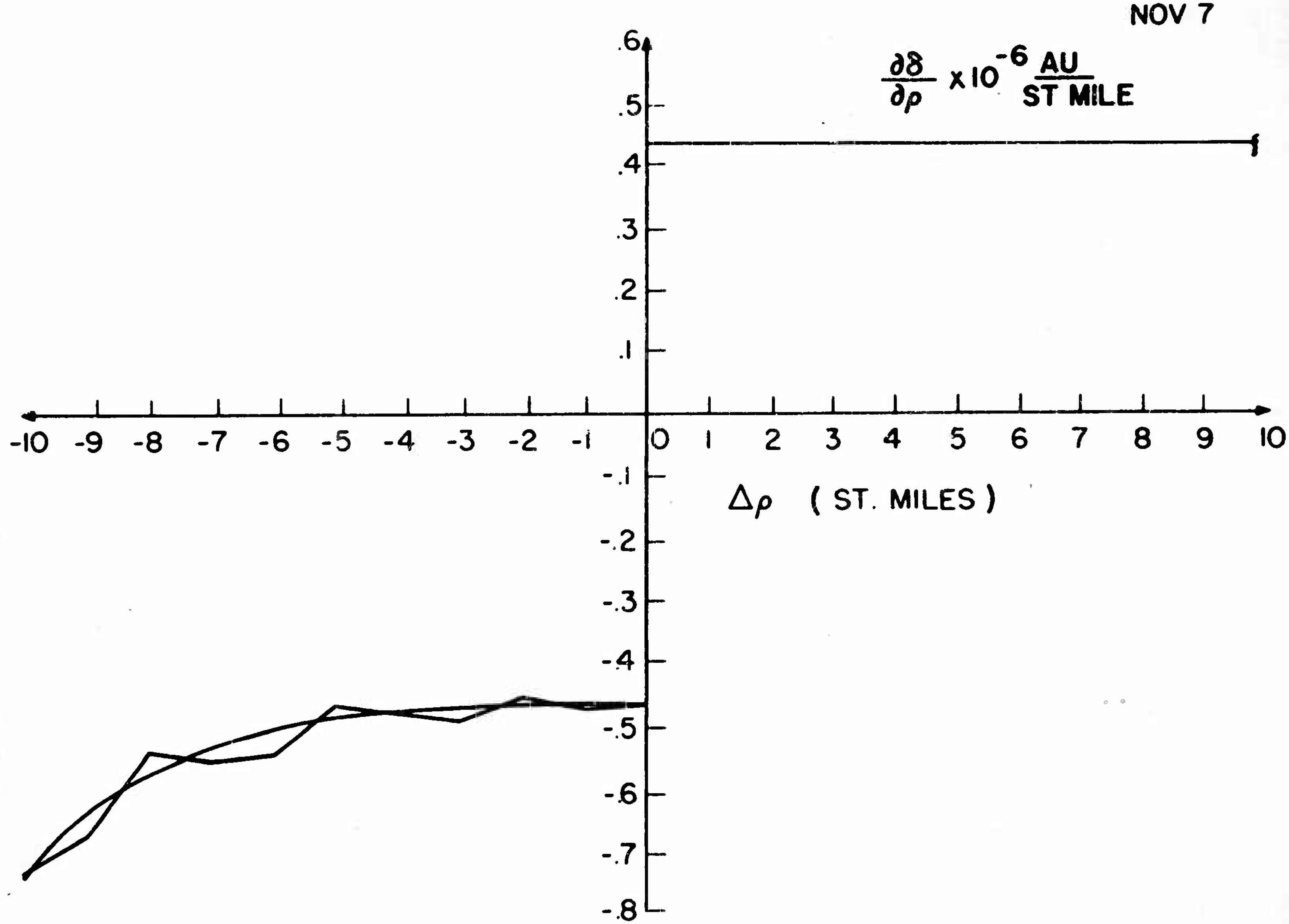


Figure 12. Error Coefficient as a Function of Error in Initial Position

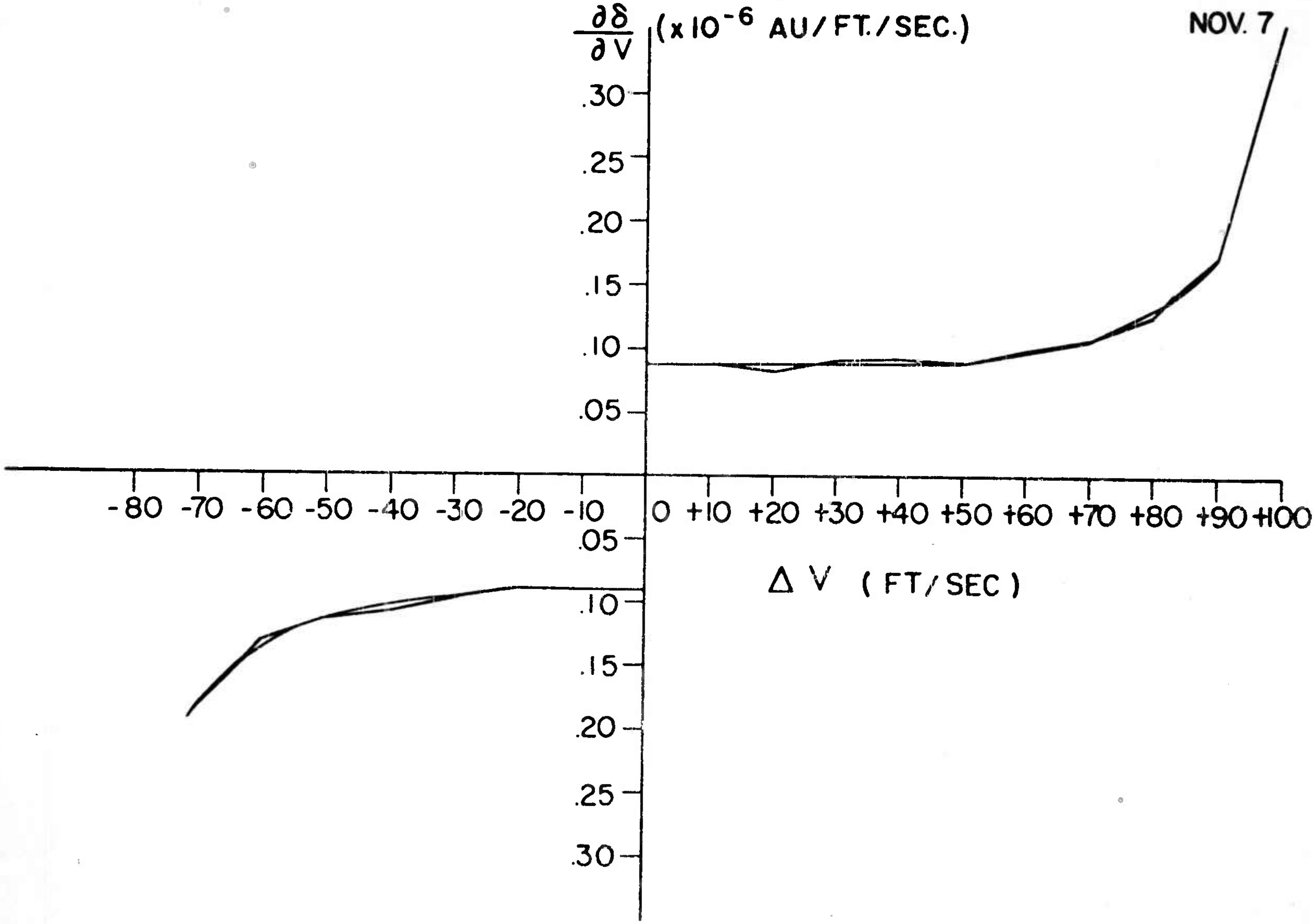


Figure 13. Error Coefficient as a Function of Error in Initial Velocity

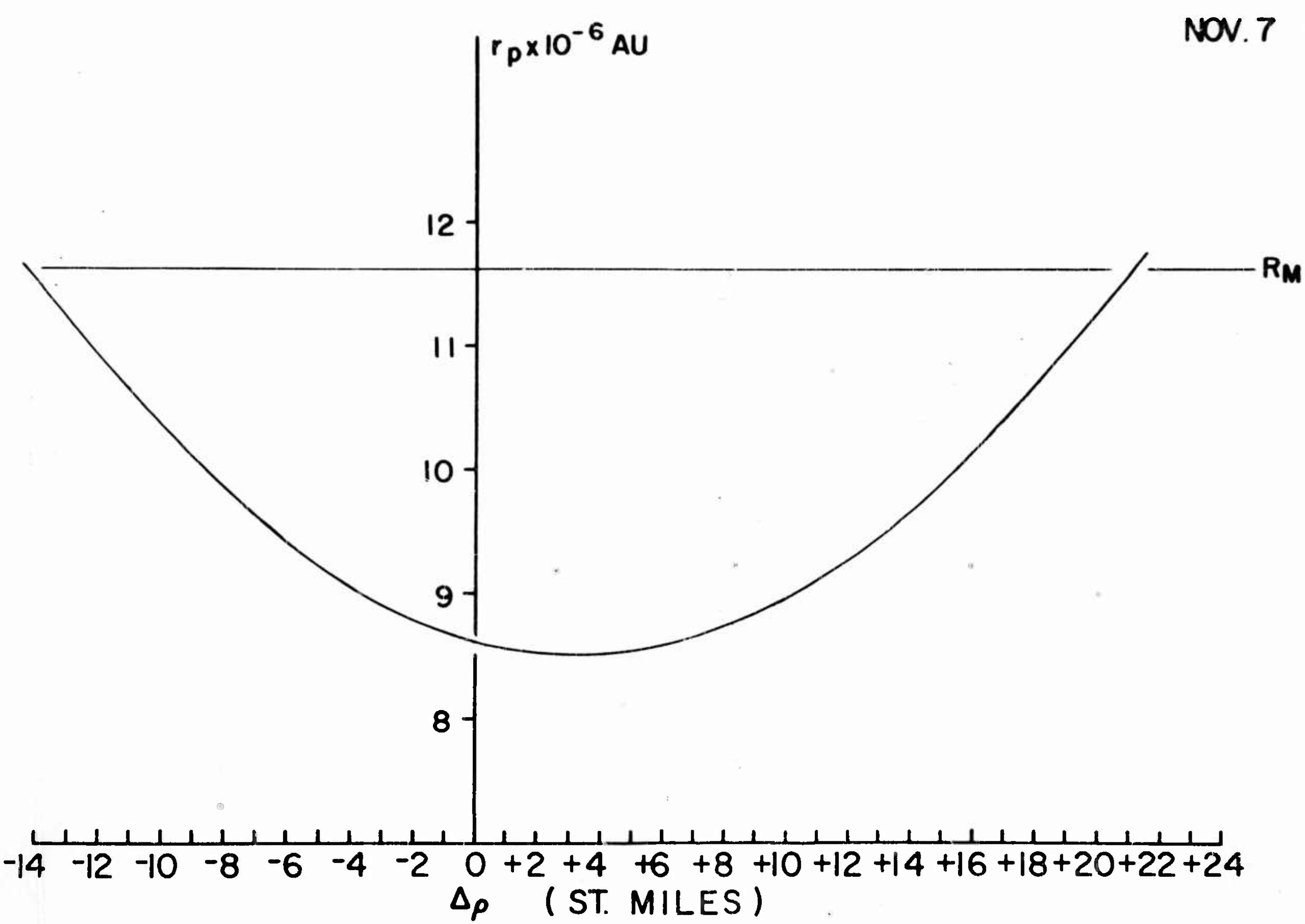


Figure 14. Distance of Closest Approach as a Function of Initial Position $|\rho|$

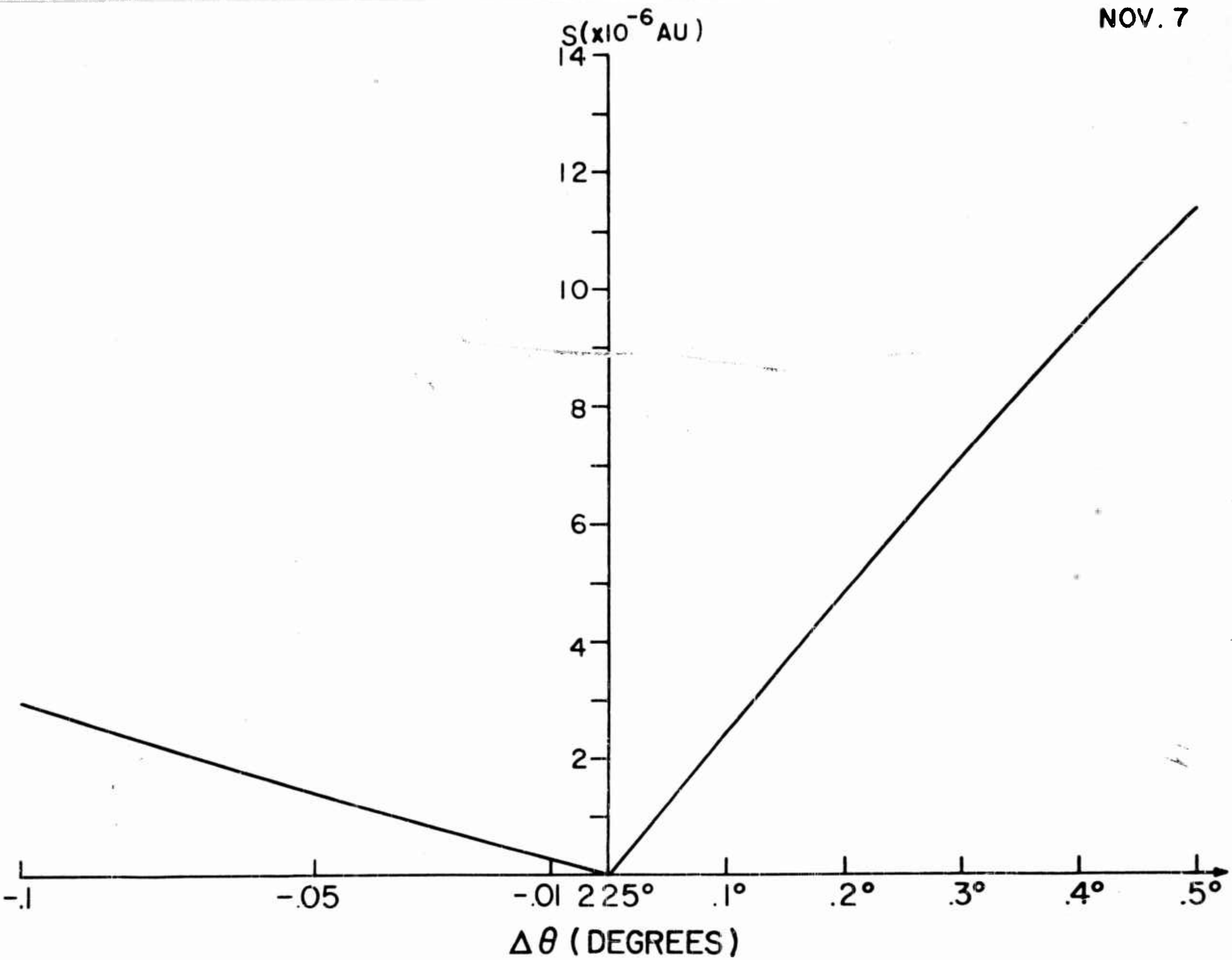


Figure 15. Miss Distance as a Function of Error in Position Angle θ

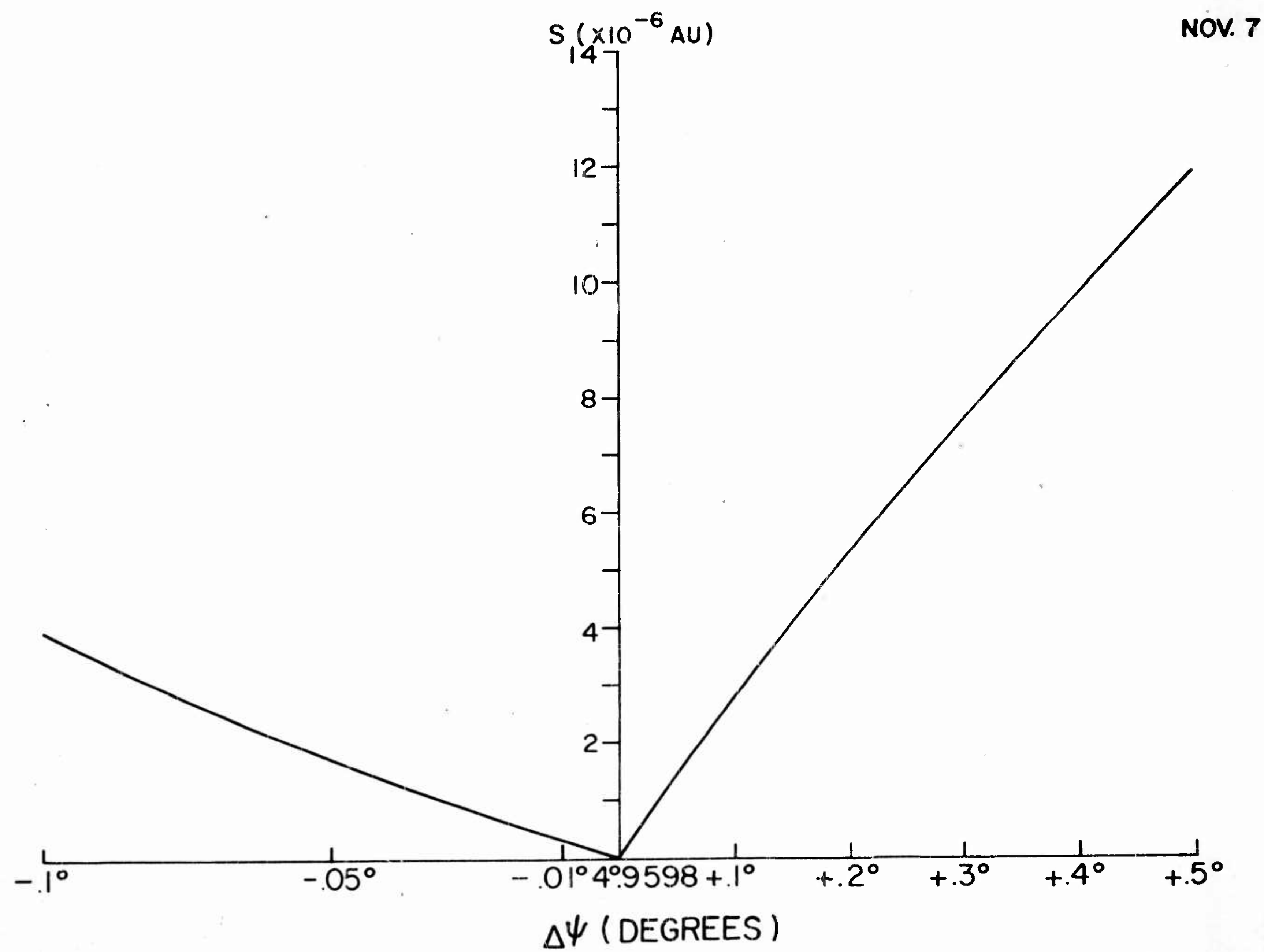


Figure 16. Miss Distance as a Function of Error in Position Angle ψ

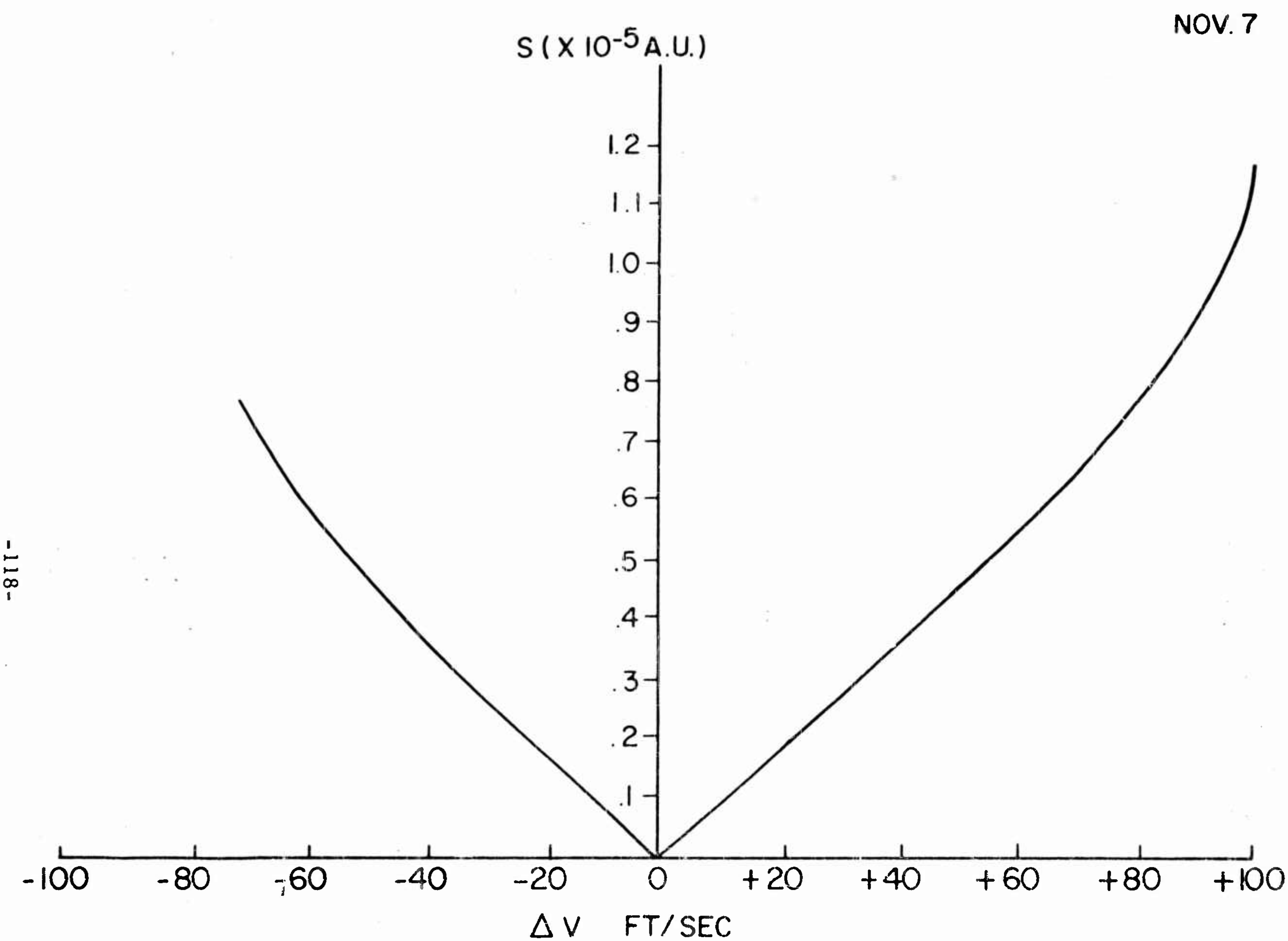


Figure 17. Miss Distance as a Function of Error in Initial Velocity

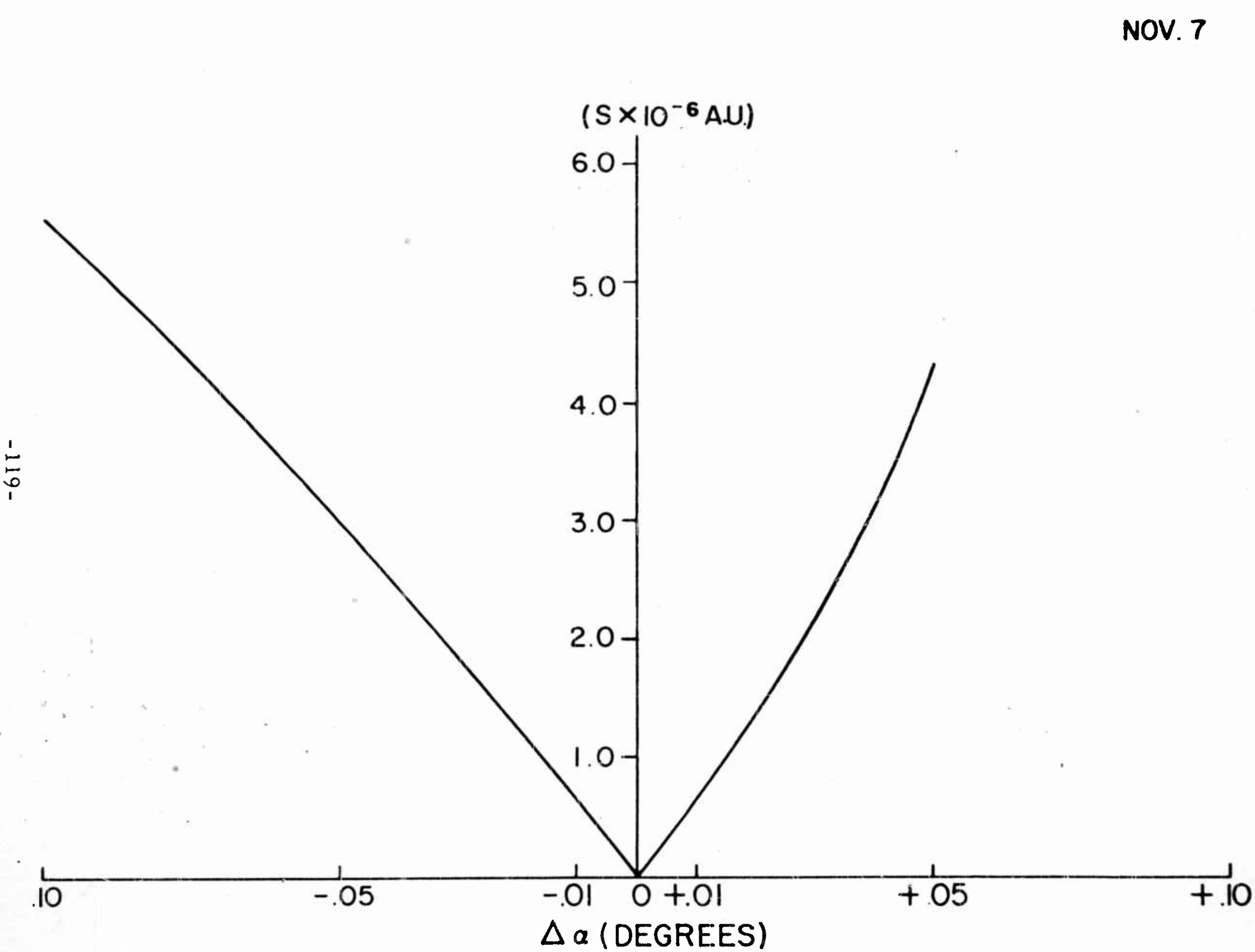


Figure 18. Miss Distance as a Function of Error in Velocity Angle α

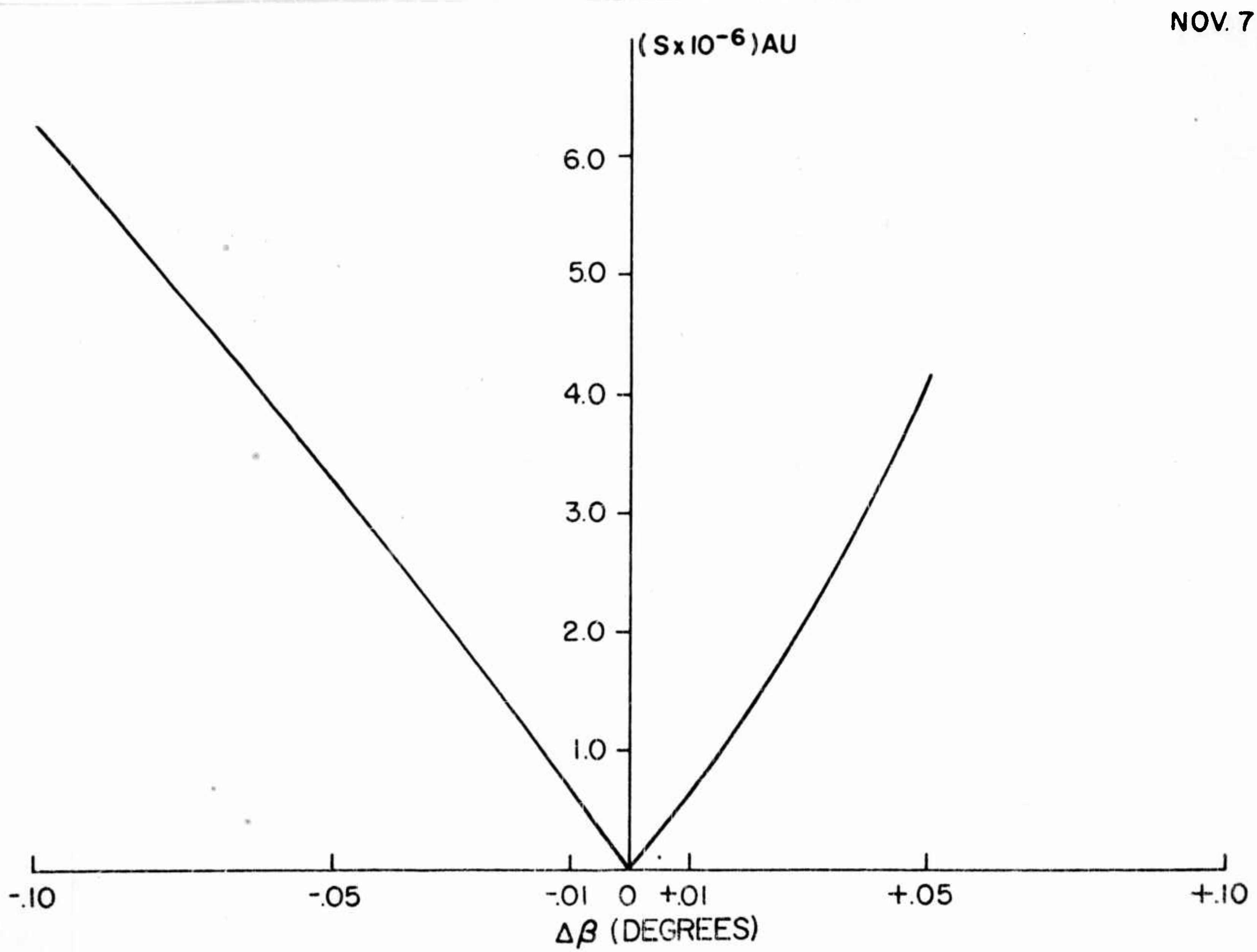


Figure 19. Miss Distance as a Function of Error in Velocity Angle β

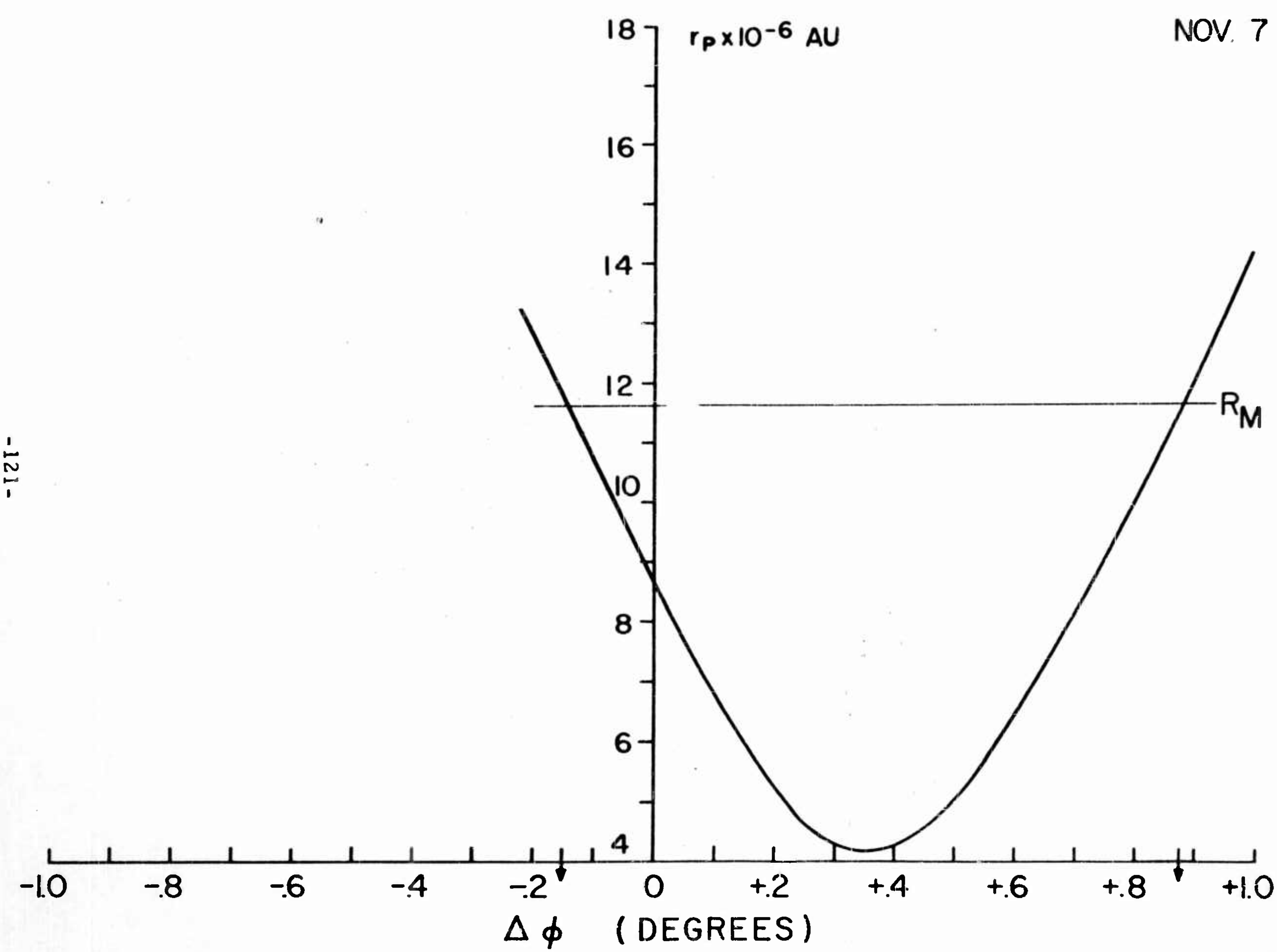


Figure 20. Distance of Closest Approach as a Function of Position Angle Φ

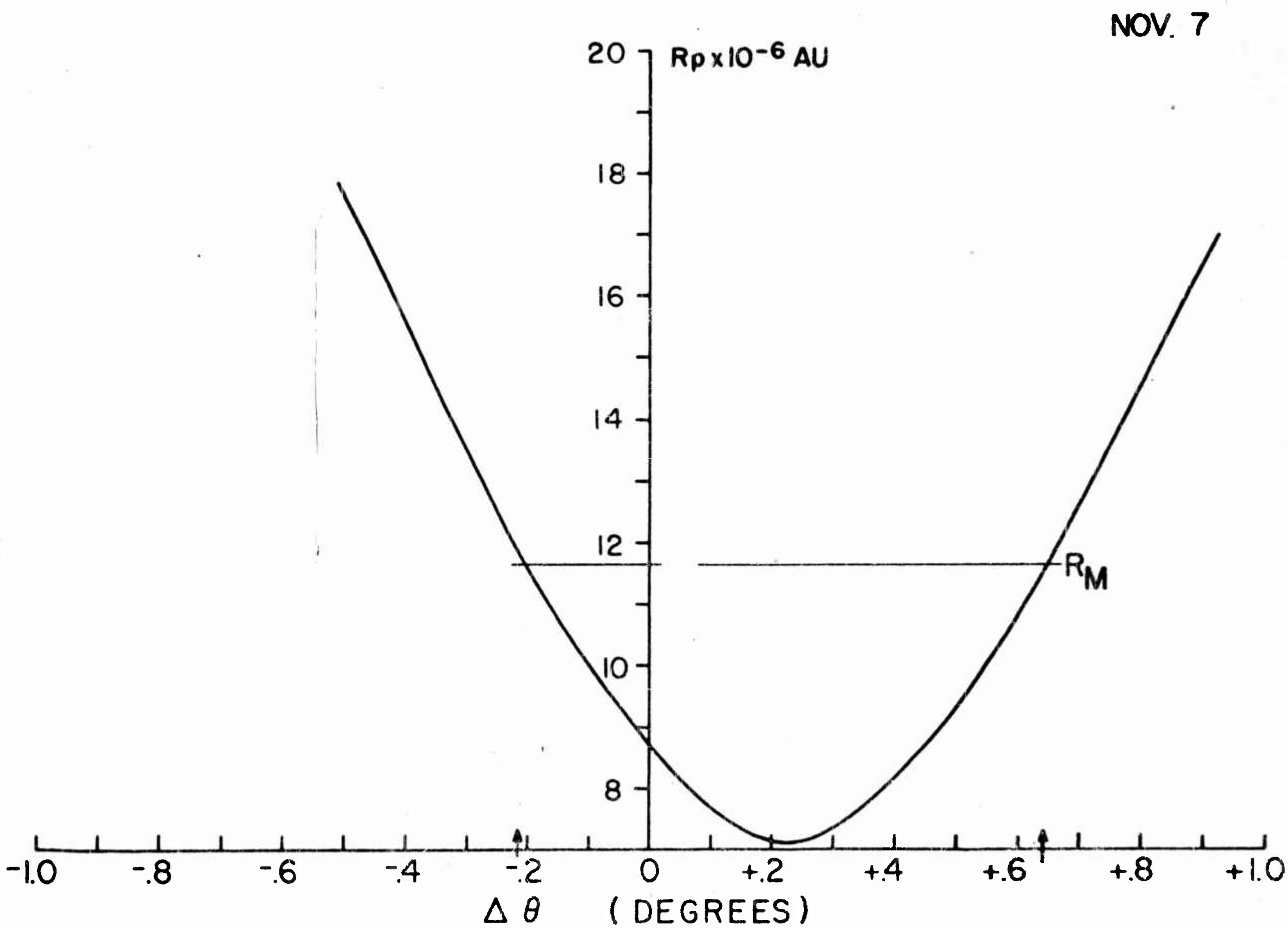


Figure 21. Distance of Closest Approach as a Function of Position Angle θ

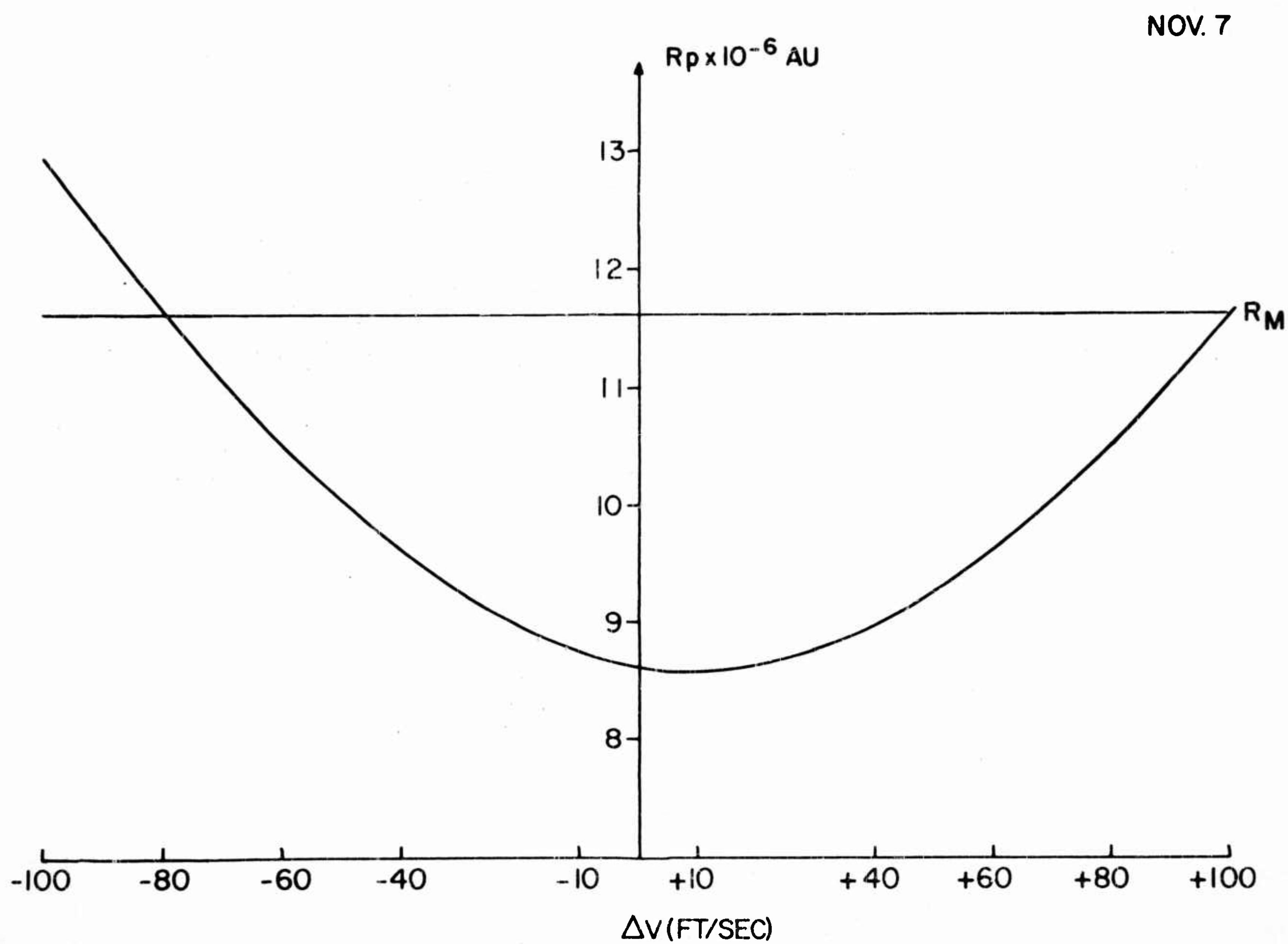


Figure 22. Distance of Closest Approach as a Function of Initial Velocity V

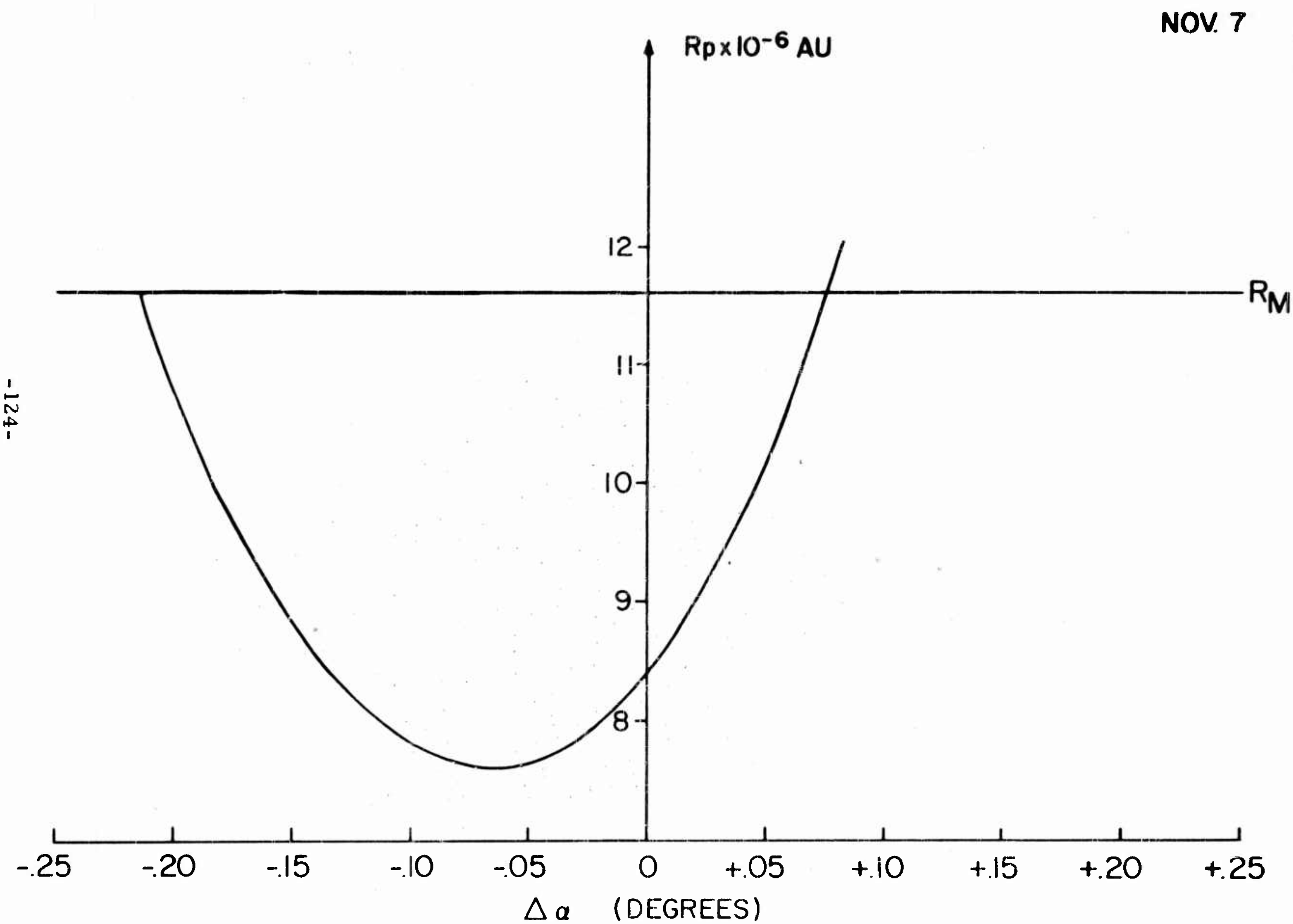


Figure 23. Distance of Closest Approach as Function of Velocity Angle α

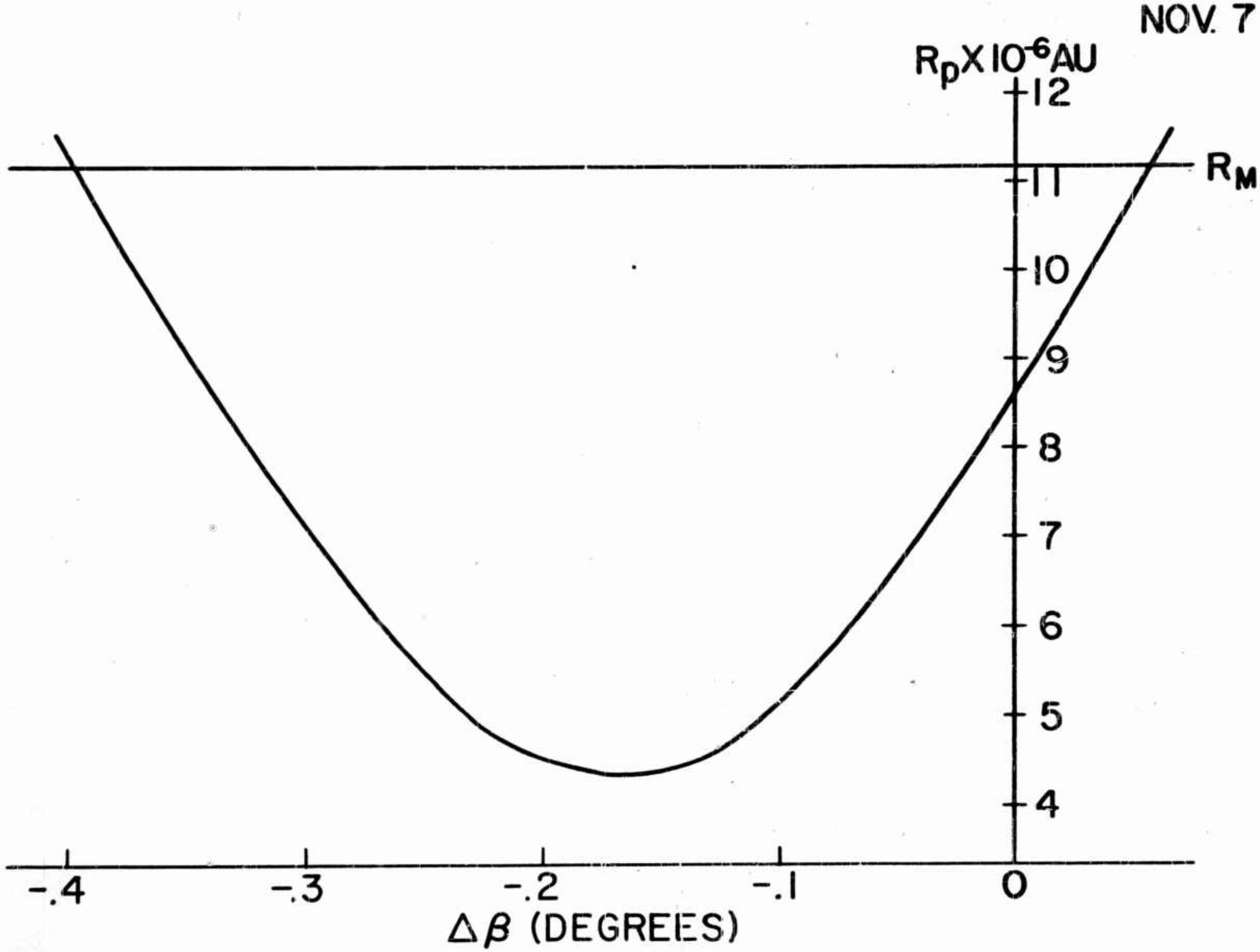


Figure 24. Distance of Closest Approach as a Function of Error in Velocity Angle β

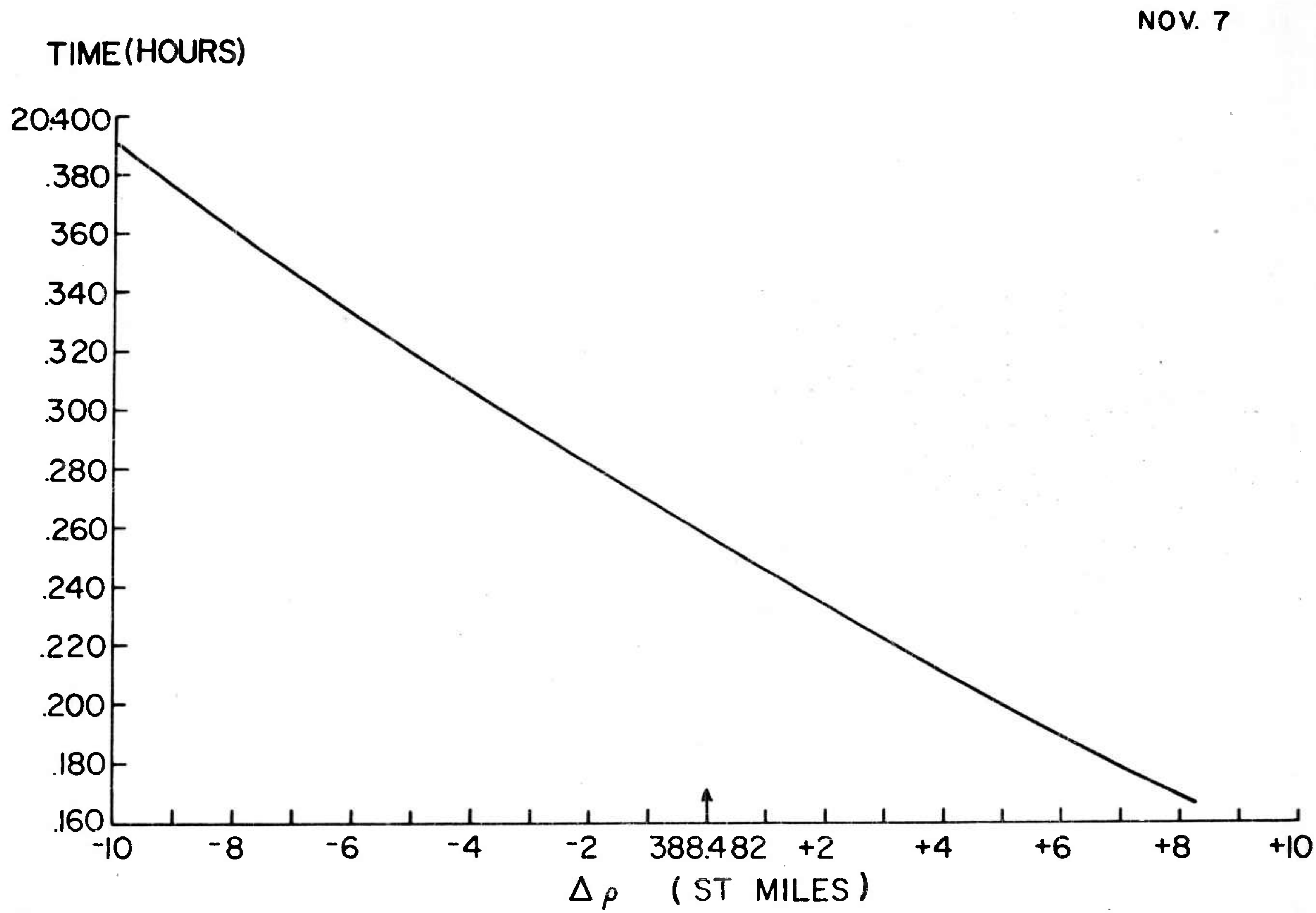


Figure 25. Flight Time as a Function of Error in Initial Position $|\rho|$

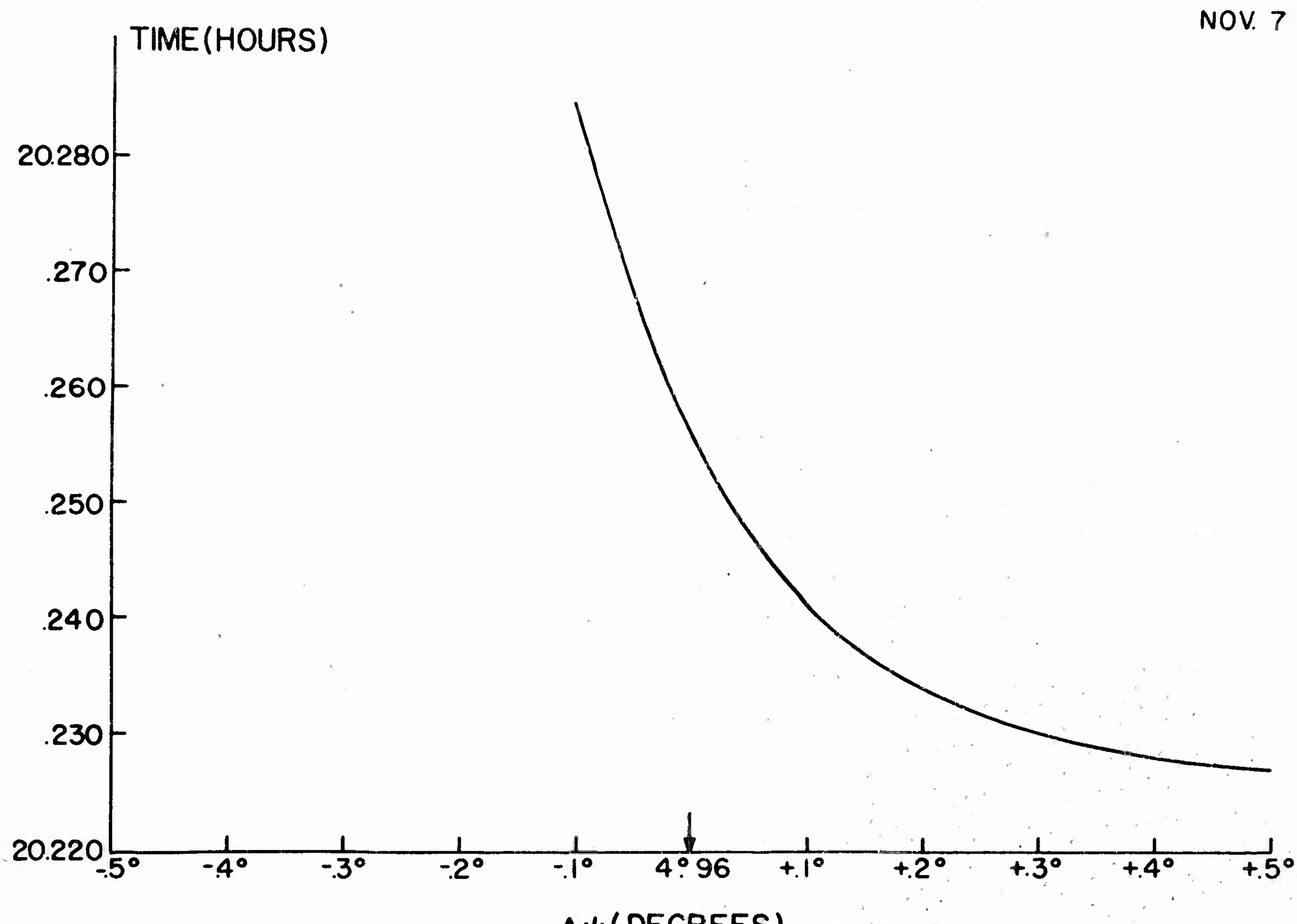


Figure 26. Error in Position Angle ψ

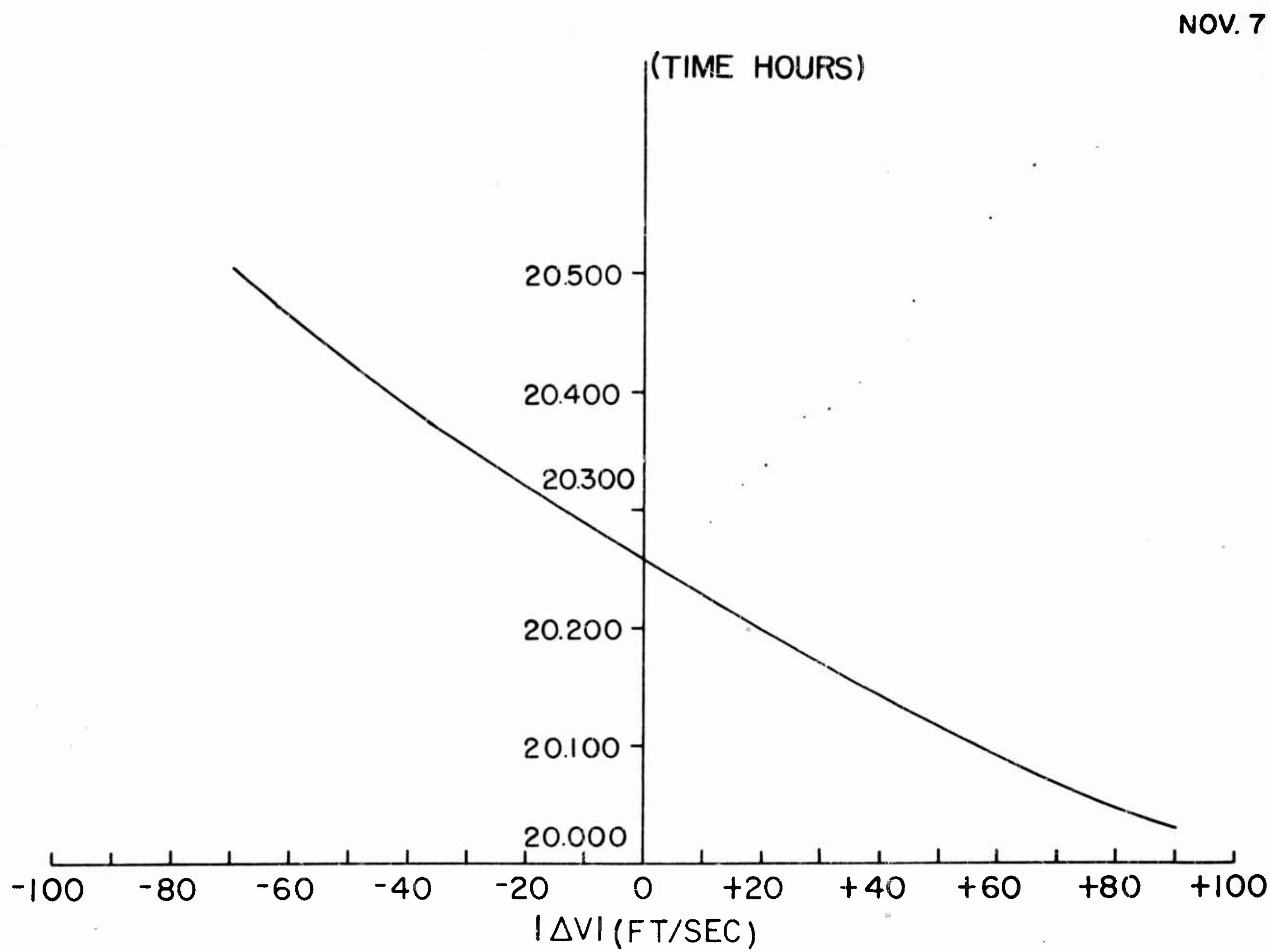


Figure 27. Flight Time as a Function of Error in Initial Velocity

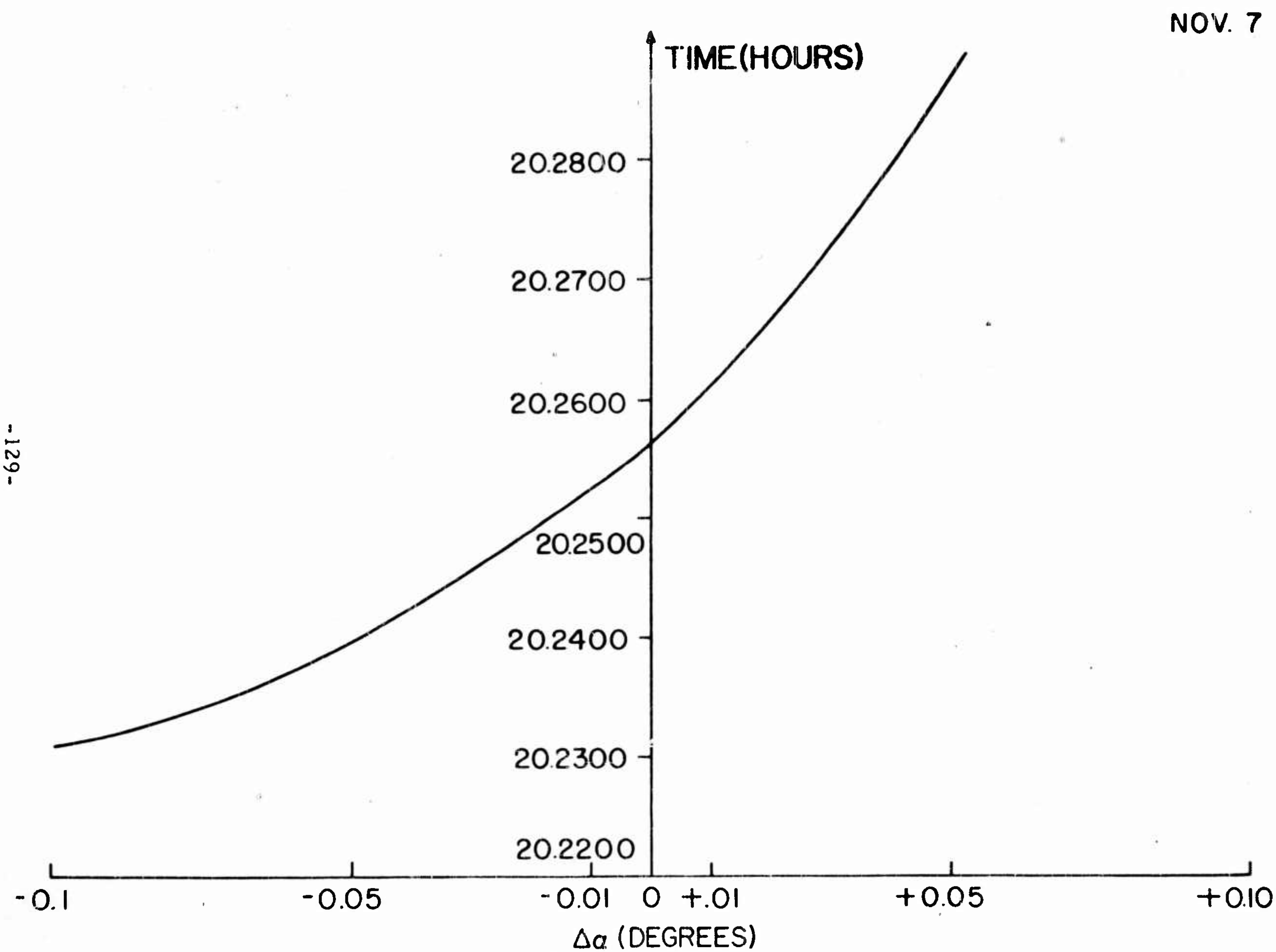


Figure 28. Flight Time as a Function of Error in Velocity Angle α

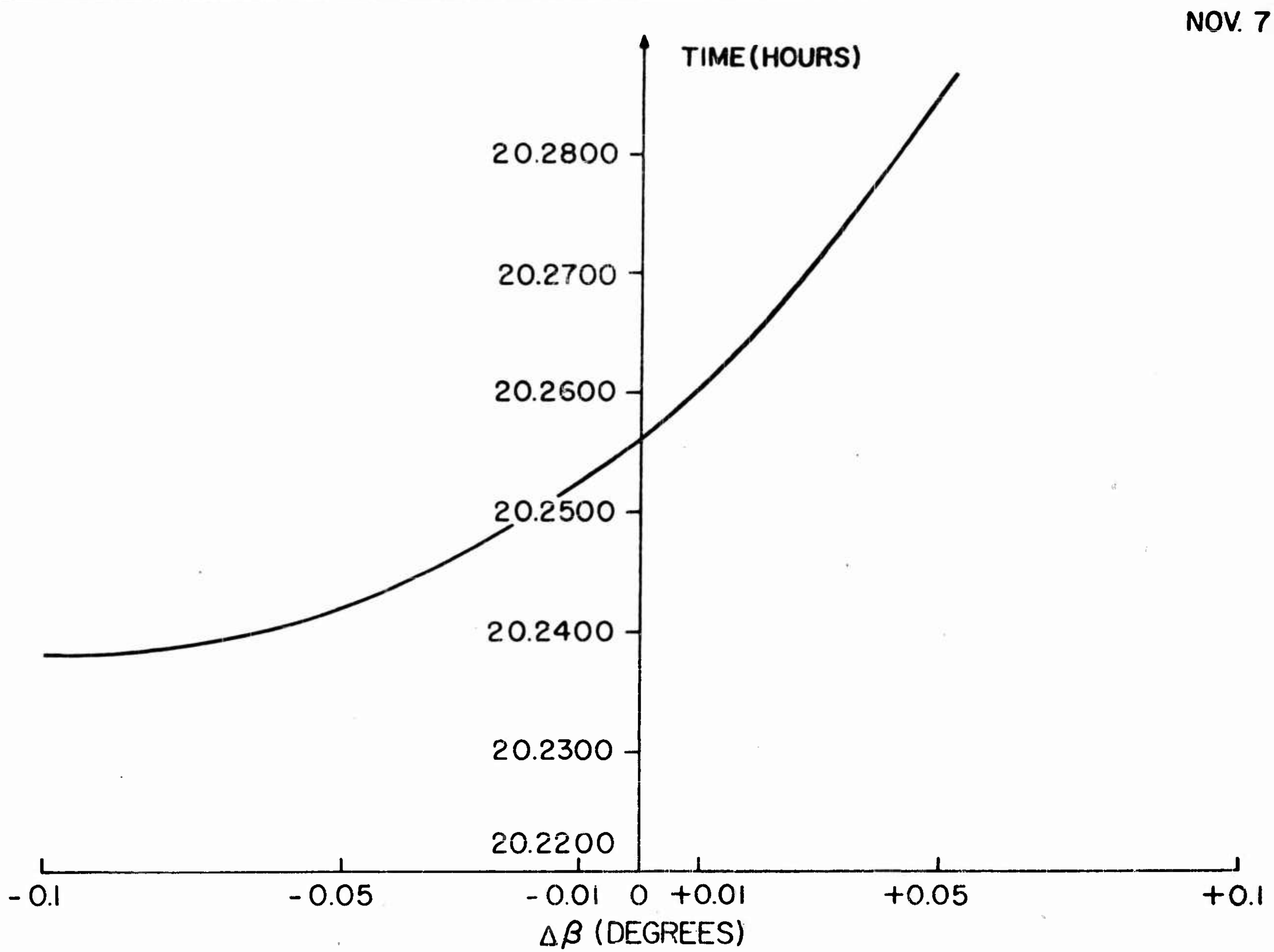


Figure 29. Flight Time as a Function of Error in Velocity Angle β

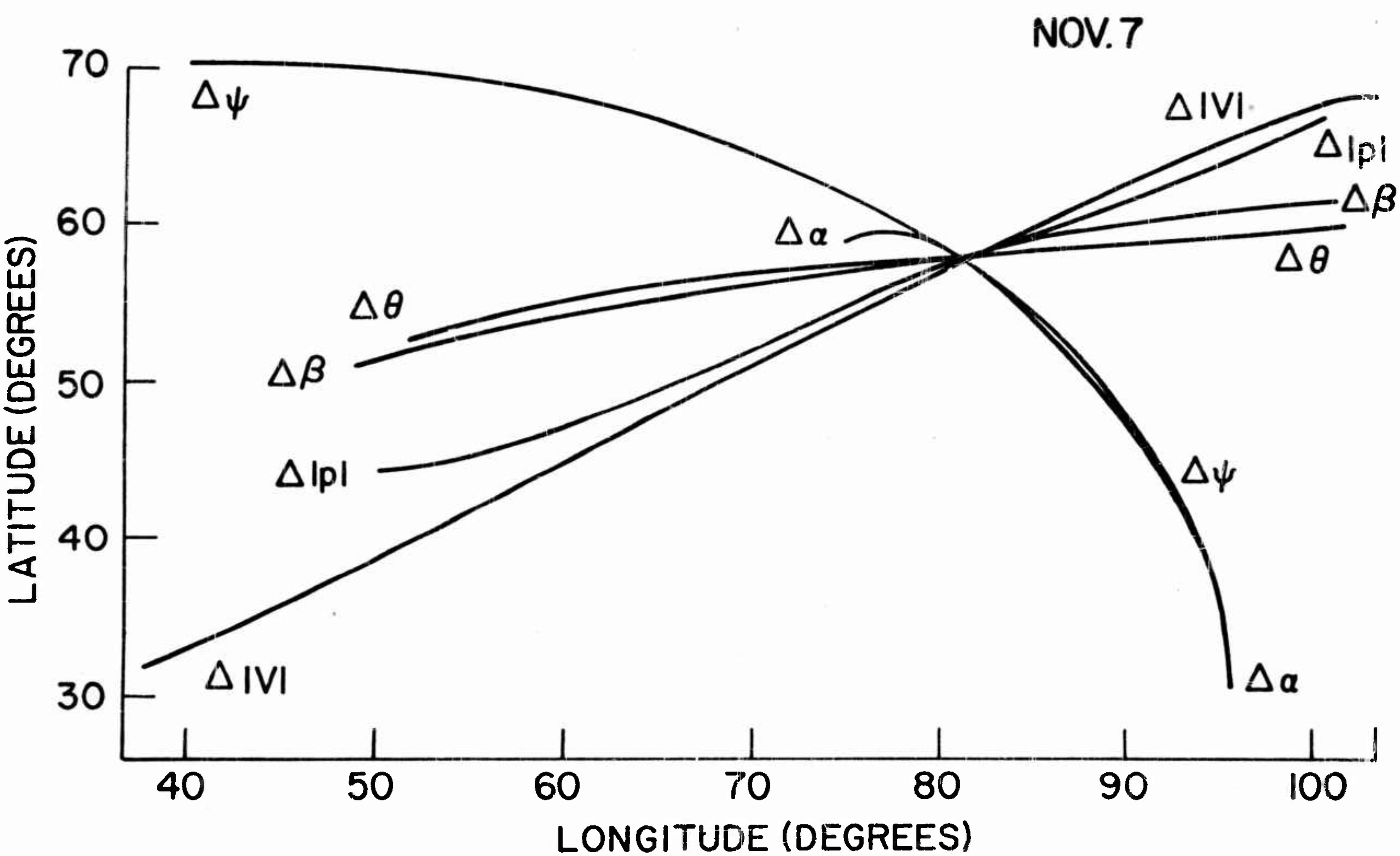


Figure 30. Latitude and Longitude of Error Trajectories Taken About the Nominal at the Surface of Moon

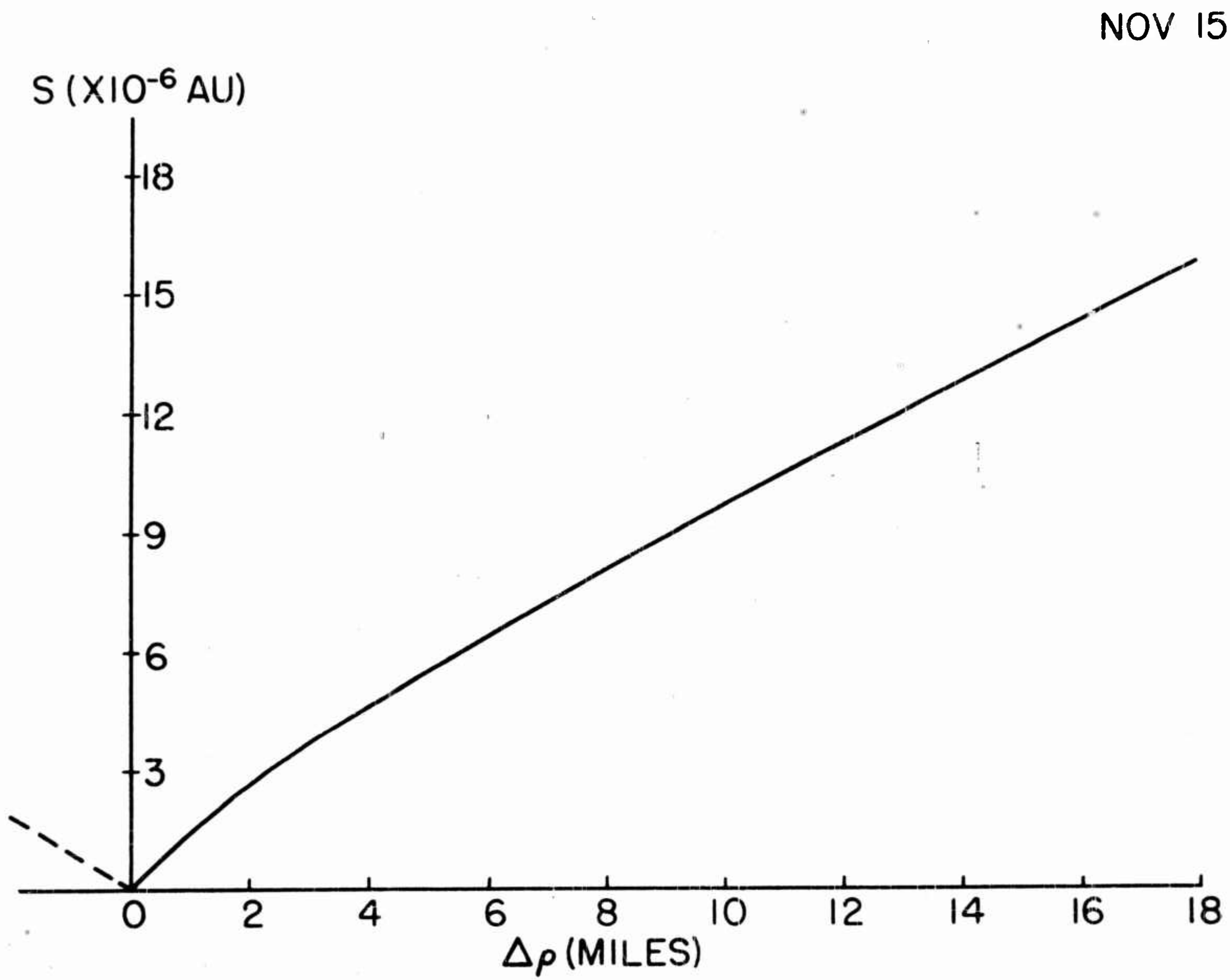


Figure 31. Miss Distance as a Function of Error in Burnout Altitude

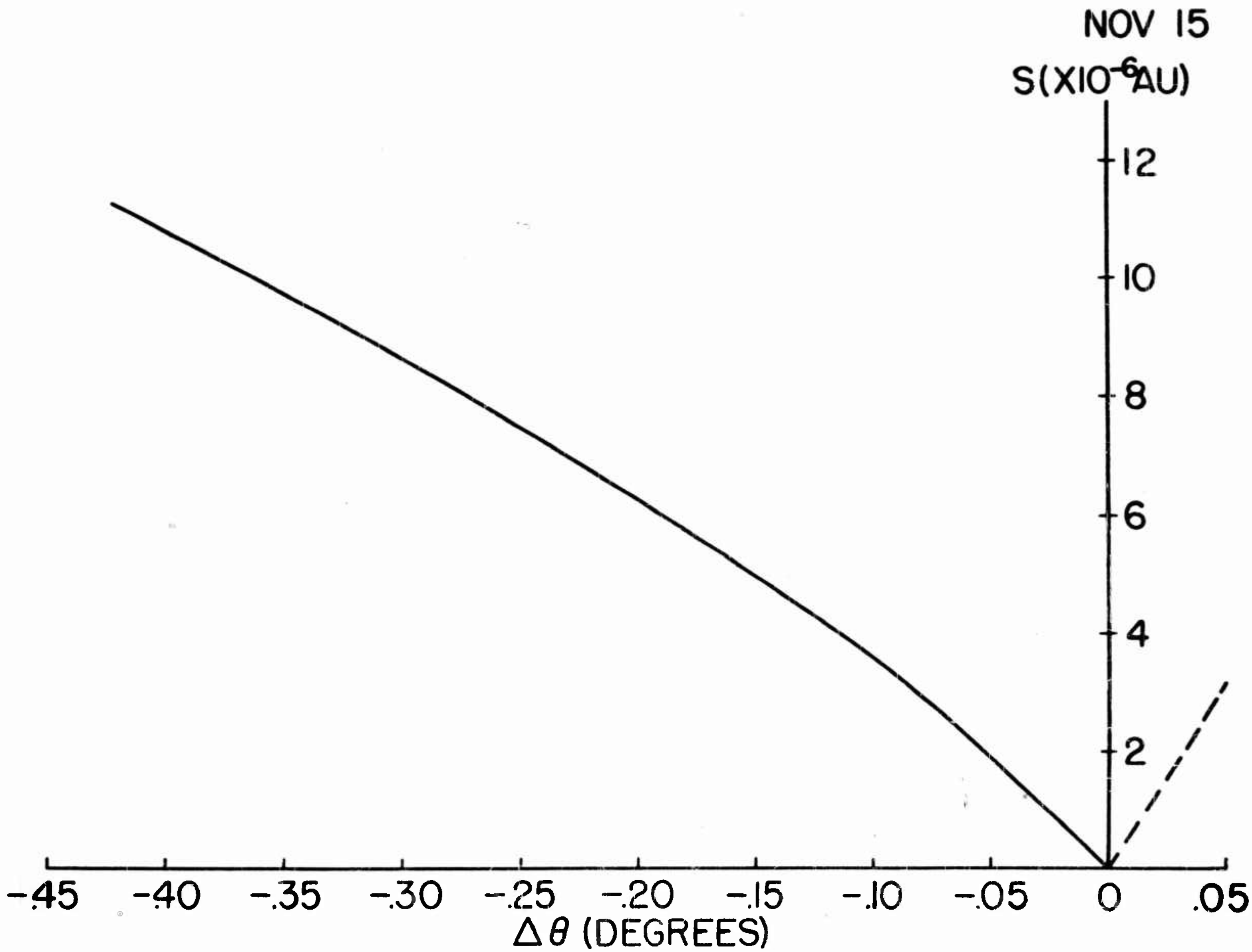


Figure 32. Miss Distance as a Function of Error in θ

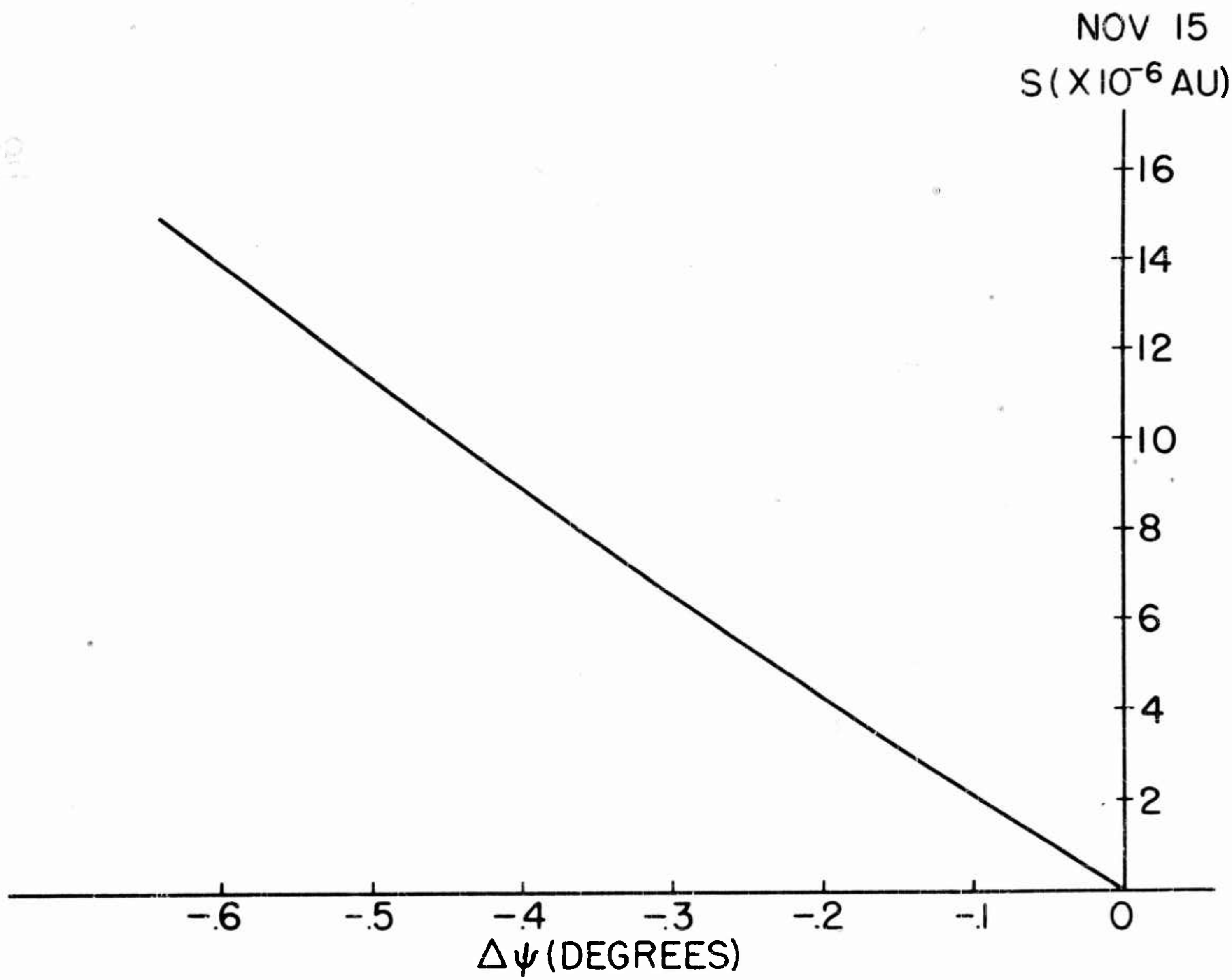


Figure 33. Miss Distance as a Function of Error in ψ

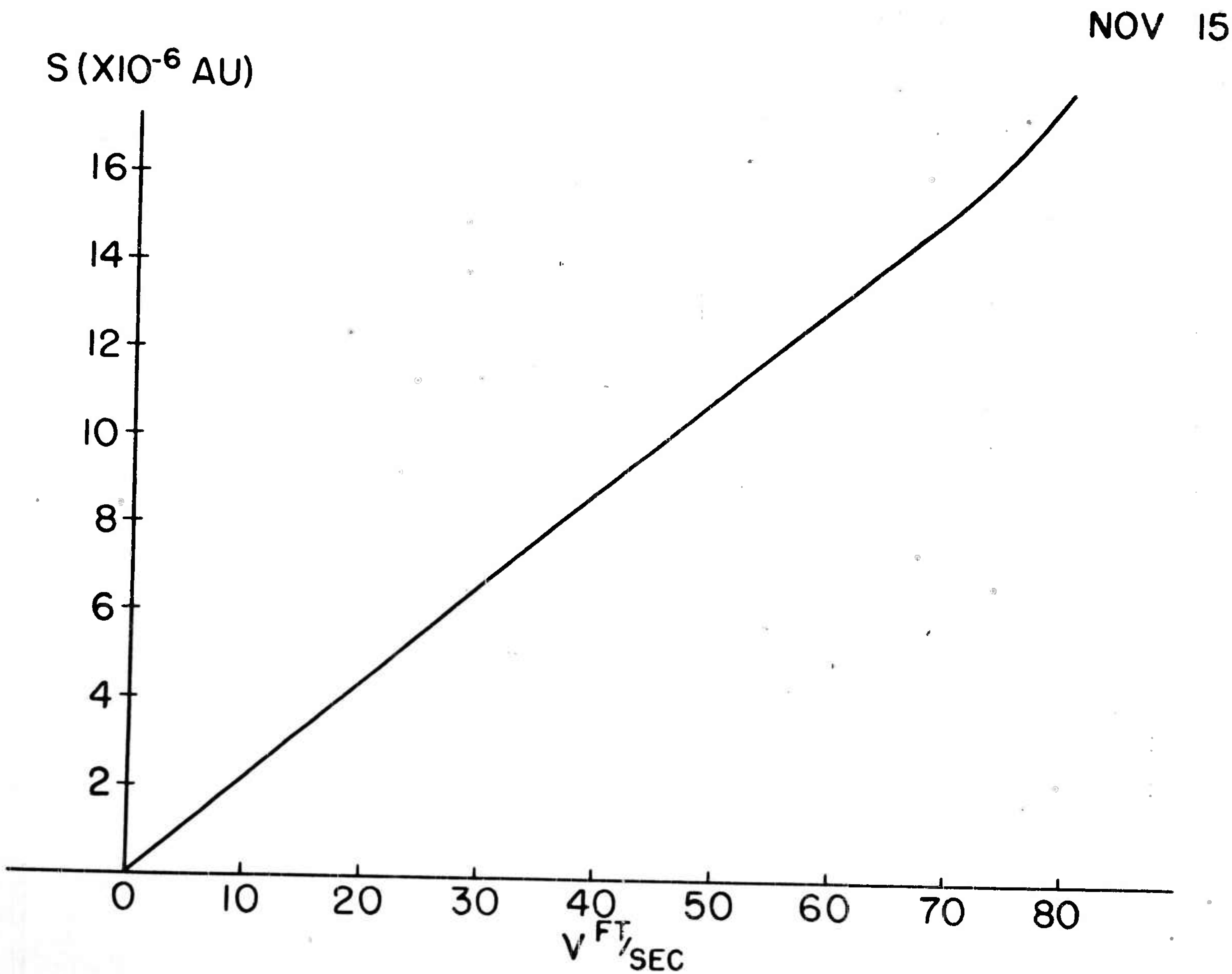


Figure 34. Miss Distance as a Function of Error in Initial Velocity

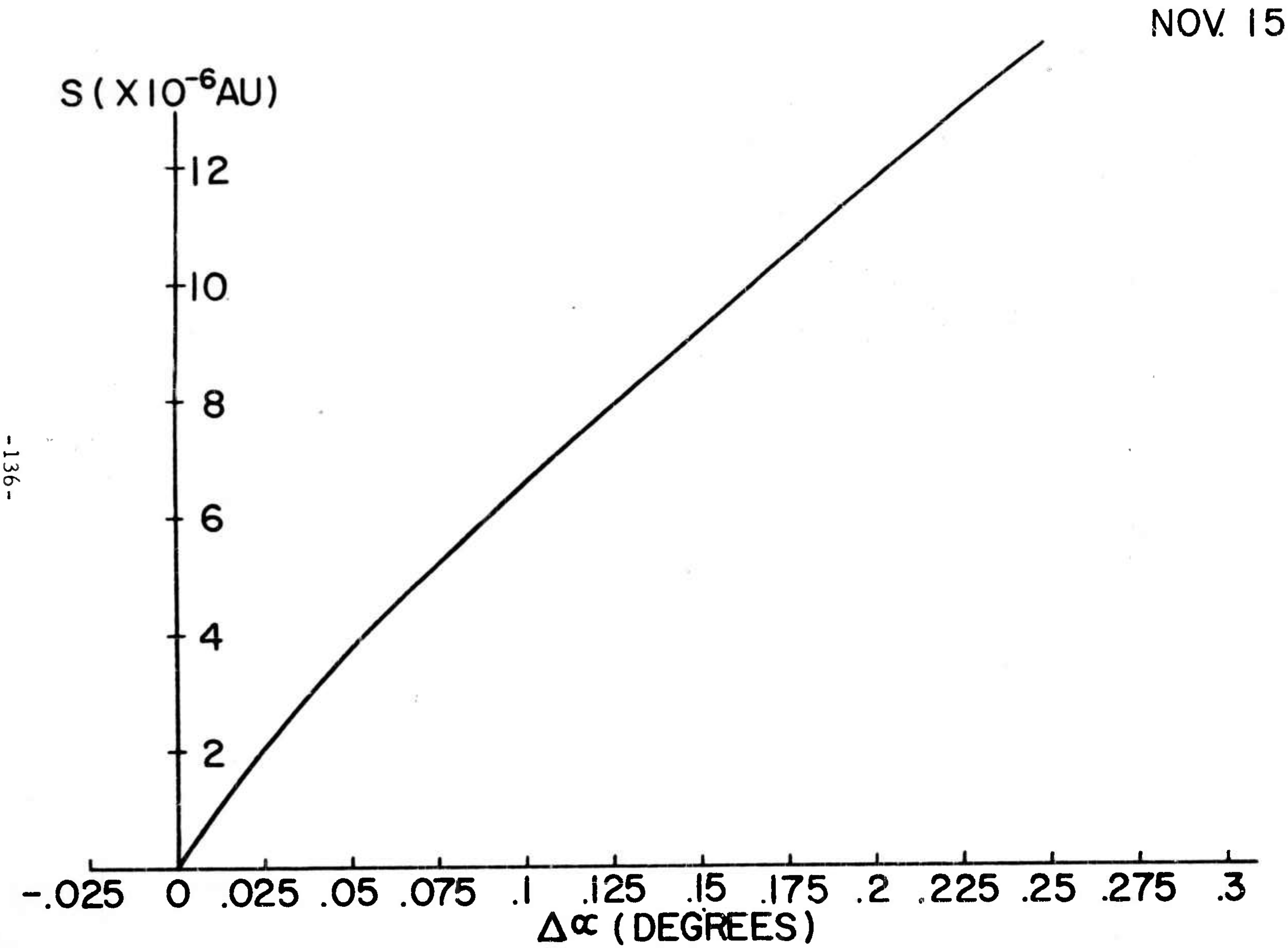


Figure 35. Miss Distance as a Function of Error in Velocity Angle α

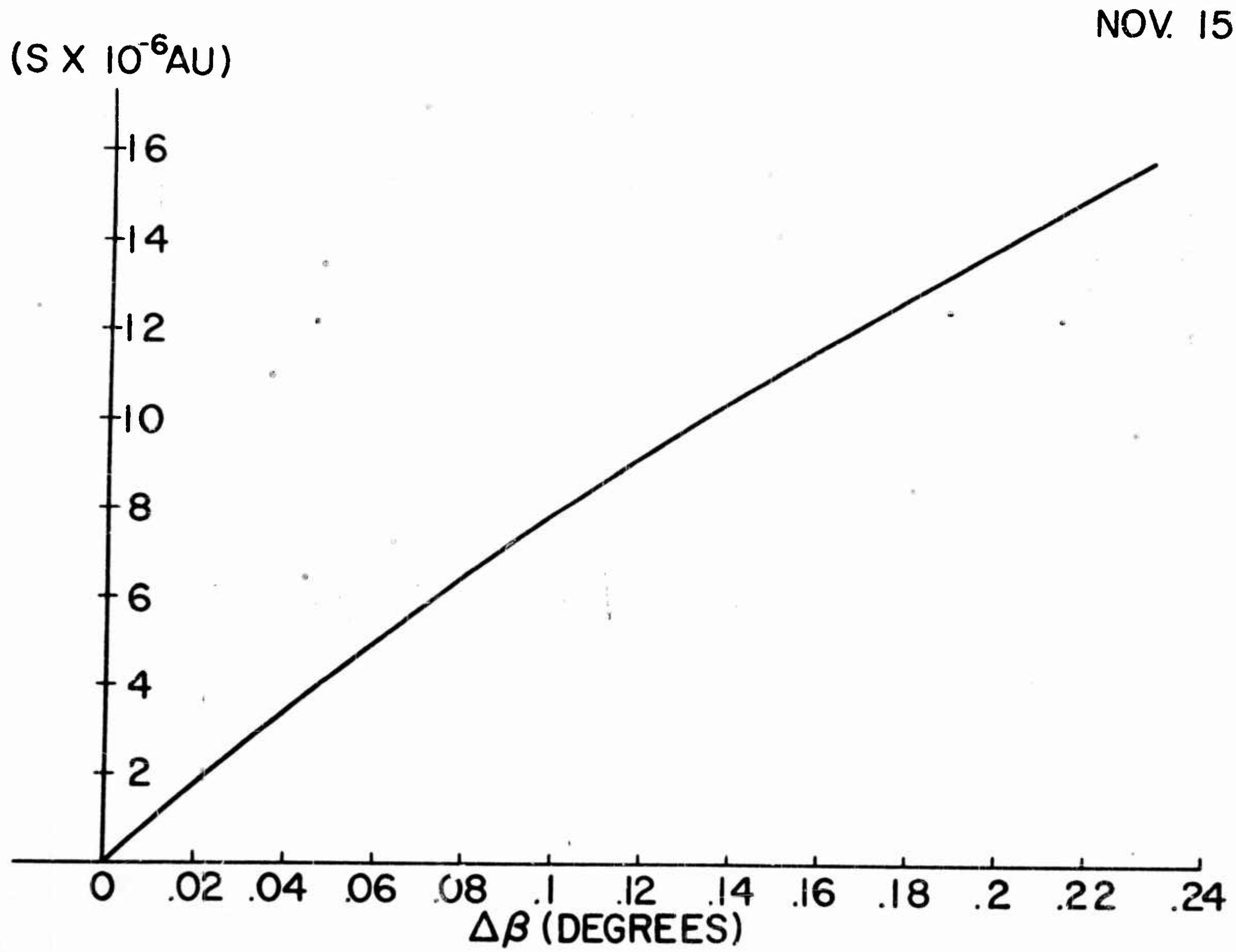


Figure 36. Miss Distance as a Function of Error in Velocity Angle β

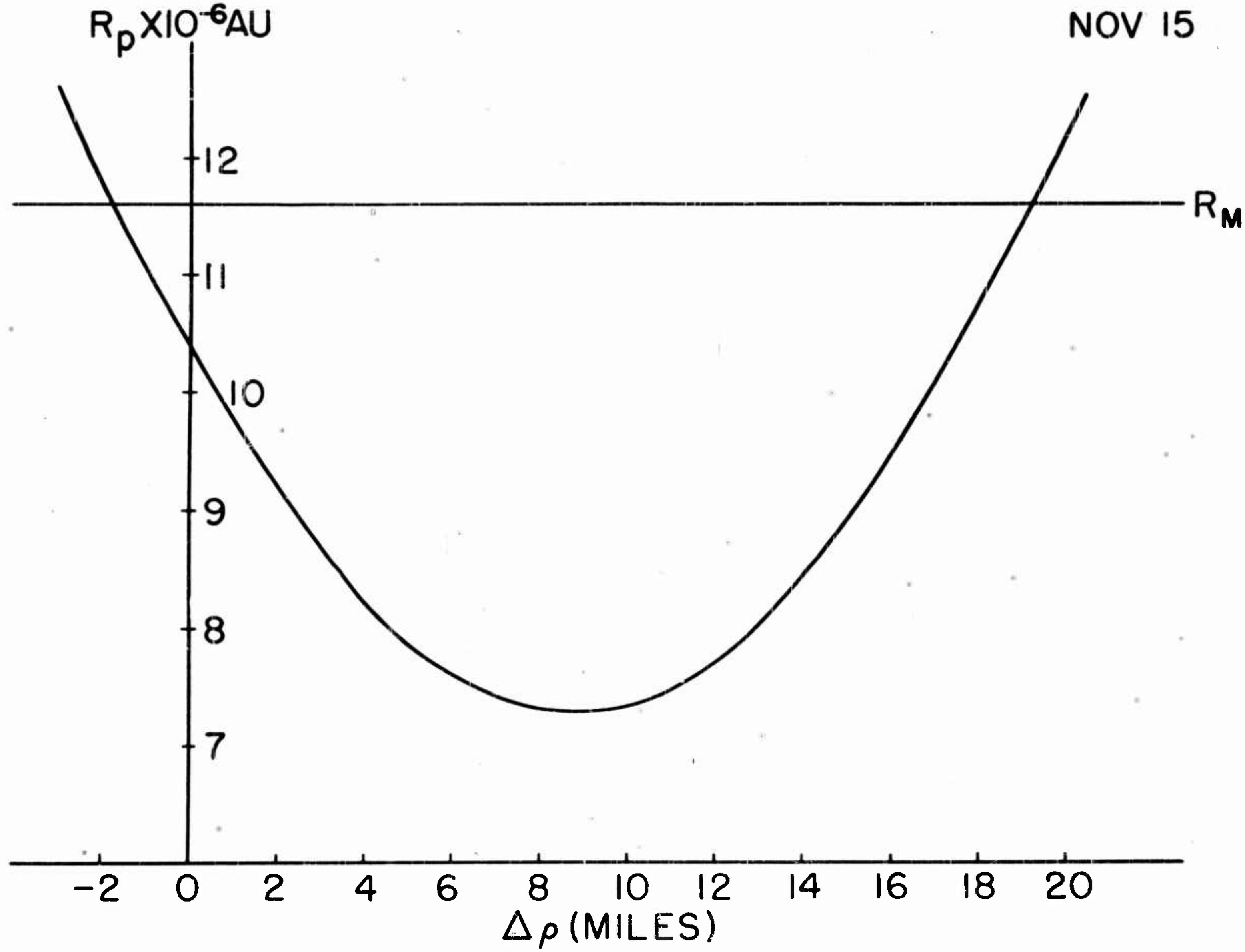


Figure 37. Distance of Closest Approach as a Function of Error in Burnout Altitude

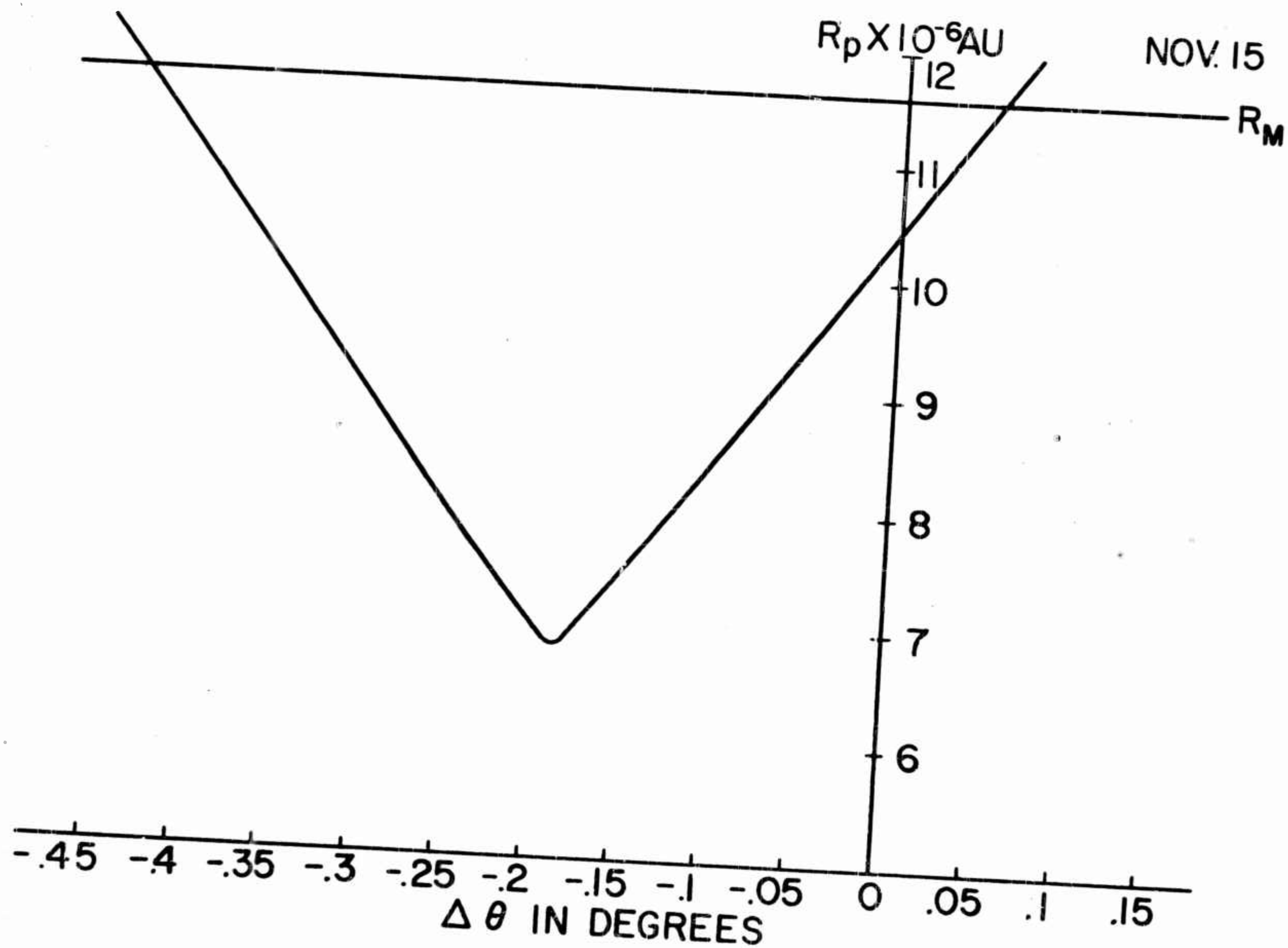


Figure 38. Distance of Closest Approach as a Function of Position Angle θ

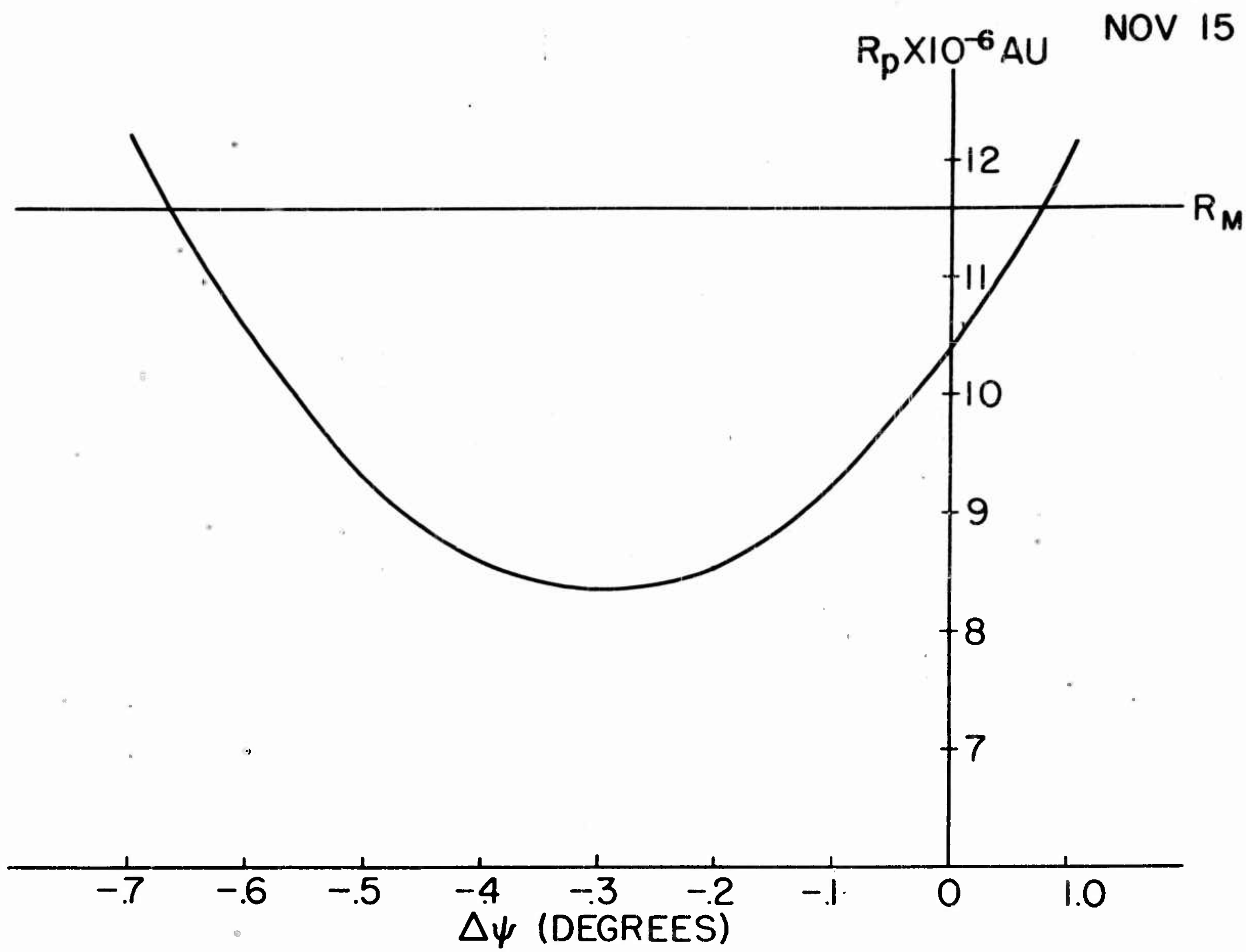


Figure 39. Distance of Closest Approach as a Function of Position Angle ψ

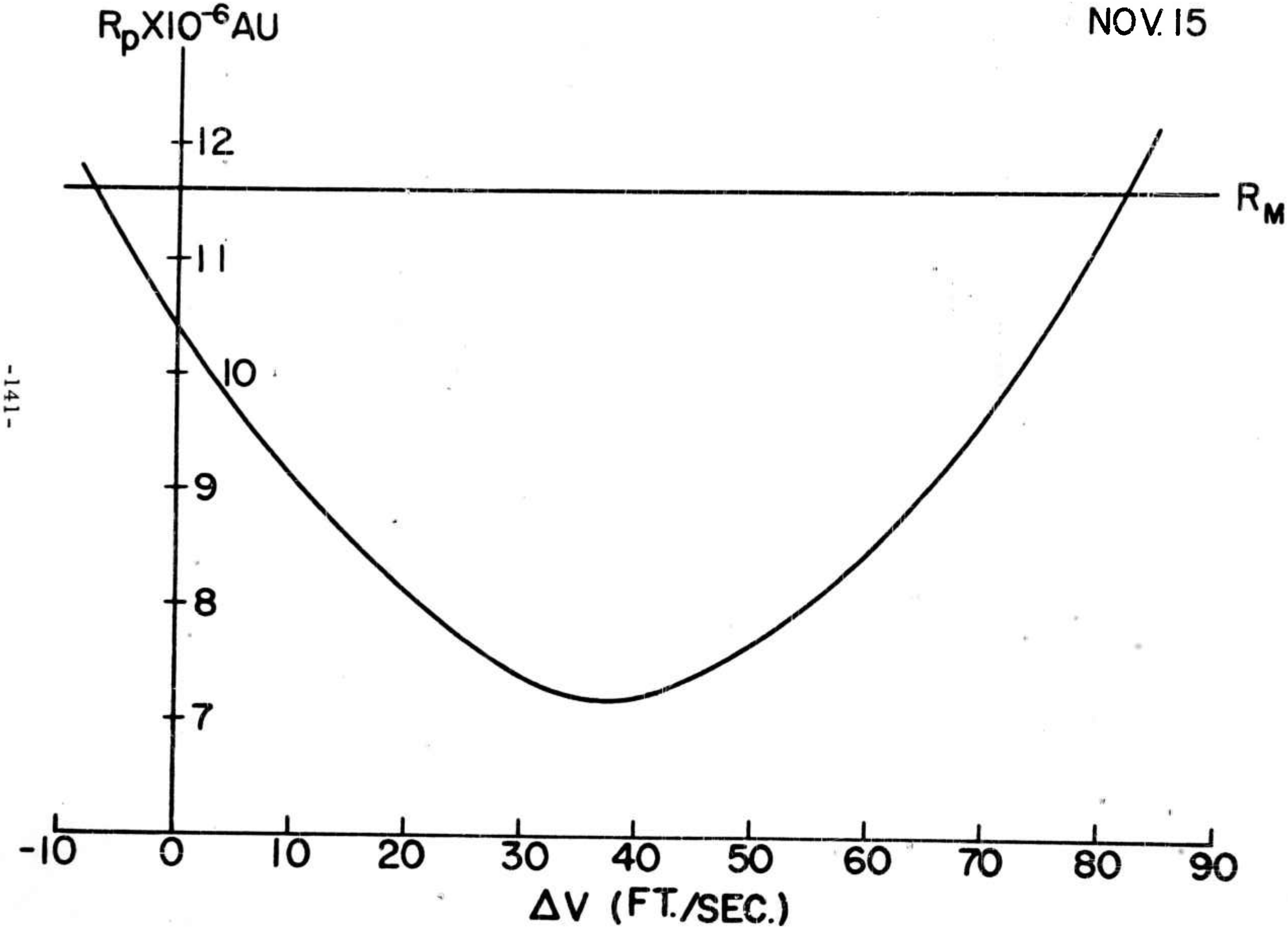


Figure 40. Distance of Closest Approach as a Function of Error in Initial Velocity

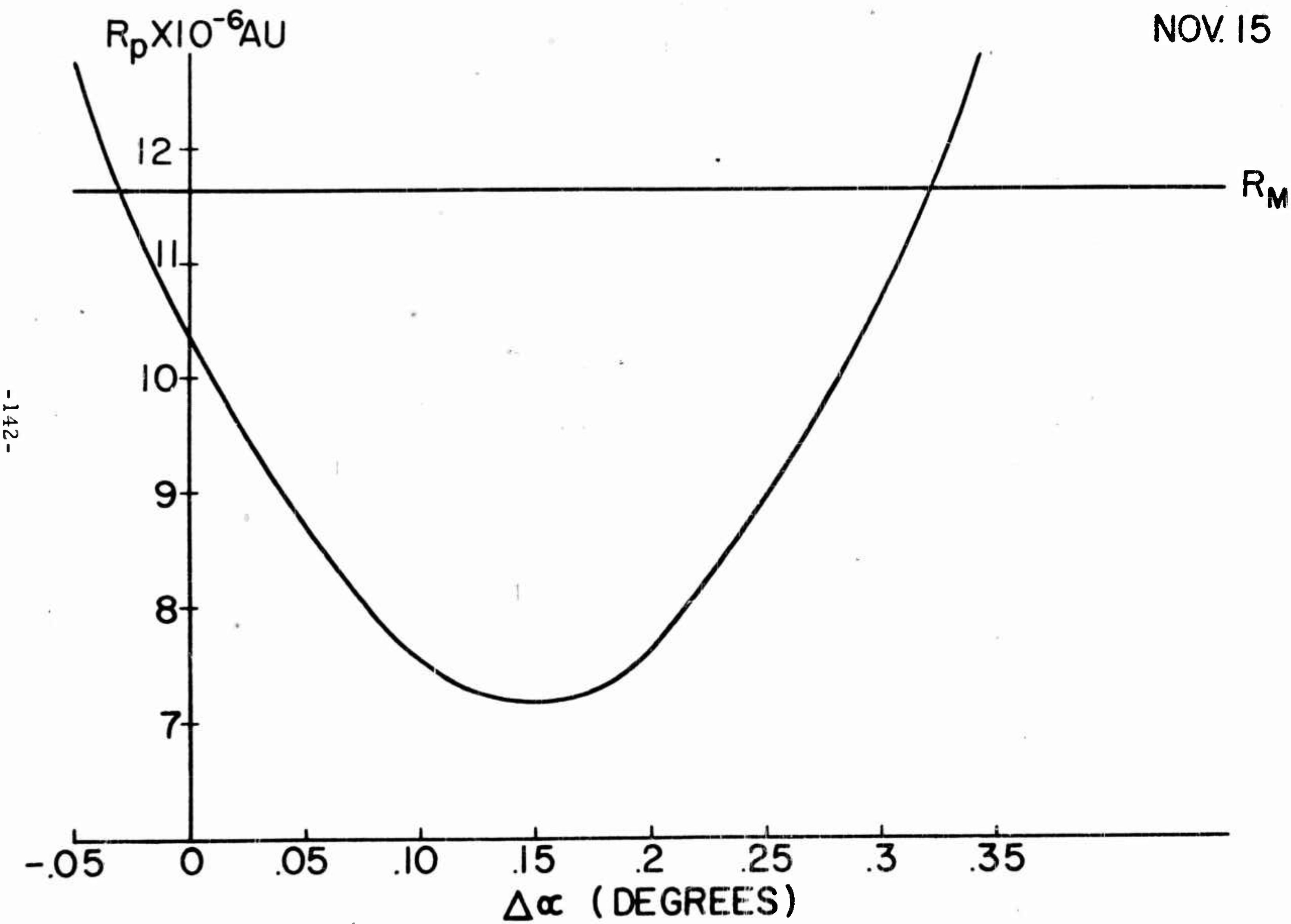


Figure 41. Distance of Closest Approach as a Function of Velocity Angle α

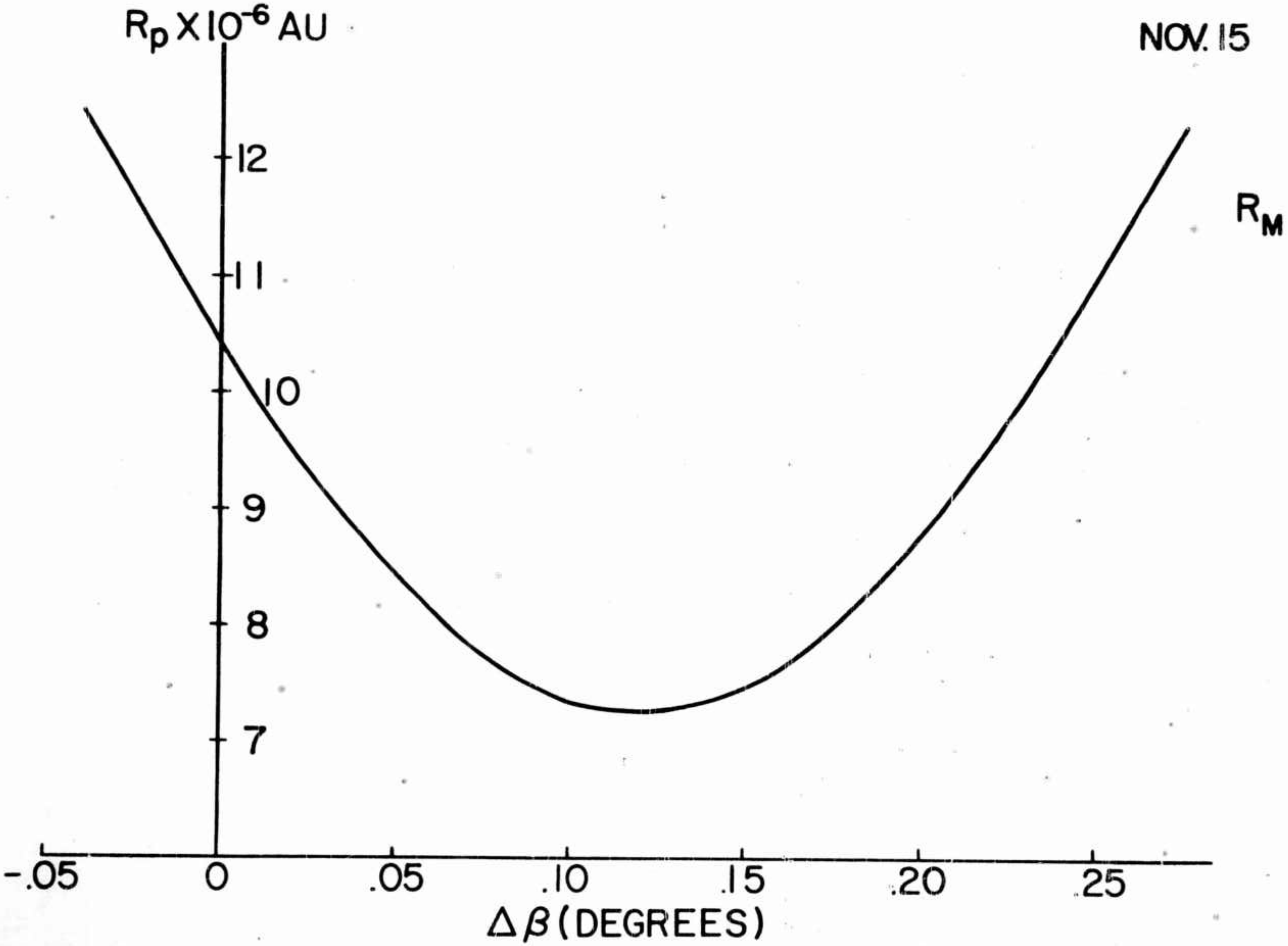


Figure 42. Distance of Closest Approach as a Function of Velocity Angle β

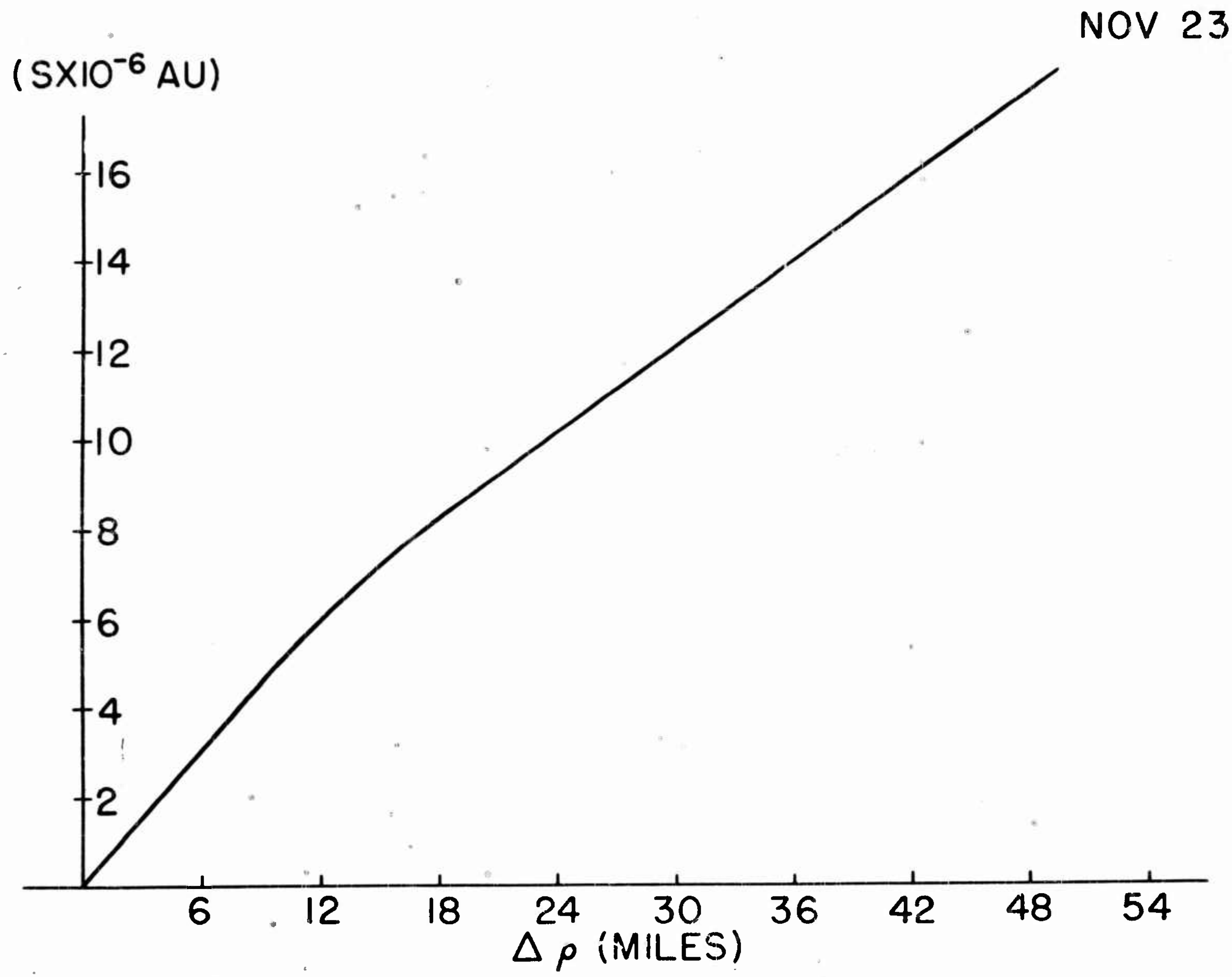


Figure 43. Miss Distance as a Function of Error in Burnout Altitude

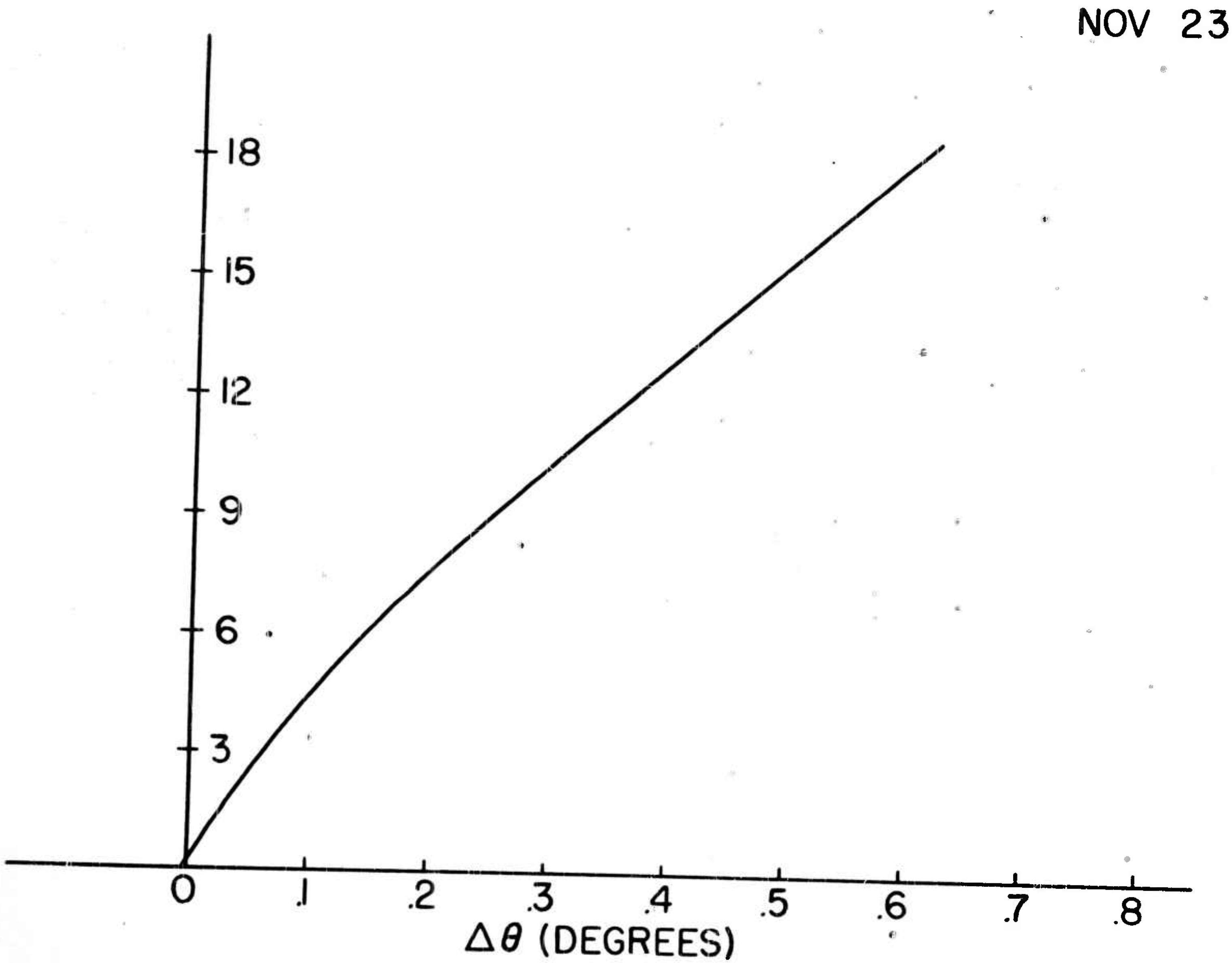


Figure 44. Miss Distance as a Function of Error in θ

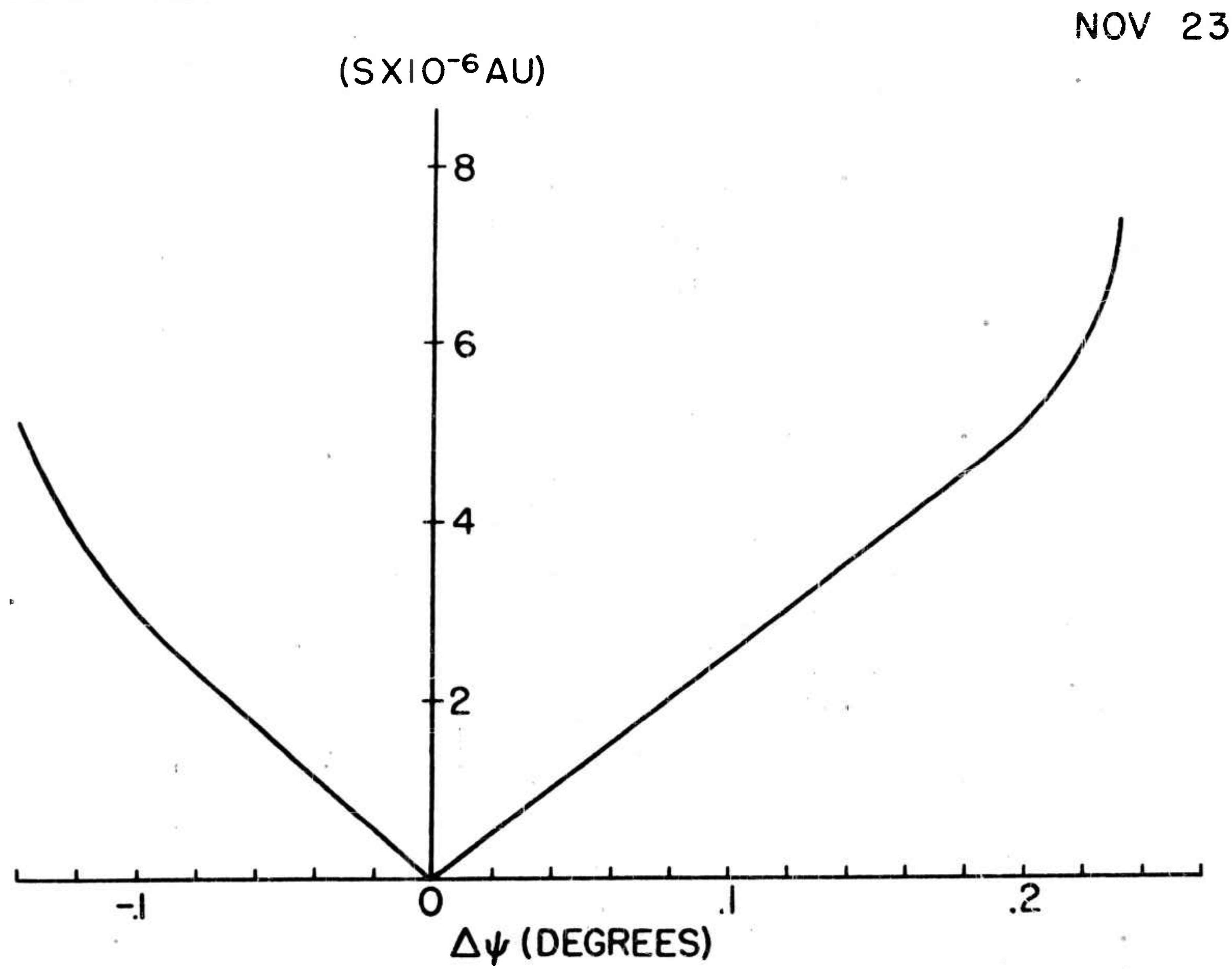


Figure 45. Miss Distance as a Function of Error in ψ

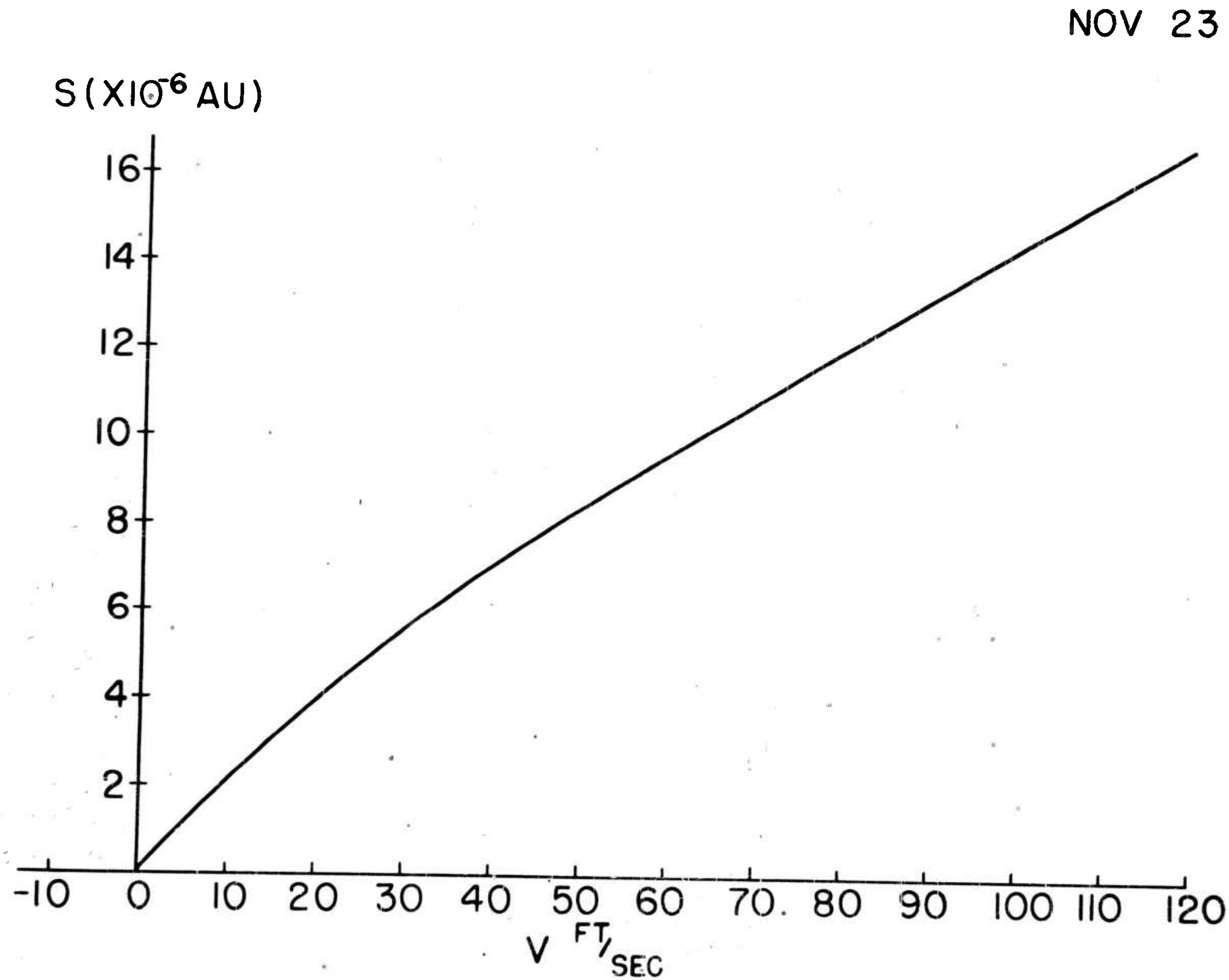


Figure 46. Miss Distance as a Function of Error in Initial Velocity

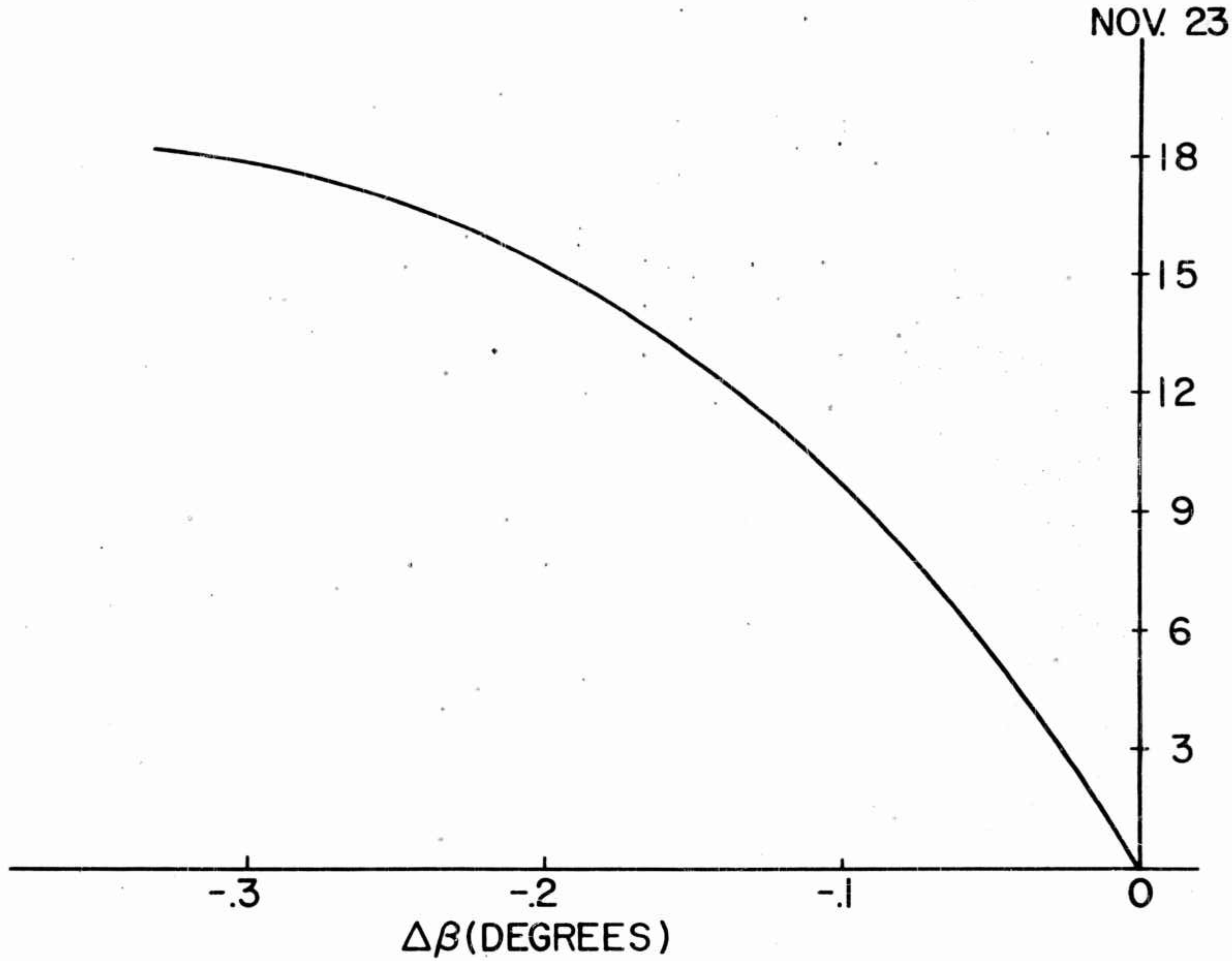


Figure 47. Miss Distance as a Function of Error in Velocity Angle β

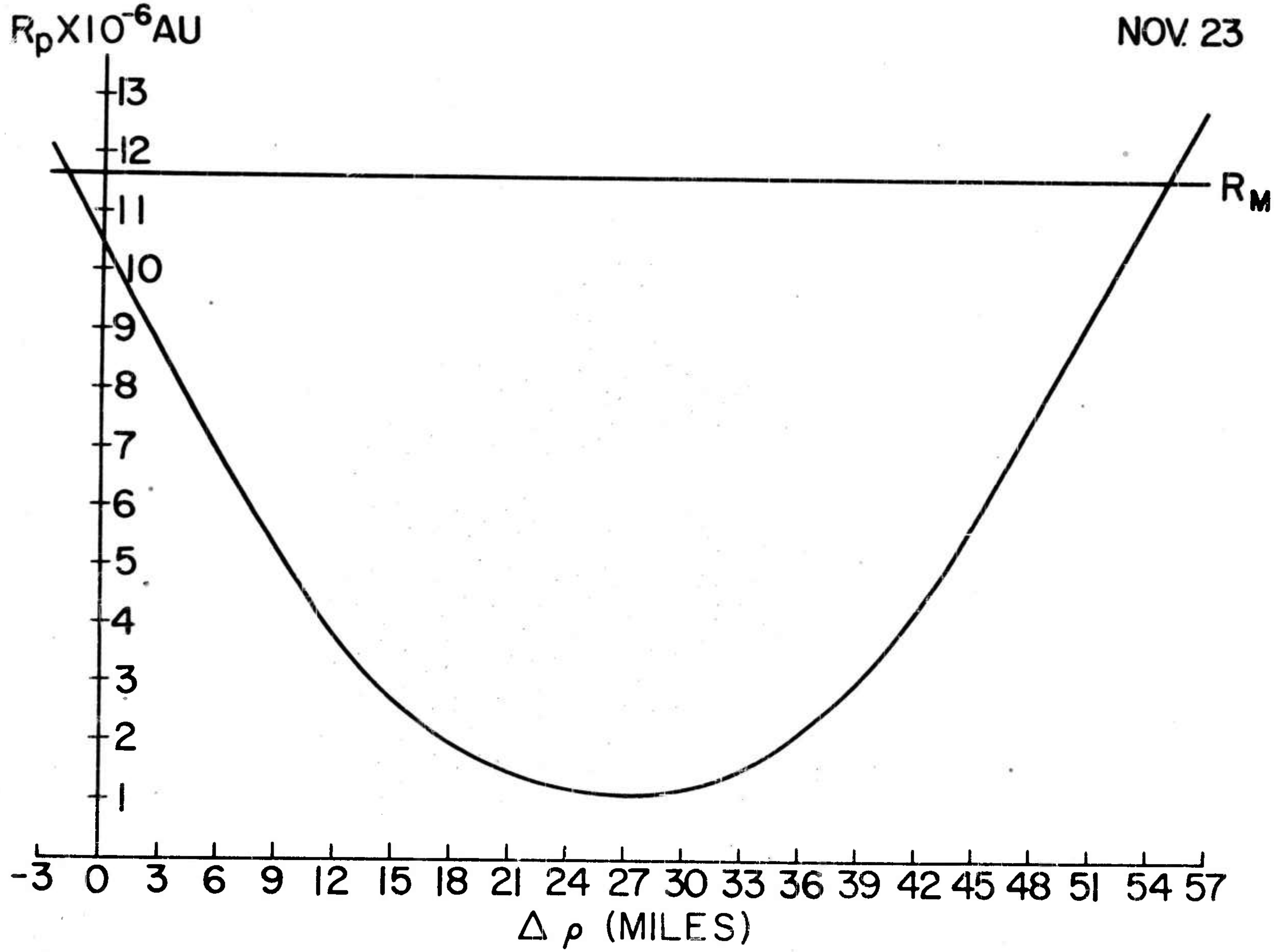


Figure 48. Distance of Closest Approach as a Function of Error in Burnout Altitude

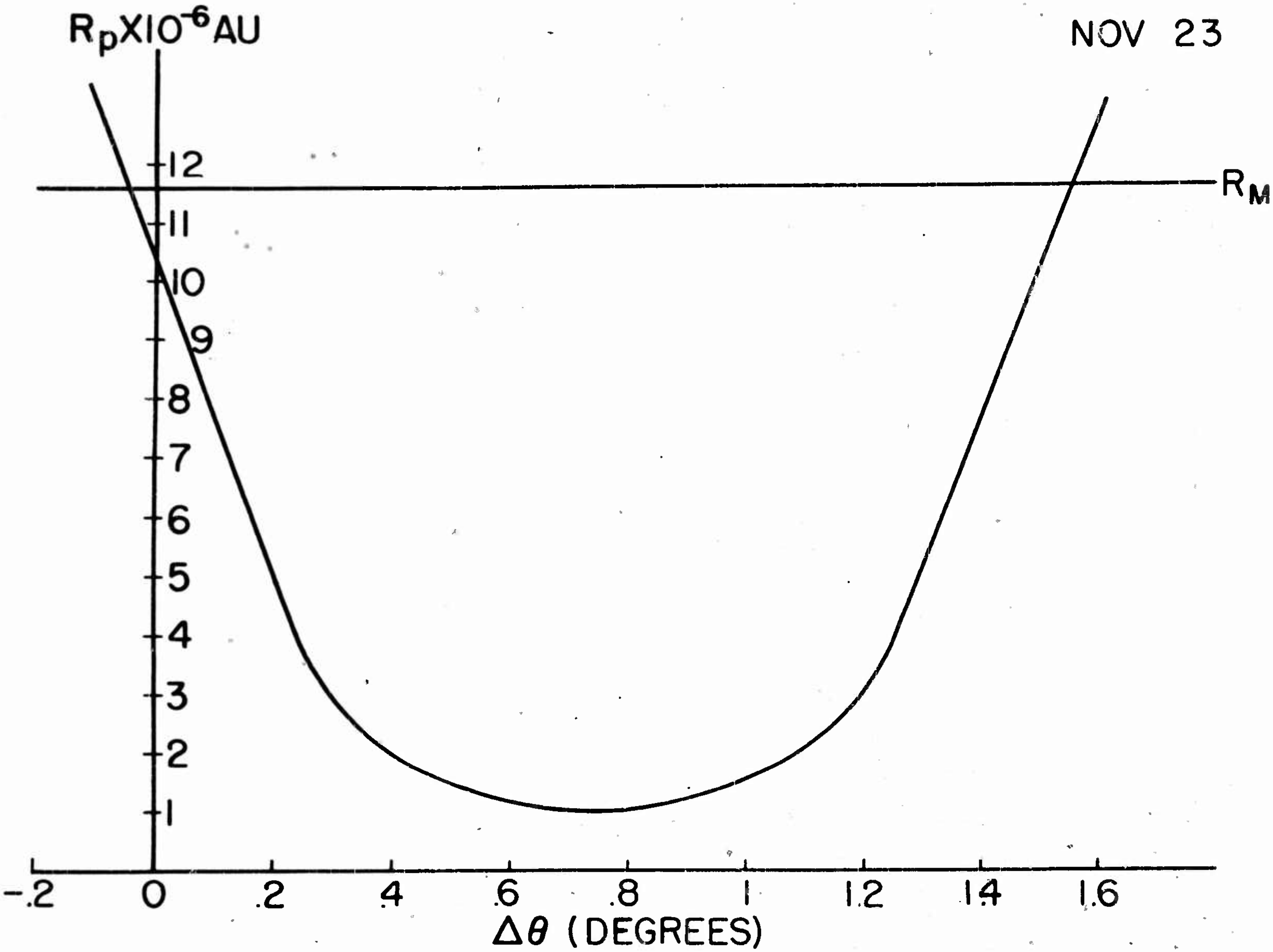


Figure 49. Distance of Closest Approach as a Function of Position Angle $\Delta\theta$.

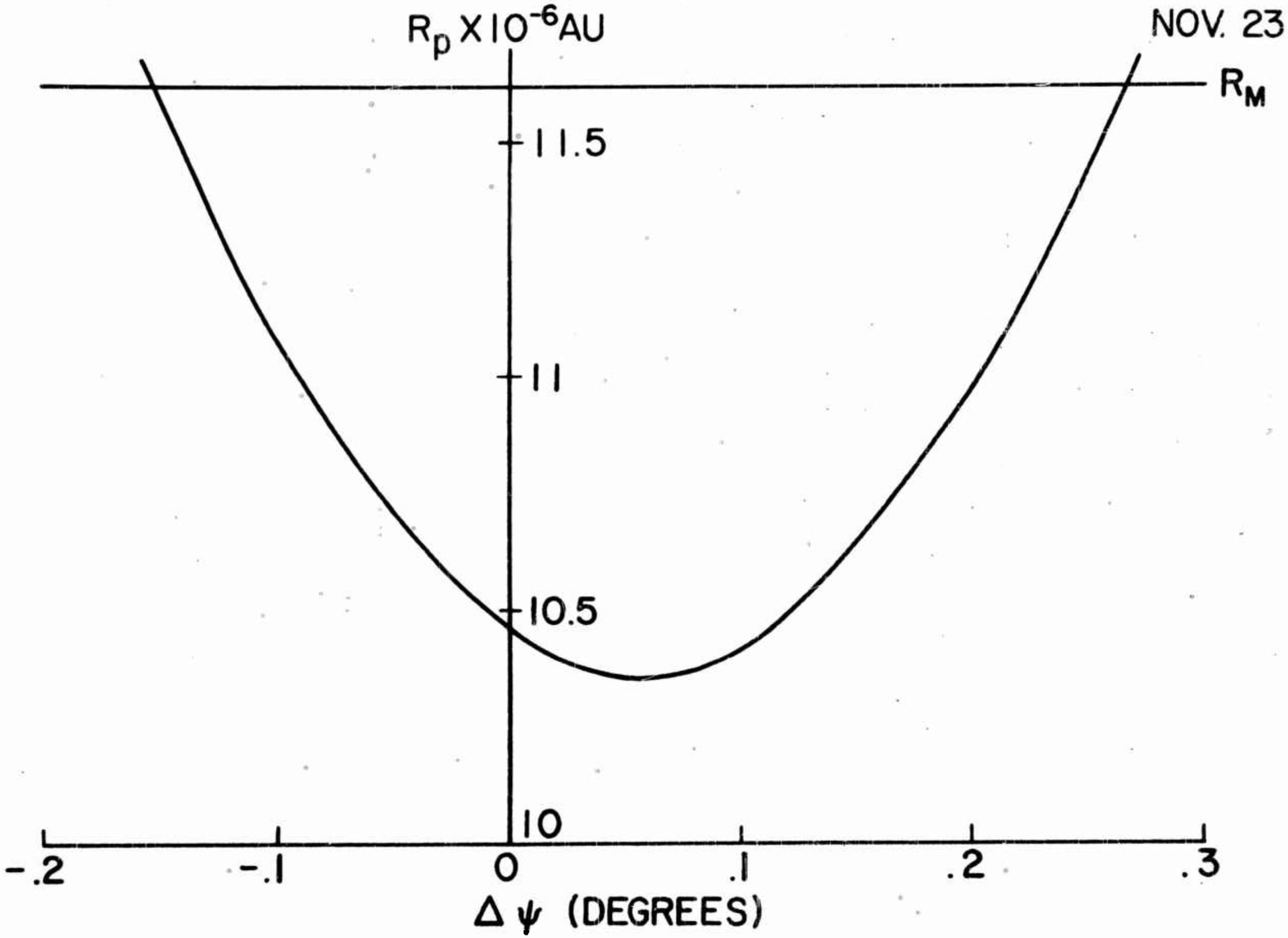


Figure 50. Distance of Closest Approach as a Function of Position Angle ψ

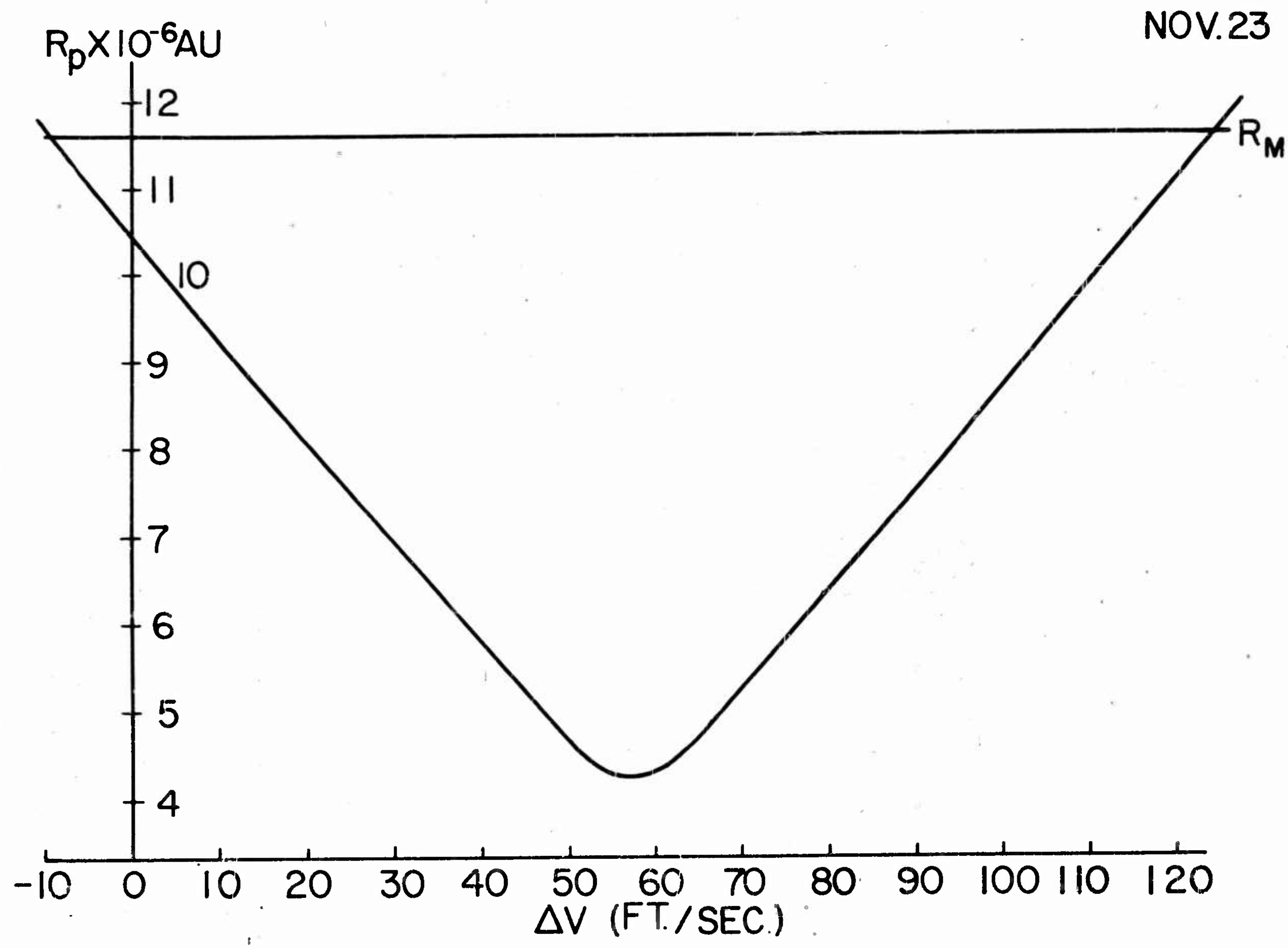


Figure 51. Distance of Closest Approach as a Function of Error in Initial Velocity

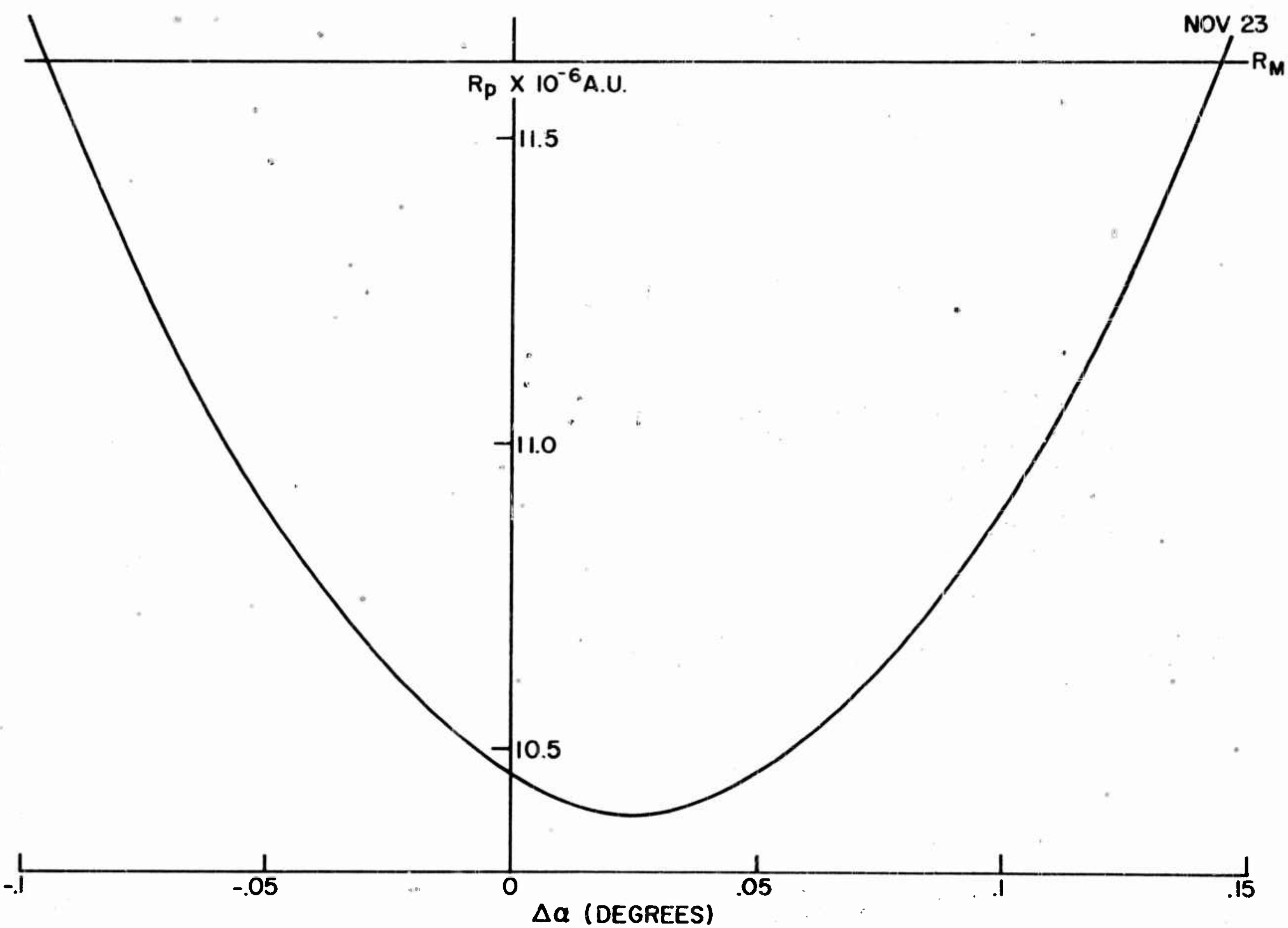


Figure 52. Distance of Closest Approach as a Function of Velocity Angle α

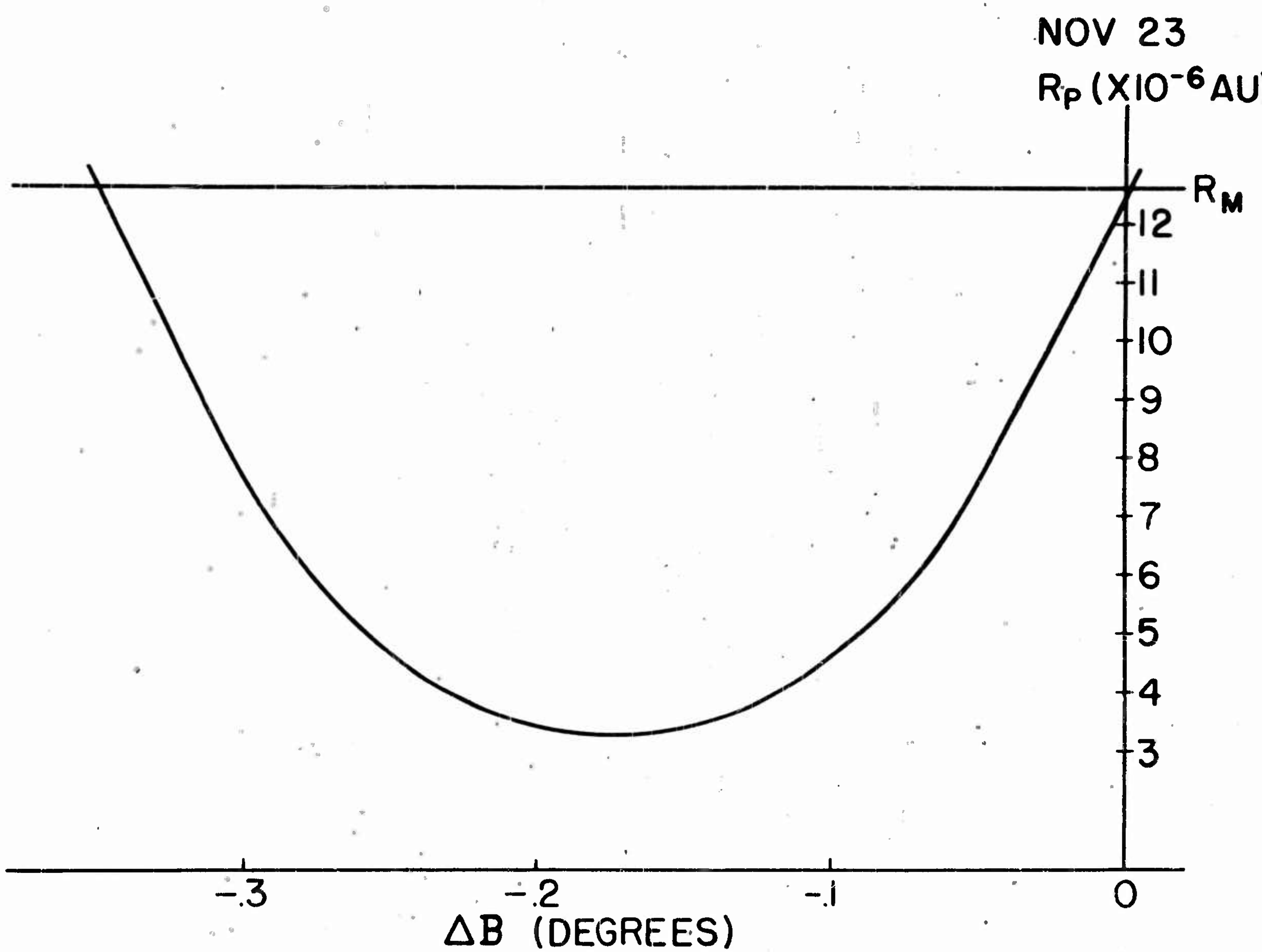


Figure 53. Distance of Closest Approach as a Function of Velocity Angle β

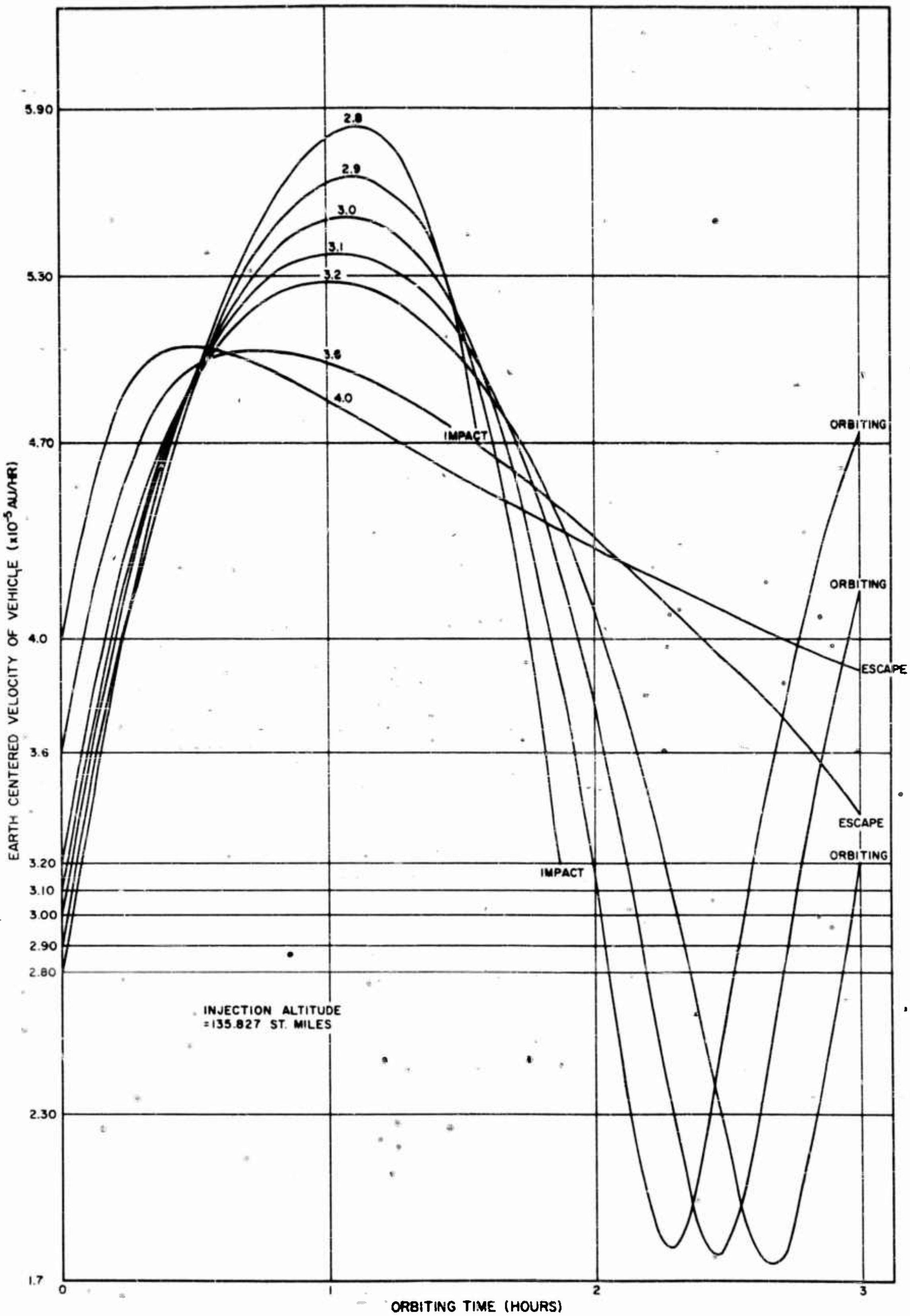
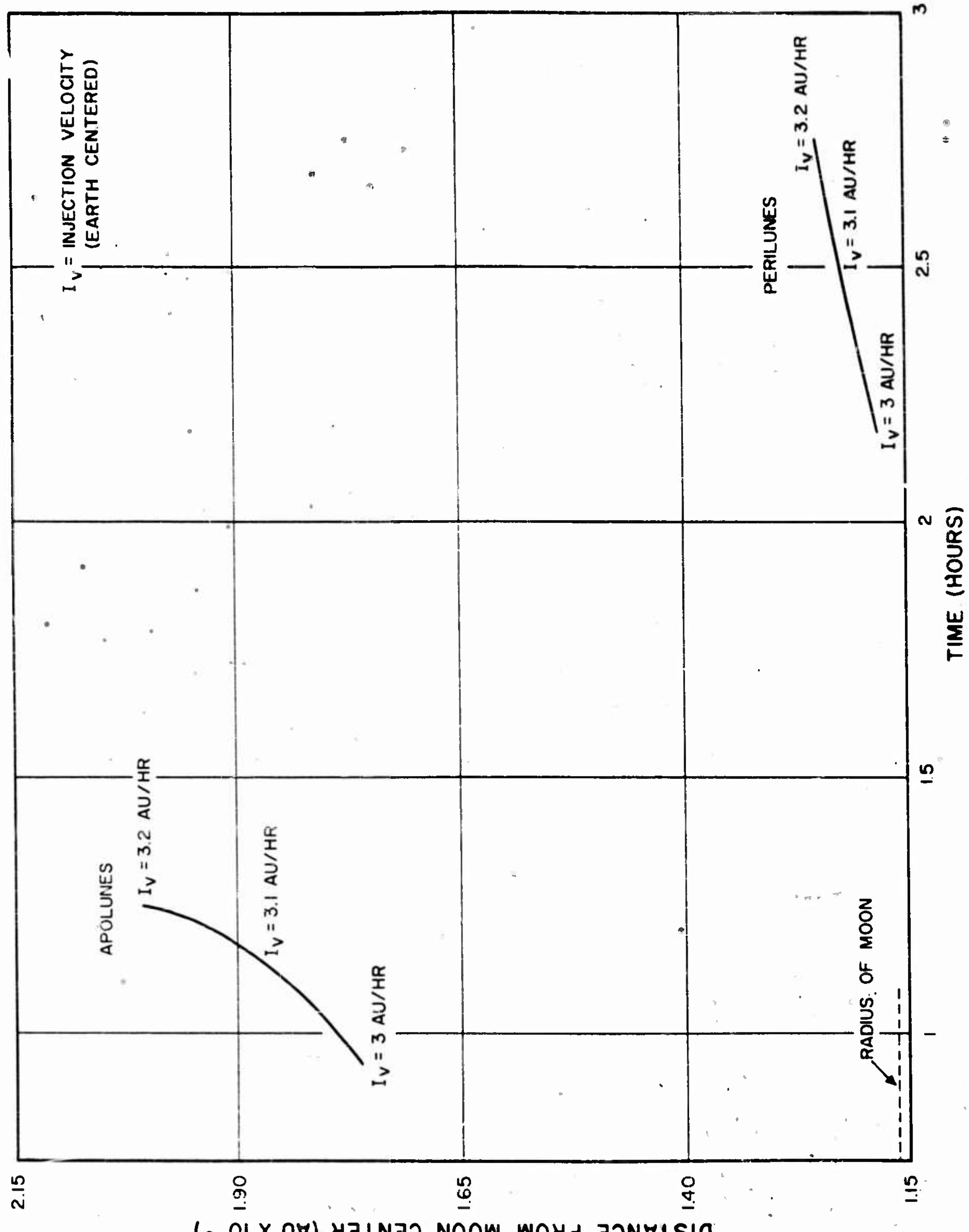

EARTH CENTERED VELOCITY OF VEHICLE (x10⁻⁵ AU/HR)
ORBITING TIME (HOURS)
5.90
5.30
4.70
4.0
3.6
3.20
3.10
3.00
2.90
2.80
2.30
1.7
0
1
2
3
2.8
2.9
3.0
3.1
3.2
3.6
4.0
IMPACT
IMPACT
ORBITING
ORBITING
ESCAPE
ESCAPE
ORBITING
INJECTION ALTITUDE
=135.827 ST. MILES

Figure 54.



I_V = INJECTION VELOCITY (EARTH CENTERED)
PERILUNES
I_V = 3.2 AU/HR
I_V = 3.1 AU/HR
I_V = 3 AU/HR
APOLUNES
I_V = 3.2 AU/HR
I_V = 3.1 AU/HR
I_V = 3 AU/HR
RADIUS OF MOON
TIME (HOURS)
1
1.5
2
2.5
3
DISTANCE FROM MOON CENTER (AU X 10^{-5})
2.15
1.90
1.65
1.40
1.15

Figure 55.

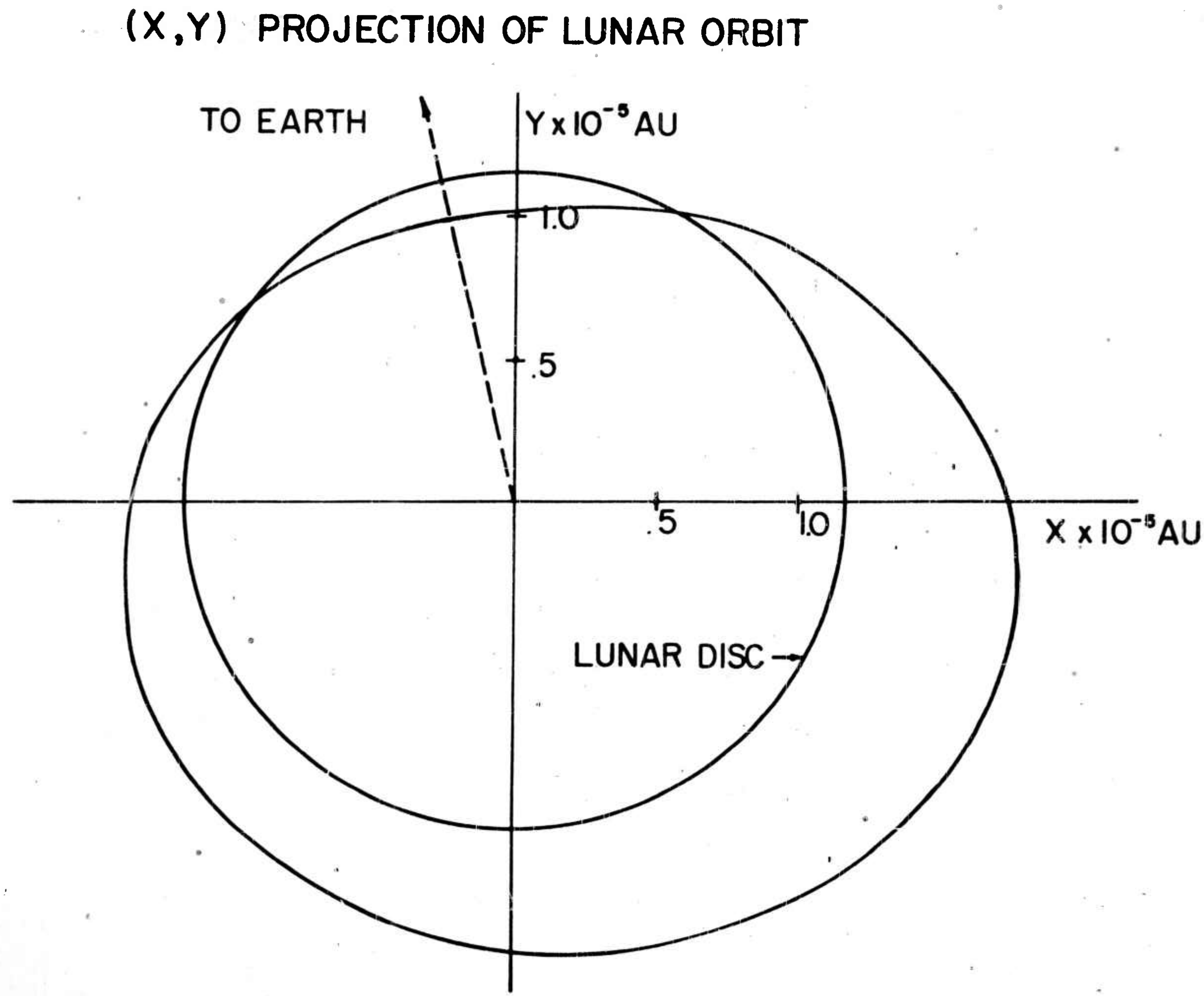

(X,Y) PROJECTION OF LUNAR ORBIT
TO EARTH
Y x 10^-5 AU
1.0
.5
.5
1.0
X x 10^-5 AU
LUNAR DISC

Figure 56.

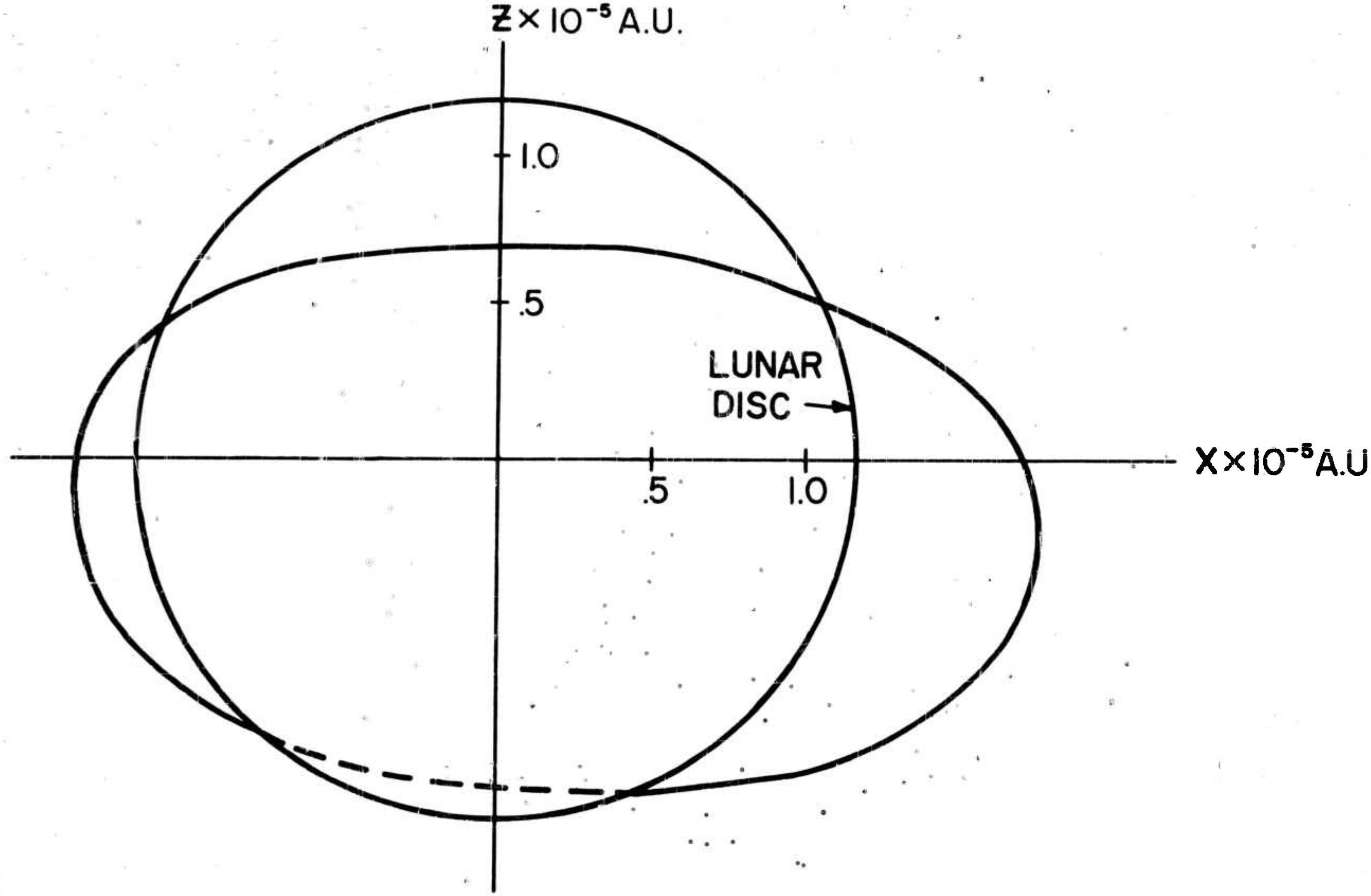


Figure 57.

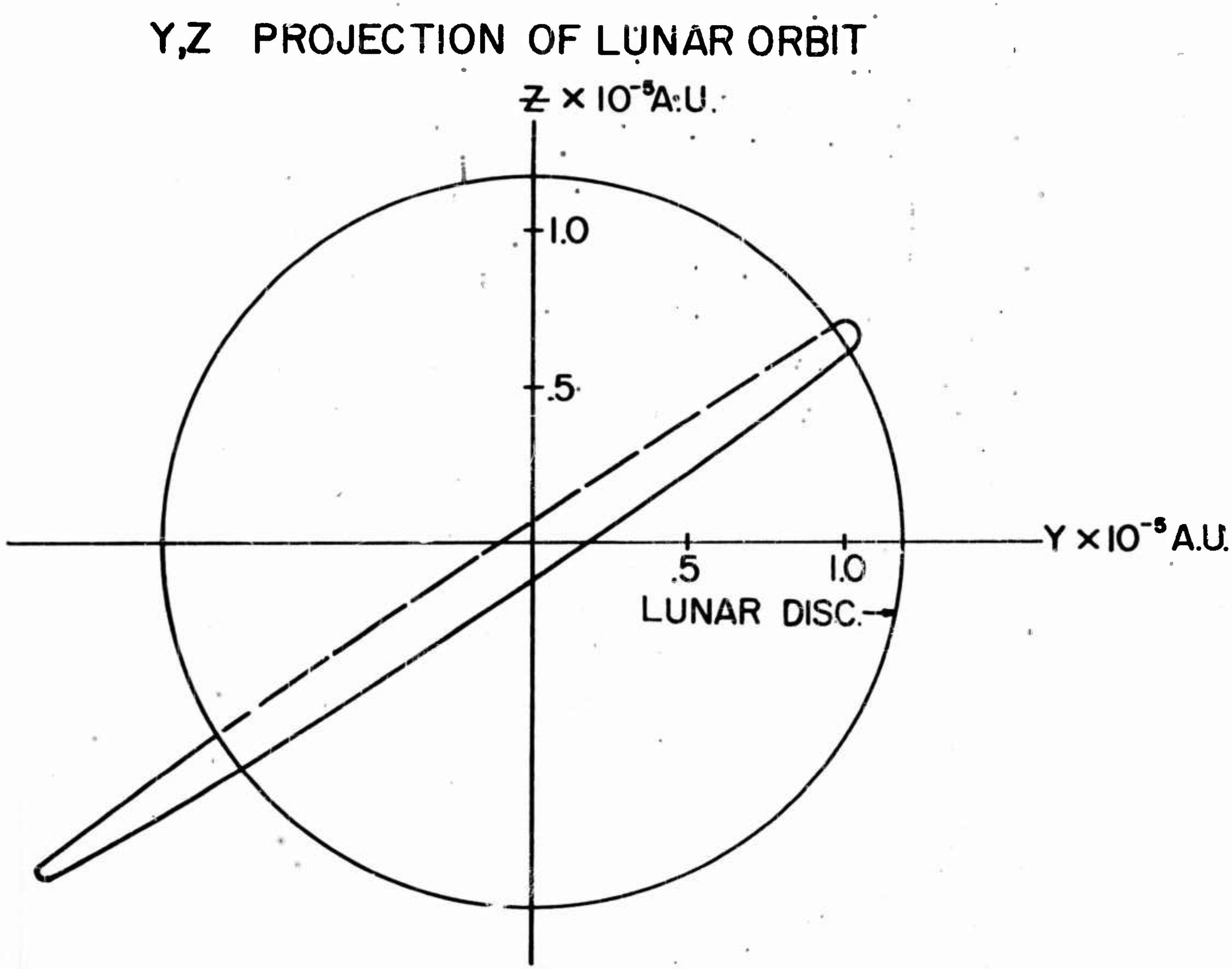

Y,Z PROJECTION OF LUNAR ORBIT
Z × 10⁻⁵ A.U.
1.0
.5
Y × 10⁻⁵ A.U.
.5
1.0
LUNAR DISC.

Figure 58.

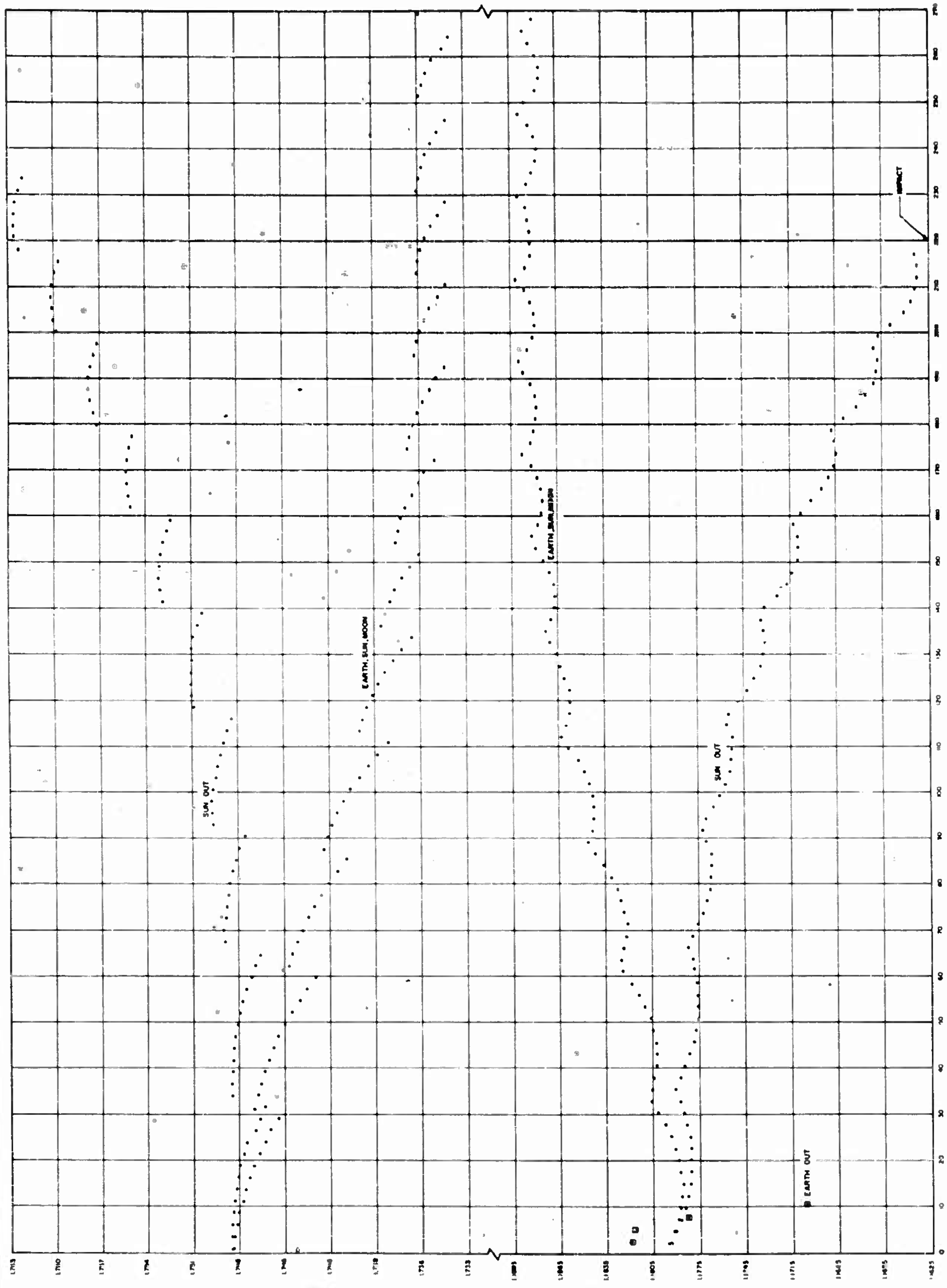

SUN OUT
EARTH, SUN, MOON
EARTH, SUN, MOON
SUN OUT
EARTH OUT
IMPACT
ORBITING TIME (HOURS)
APOLUNES
PERILUNES

Figure 59A.

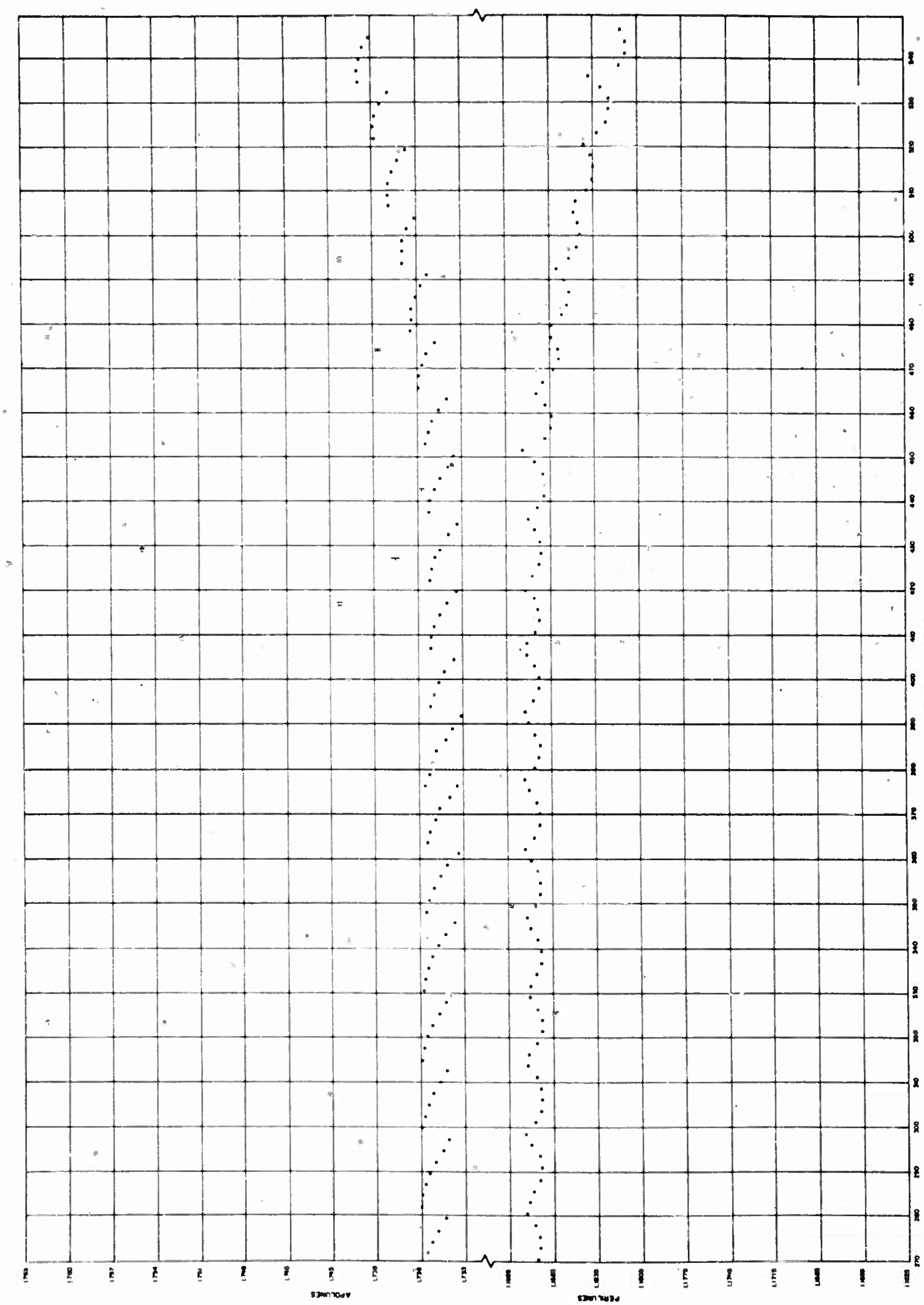


Figure 59B.

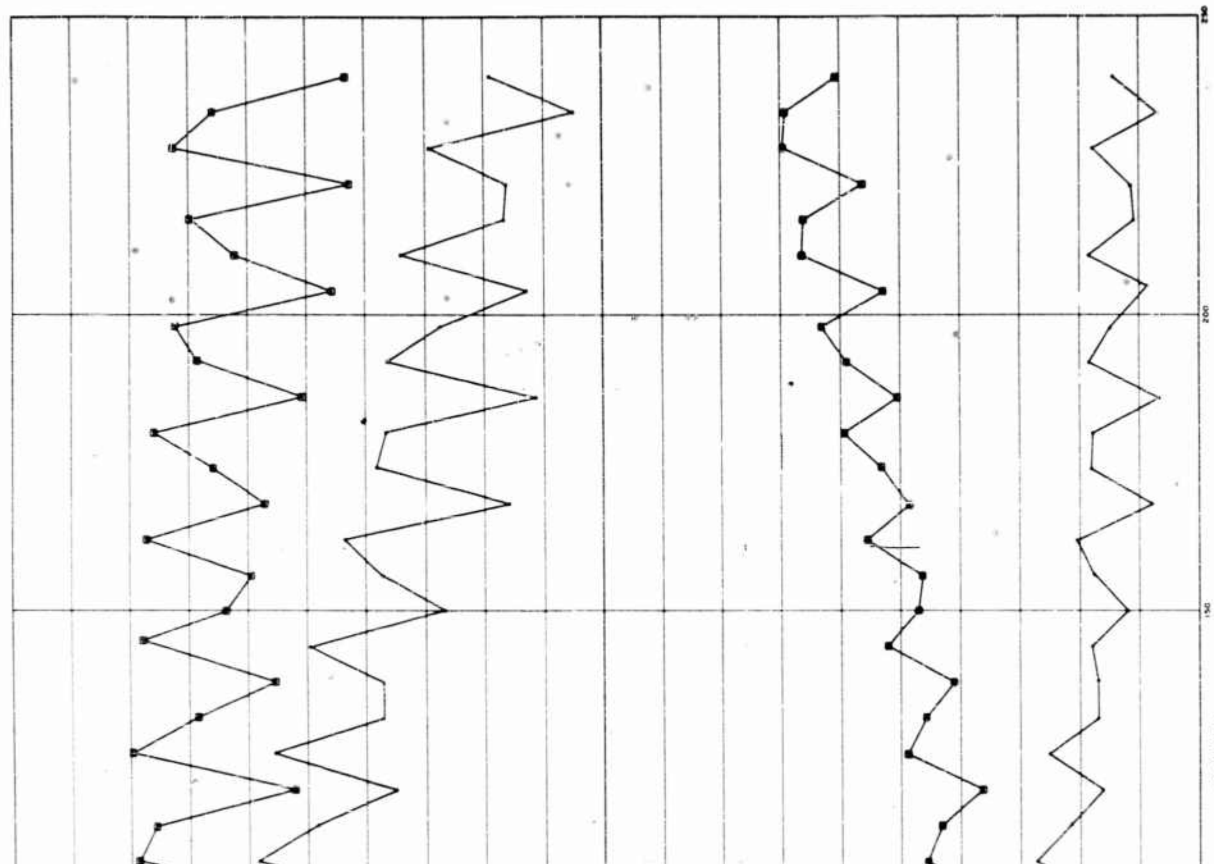
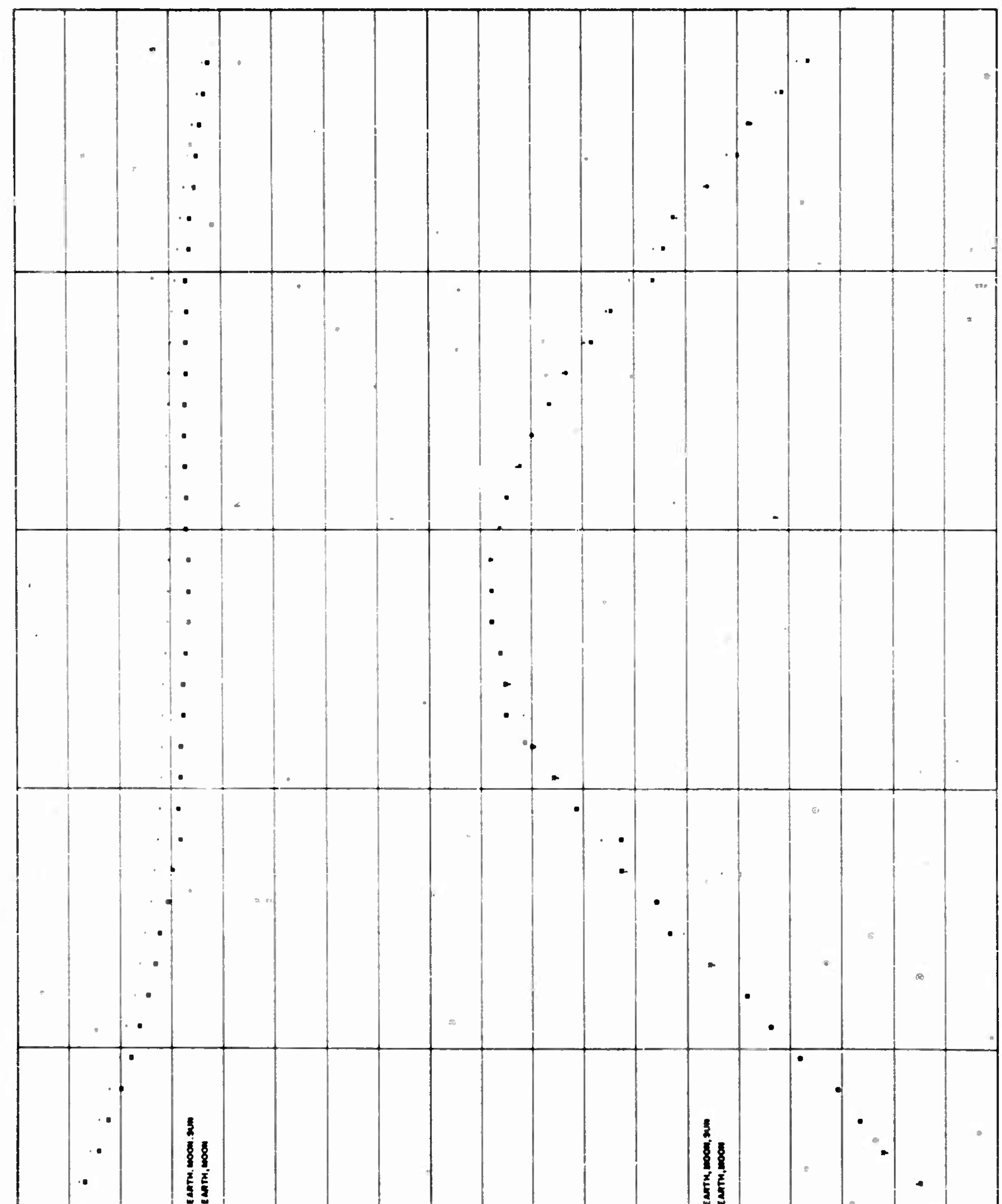


Figure 60.



EARTH, MOON, SUN
EARTH, MOON
EARTH, MOON, SUN
EARTH, MOON
Ω (DEGREES)
i (DEGREES)
TIME (HOURS)

Figure 61.

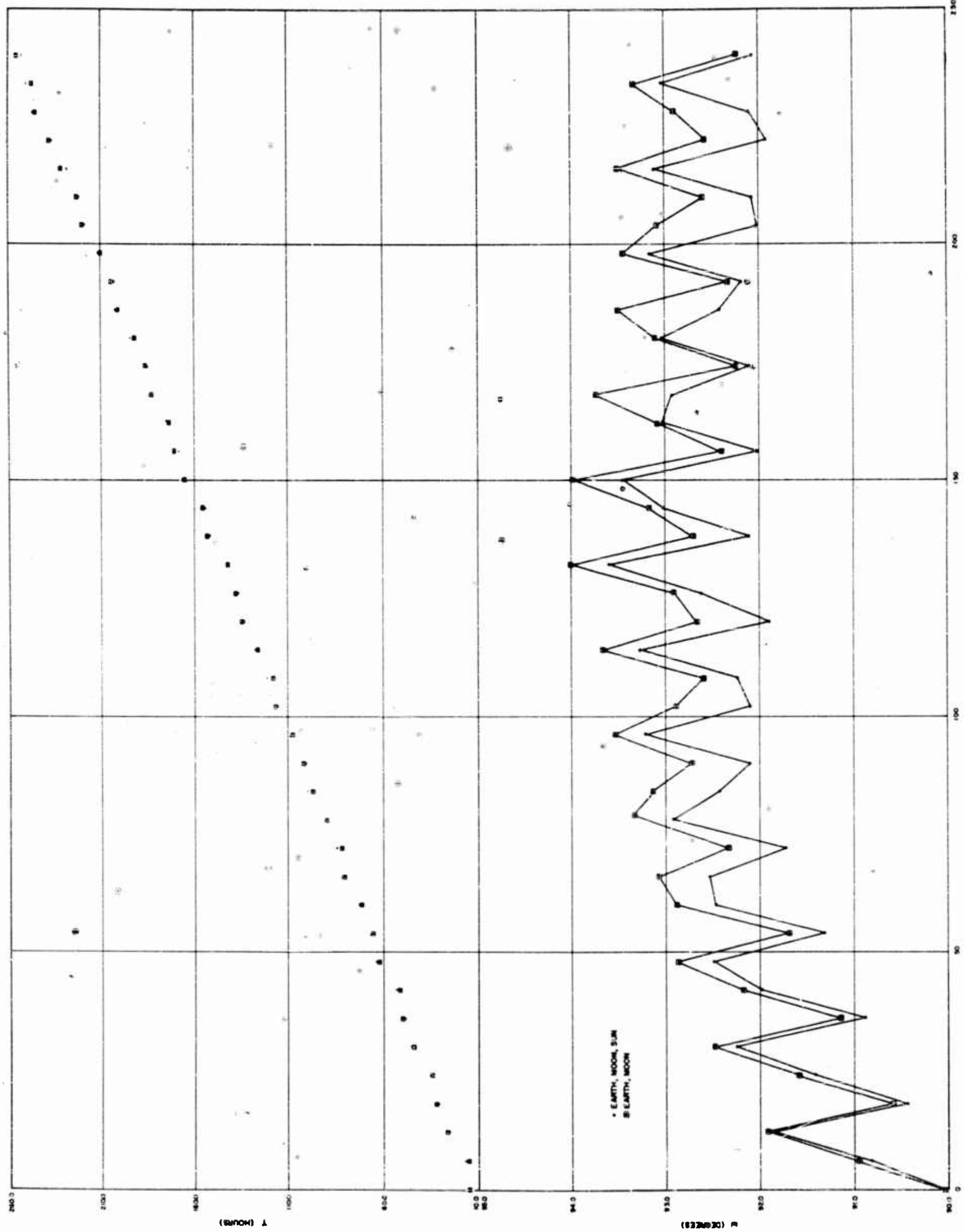


Figure 62.

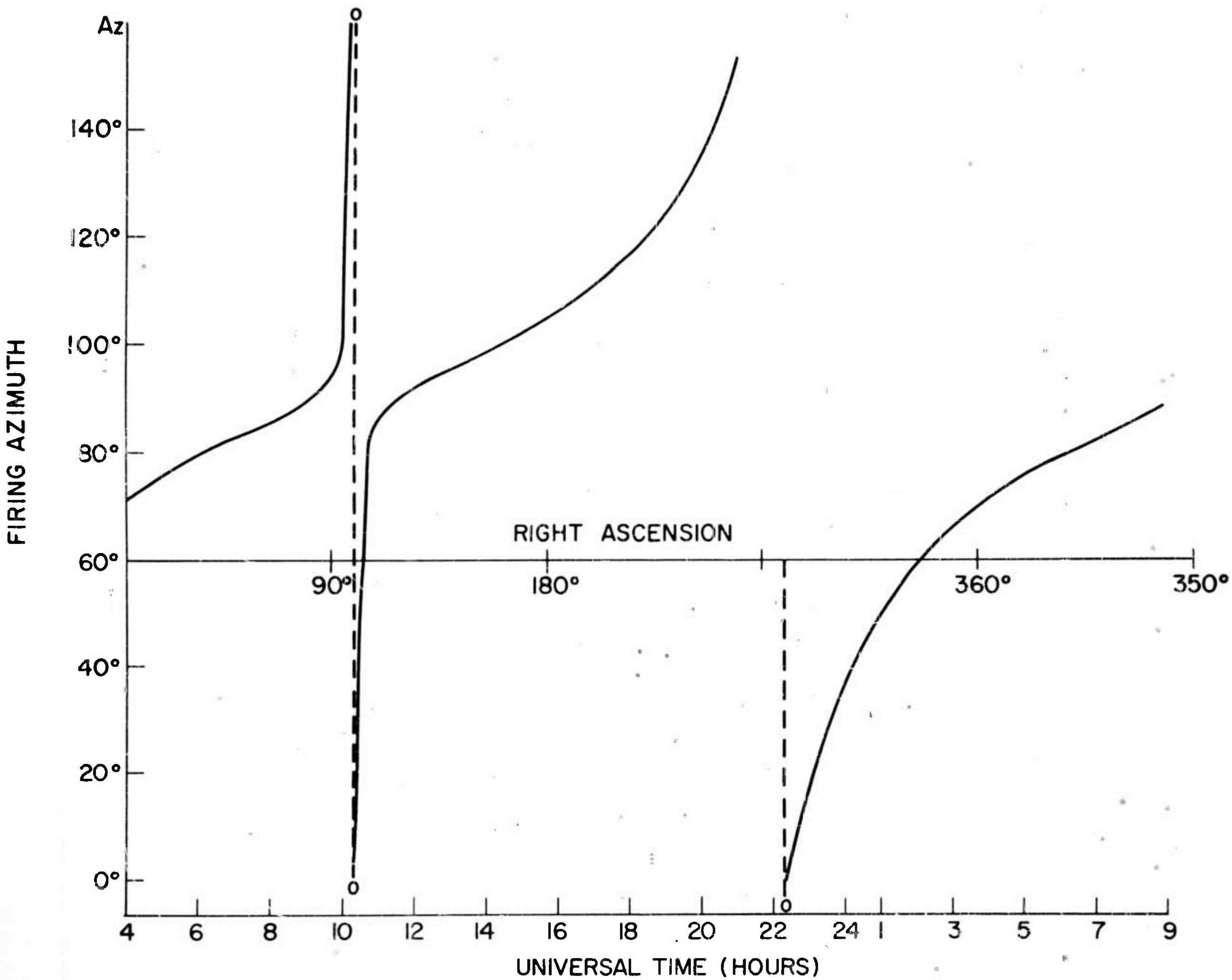
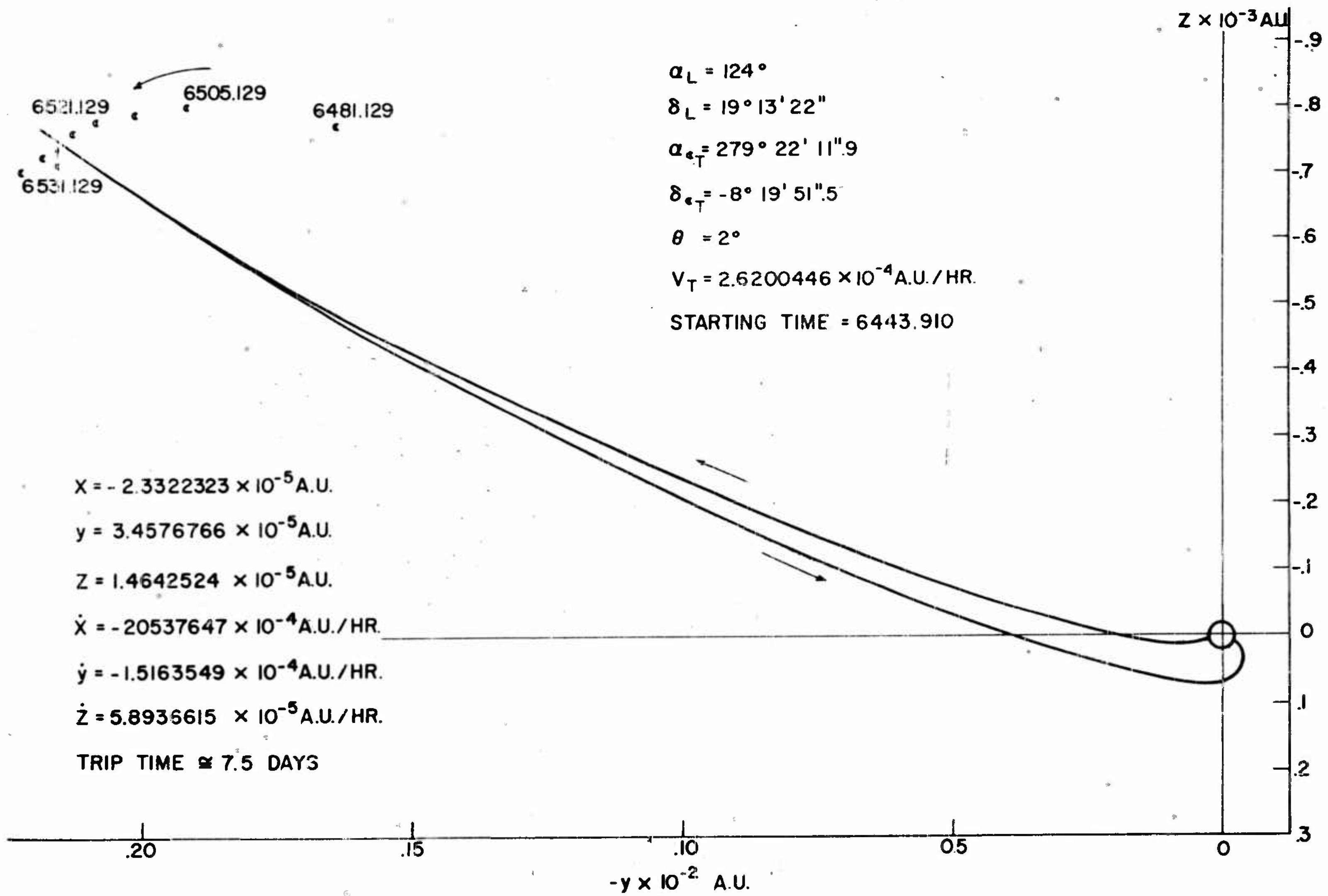
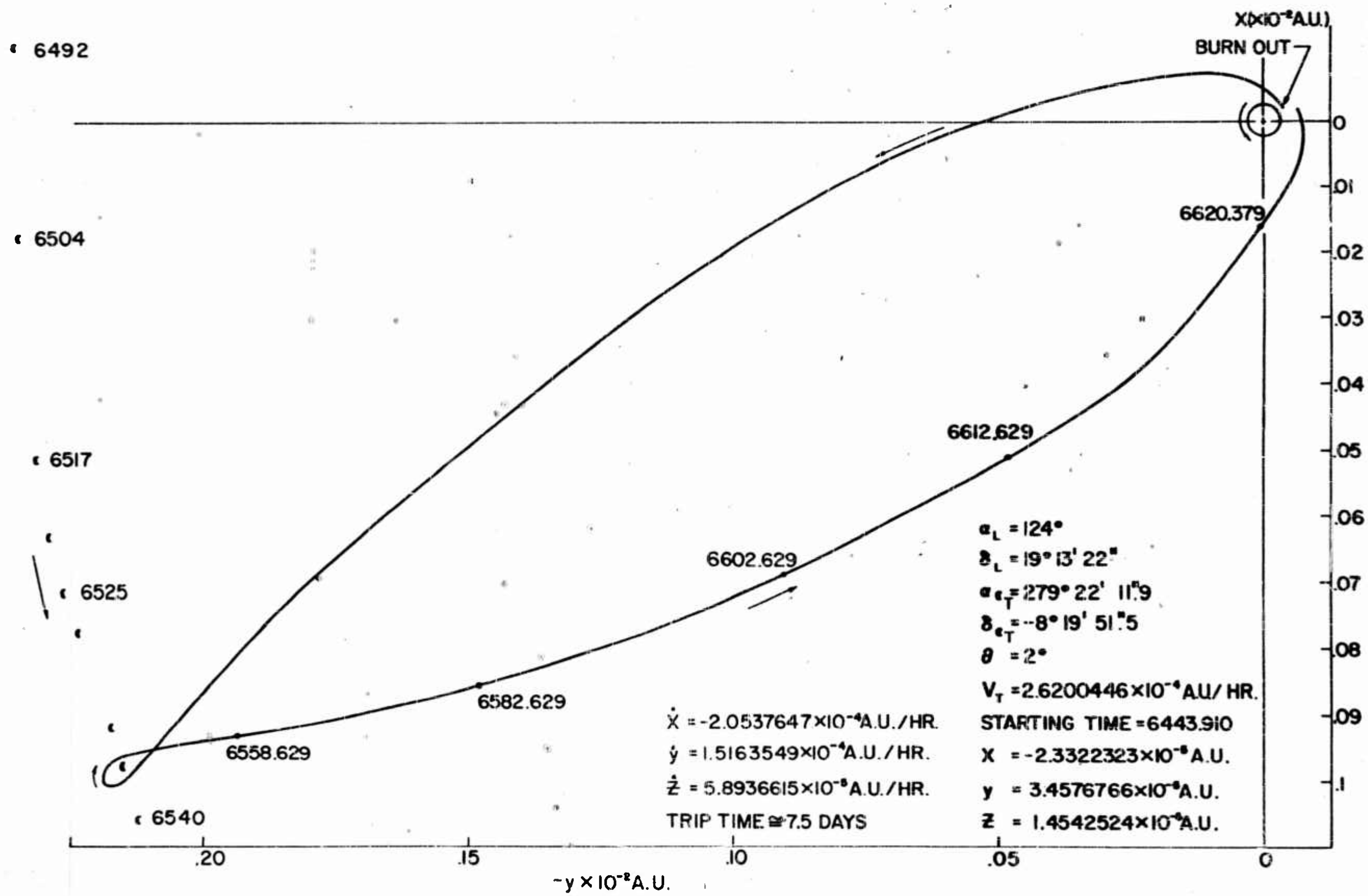

Az
140°
120°
100°
80°
60°
40°
20°
0°
FIRING AZIMUTH
RIGHT ASCENSION
90°
180°
360°
350°
0
0
0
4
6
8
10
12
14
16
18
20
22
24
1
3
5
7
9
UNIVERSAL TIME (HOURS)

Figure 63.



Z × 10^-3 A.U.
-.9
-.8
-.7
-.6
-.5
-.4
-.3
-.2
-.1
0
.1
.2
.3
α_L = 124°
δ_L = 19° 13' 22"
α_εT = 279° 22' 11".9
δ_εT = -8° 19' 51".5
θ = 2°
V_T = 2.6200446 × 10^-4 A.U./HR.
STARTING TIME = 6443.910
6521.129
6505.129
6481.129
6531.129
X = -2.3322323 × 10^-5 A.U.
y = 3.4576766 × 10^-5 A.U.
Z = 1.4642524 × 10^-5 A.U.
Ẋ = -20537647 × 10^-4 A.U./HR.
ẏ = -1.5163549 × 10^-4 A.U./HR.
Ż = 5.8936615 × 10^-5 A.U./HR.
TRIP TIME ≅ 7.5 DAYS
.20
.15
.10
0.5
0
-y × 10^-2 A.U.

Figure 64.



X(×10⁻² A.U.)
BURN OUT
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.1
6620.379
6612.629
6602.629
6582.629
6558.629
6492
6504
6517
6525
6540
α_L = 124°
δ_L = 19° 13' 22"
α_{e_T} = 279° 22' 11".9
δ_{e_T} = -8° 19' 51".5
θ = 2°
V_T = 2.6200446×10⁻⁴ A.U./HR.
STARTING TIME = 6443.910
X = -2.3322323×10⁻⁵ A.U.
y = 3.4576766×10⁻⁵ A.U.
z = 1.4542524×10⁻⁵ A.U.
$\dot{X}$ = -2.0537647×10⁻⁴ A.U./HR.
$\dot{y}$ = 1.5163549×10⁻⁴ A.U./HR.
$\dot{z}$ = 5.8936615×10⁻⁵ A.U./HR.
TRIP TIME ≅ 7.5 DAYS
.20
.15
.10
.05
0
-y × 10⁻² A.U.

Figure 65.

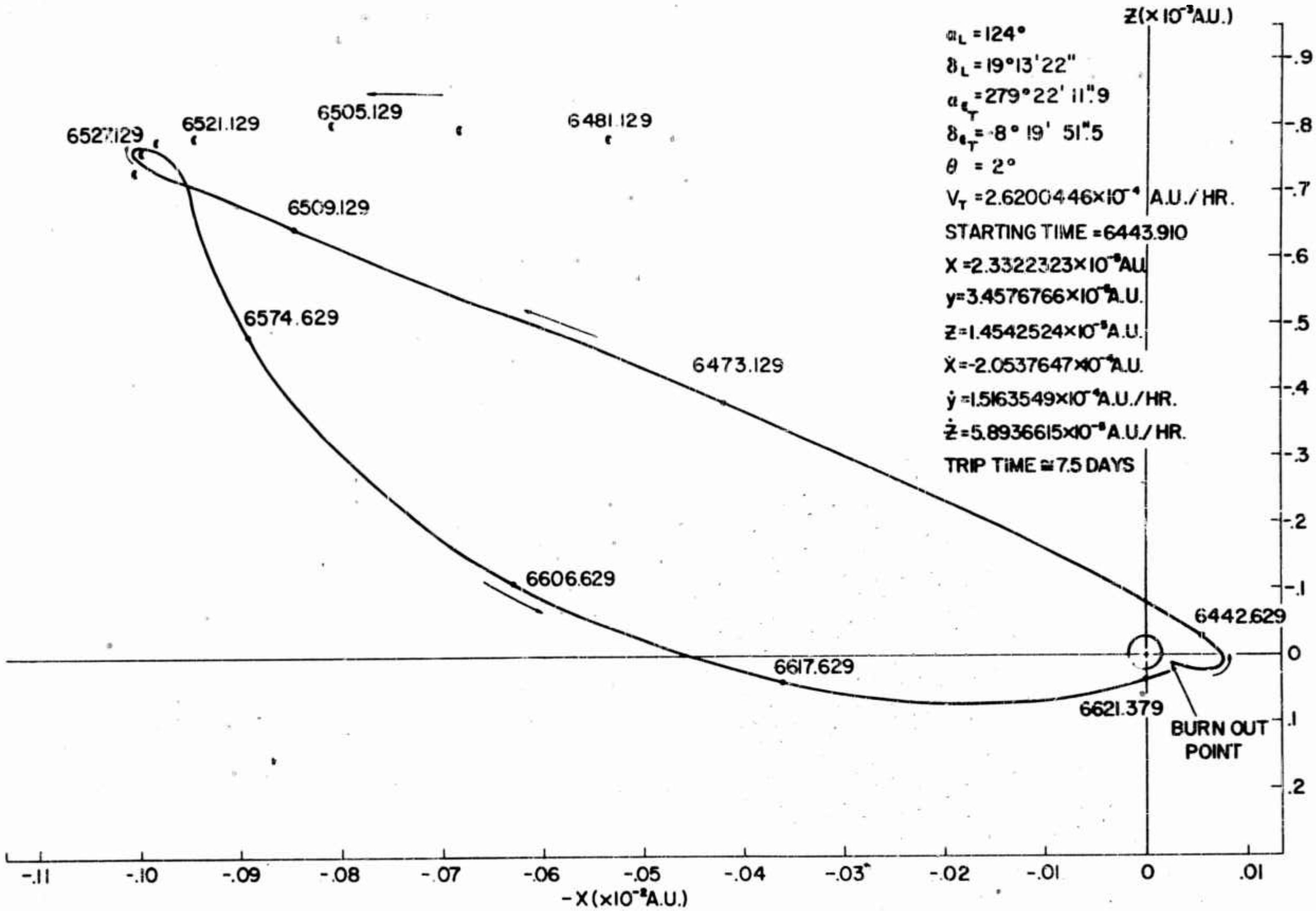

Z(×10⁻³A.U.)
α_L = 124°
δ_L = 19°13'22"
α_{e_T} = 279°22' 11".9
δ_{e_T} = -8° 19' 51".5
θ = 2°
V_T = 2.6200446×10⁻⁴ A.U./HR.
STARTING TIME = 6443.910
X = 2.3322323×10⁻⁶ A.U.
y = 3.4576766×10⁻⁶ A.U.
z = 1.4542524×10⁻⁶ A.U.
$\dot{X}$ = -2.0537647×10⁻⁴ A.U.
$\dot{y}$ = 1.5163549×10⁻⁴ A.U./HR.
$\dot{z}$ = 5.8936615×10⁻⁶ A.U./HR.
TRIP TIME ≅ 7.5 DAYS
6527.129
6521.129
6505.129
6481.129
6509.129
6574.629
6473.129
6606.629
6617.629
6442.629
6621.379
BURN OUT POINT
-X (×10⁻²A.U.)

Figure 66.

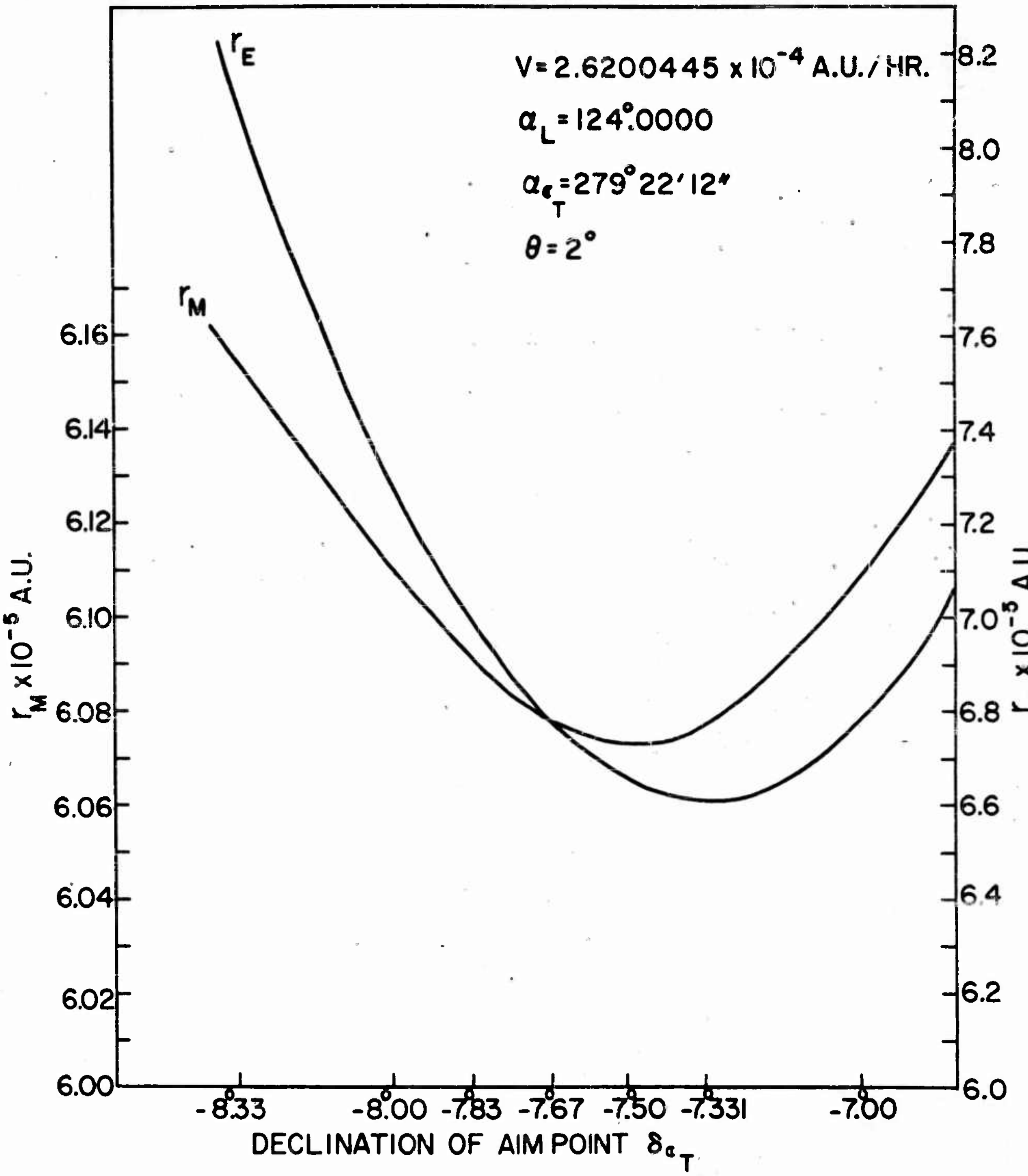


Figure 67. Distance of Closest Approach to the Moon and Earth as a Function

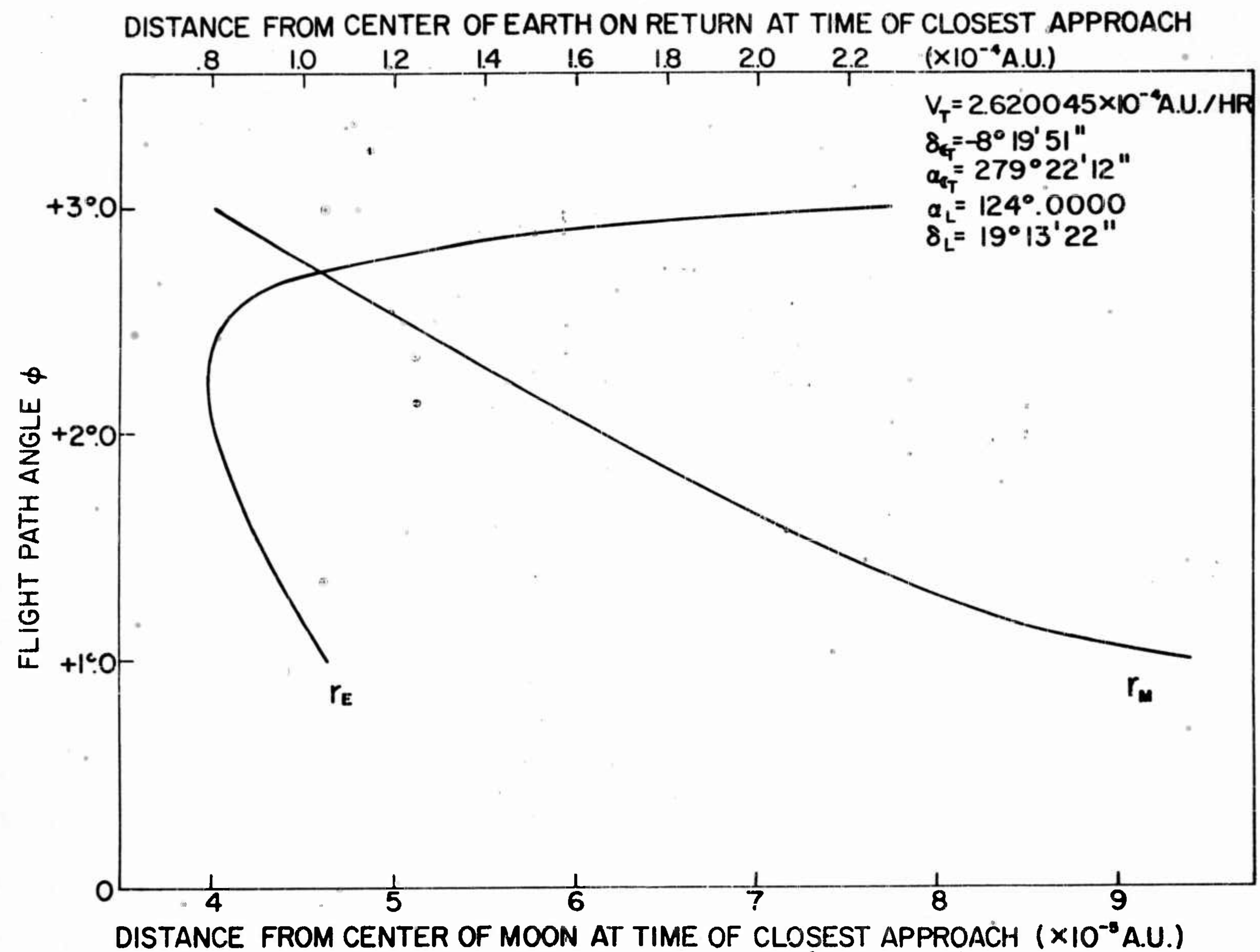

DISTANCE FROM CENTER OF EARTH ON RETURN AT TIME OF CLOSEST APPROACH
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
(×10⁻⁴ A.U.)
$V_T = 2.620045 \times 10^{-4}$ A.U./HR
$\delta_{\mathbb{C}T} = -8° 19' 51''$
$\alpha_{\mathbb{C}T} = 279° 22' 12''$
$\alpha_L = 124°.0000$
$\delta_L = 19° 13' 22''$
+3°.0
+2°.0
+1°.0
0
FLIGHT PATH ANGLE ϕ
r_E
r_M
4
5
6
7
8
9
DISTANCE FROM CENTER OF MOON AT TIME OF CLOSEST APPROACH (×10⁻⁵ A.U.)

Figure 68.

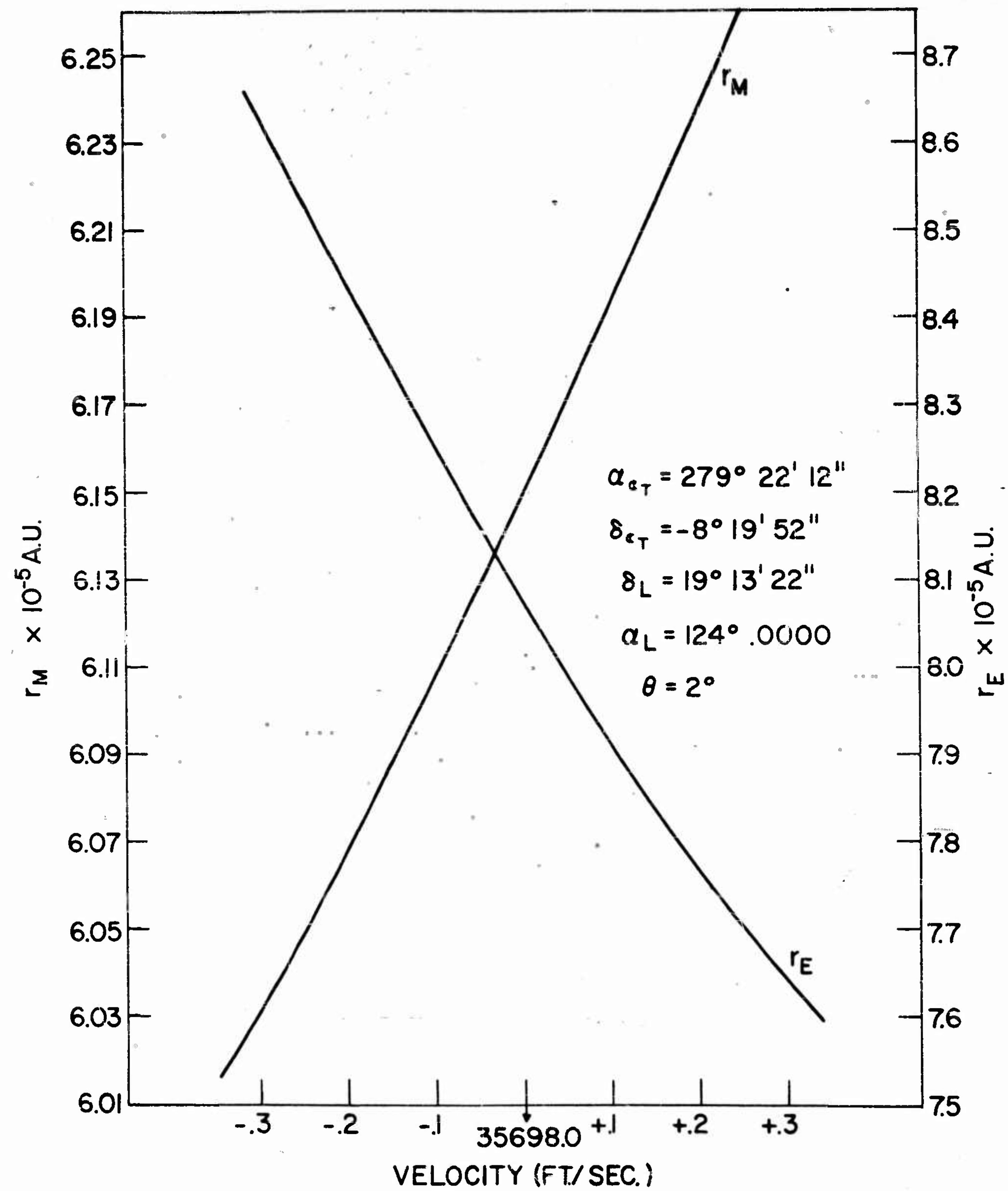

6.25
6.23
6.21
6.19
6.17
6.15
6.13
6.11
6.09
6.07
6.05
6.03
6.01
$r_M \times 10^{-5}$ A.U.
8.7
8.6
8.5
8.4
8.3
8.2
8.1
8.0
7.9
7.8
7.7
7.6
7.5
$r_E \times 10^{-5}$ A.U.
r_M
r_E
$\alpha_{\mathbb{C}_T} = 279° 22' 12''$
$\delta_{\mathbb{C}_T} = -8° 19' 52''$
$\delta_L = 19° 13' 22''$
$\alpha_L = 124° .0000$
$\theta = 2°$
-.3
-.2
-.1
35698.0
+.1
+.2
+.3
VELOCITY (FT/SEC.)

Figure 69.

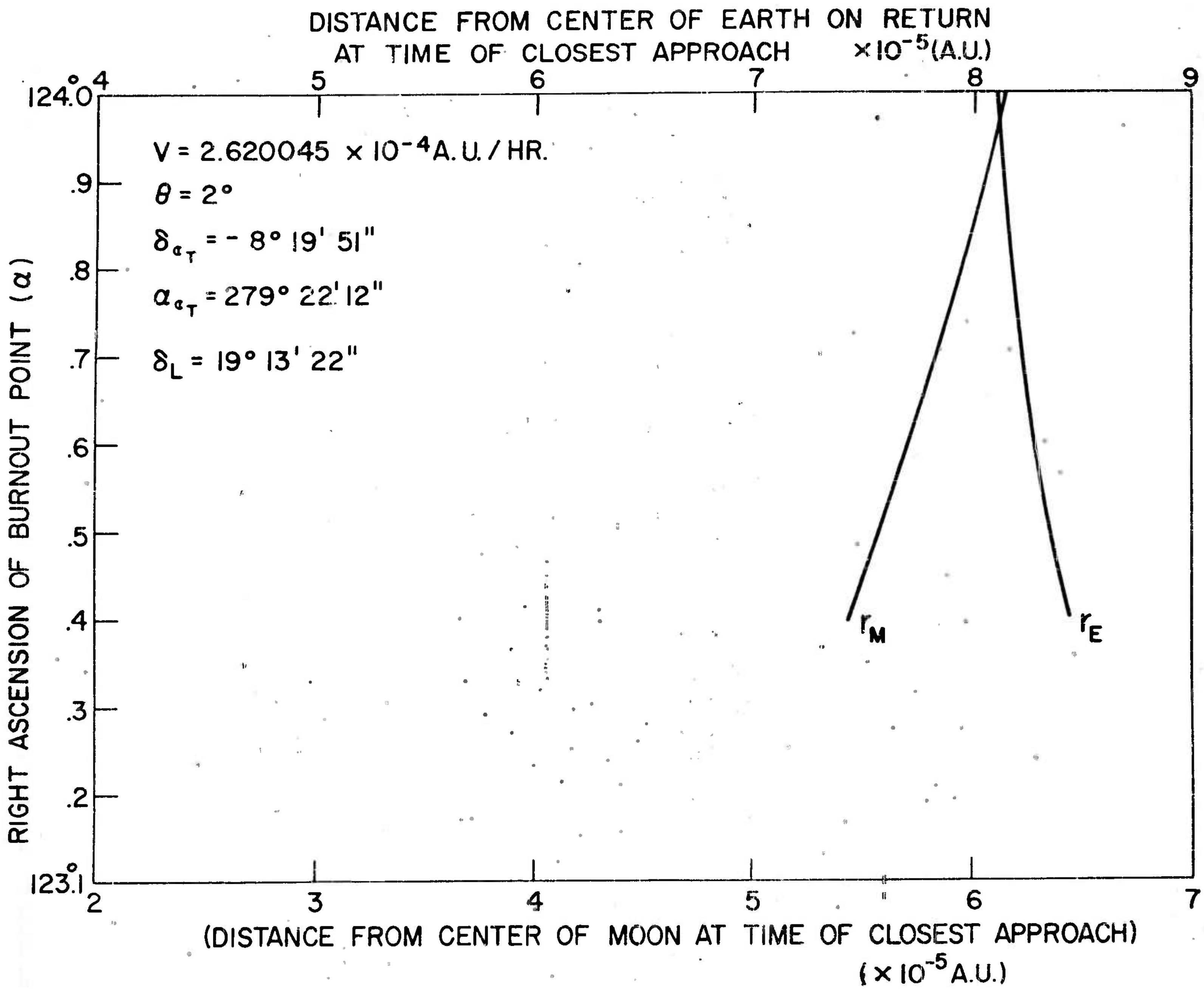
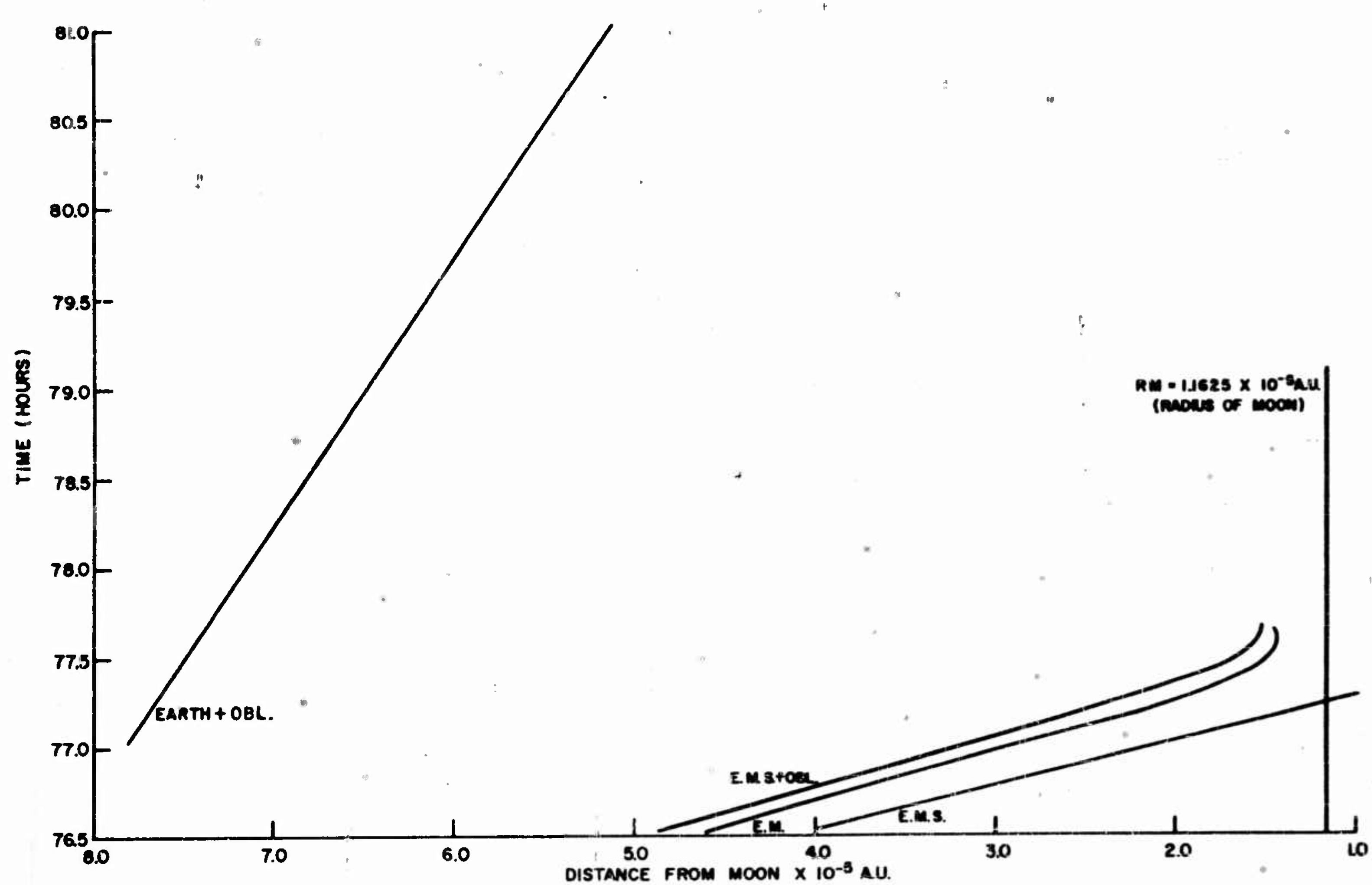

DISTANCE FROM CENTER OF EARTH ON RETURN
AT TIME OF CLOSEST APPROACH ×10^-5 (A.U.)
V = 2.620045 × 10^-4 A.U./HR.
θ = 2°
δ_αT = - 8° 19' 51"
α_αT = 279° 22' 12"
δ_L = 19° 13' 22"
r_M
r_E
RIGHT ASCENSION OF BURNOUT POINT (α)
124.0°
123.1°
(DISTANCE FROM CENTER OF MOON AT TIME OF CLOSEST APPROACH)
(× 10^-5 A.U.)

Figure 70.



TIME (HOURS)
81.0
80.5
80.0
79.5
79.0
78.5
78.0
77.5
77.0
76.5
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
DISTANCE FROM MOON X 10^-5 A.U.
EARTH + OBL.
E.M.S.+OBL.
E.M.
E.M.S.
RM = 1.1625 X 10^-5 A.U.
(RADIUS OF MOON)

Figure 71

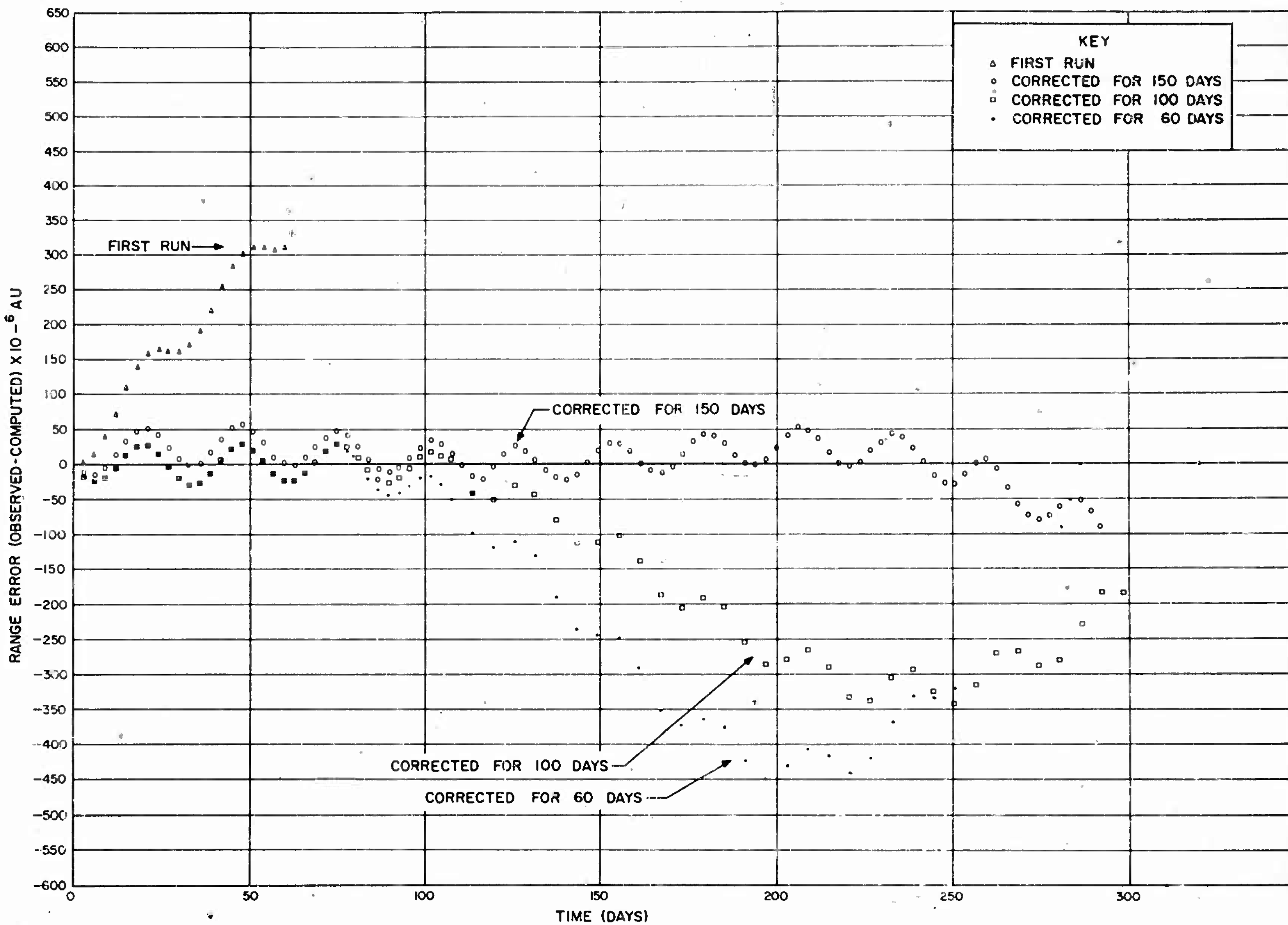


Figure 72. Pallas Ephemeris Residuals

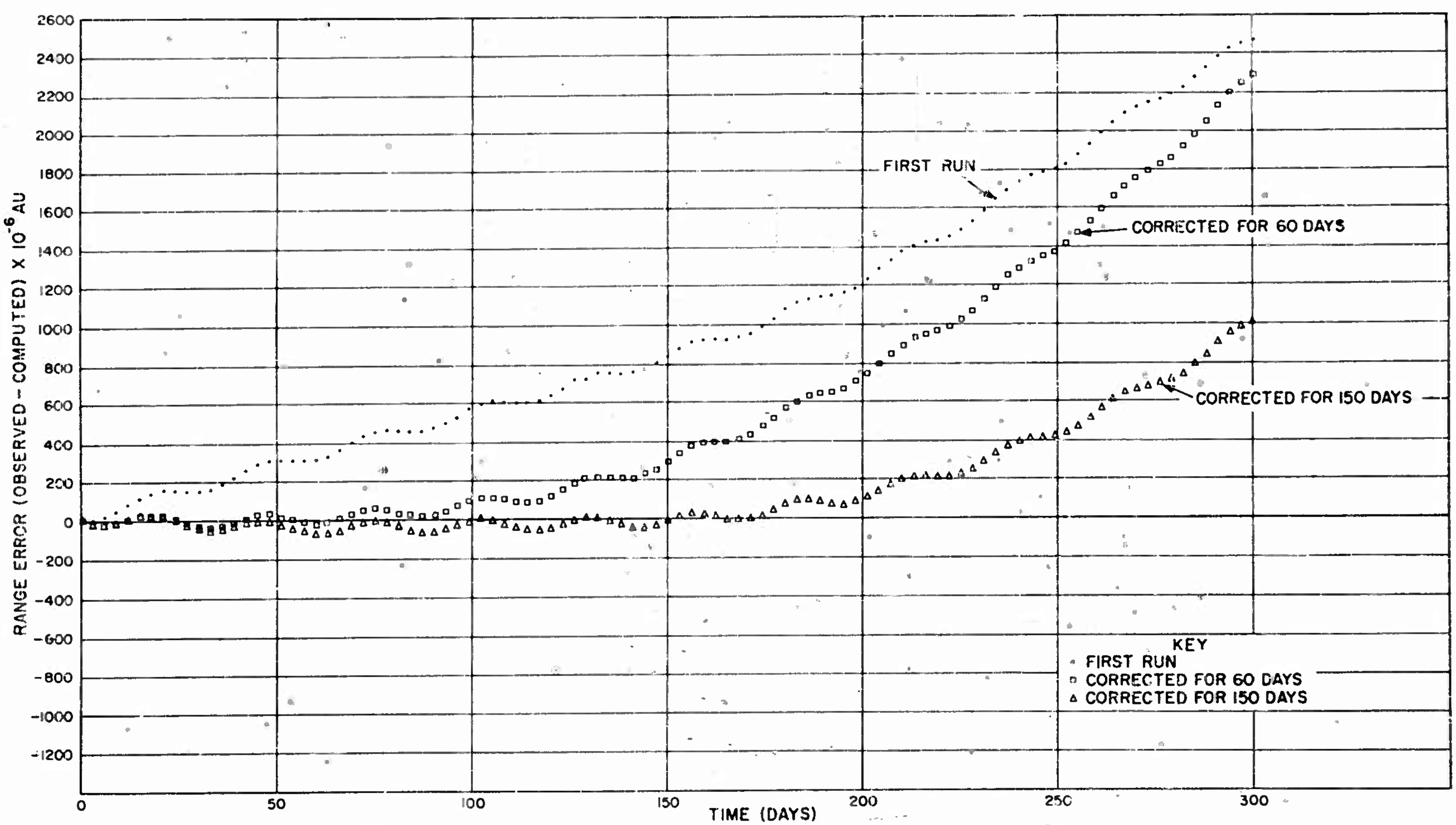


Figure 73. Vesta Ephemeris Residuals

APPENDIX A

MODIFICATION OF THE LUNAR TRAJECTORY PROGRAM TO N-BODY COMPUTER PROGRAM

This report pointed out earlier that the original Lunar Trajectory Program had been extended to cover all major bodies of the Solar System ($n = 9$). The resulting equations of motion differ in no way from those given in the Scientific Report No. 1. Thus, the equation of motion of body m_i with respect to the body m_k is given by

$$\ddot{\bar{r}}_{ik} = -k^2 \; (m_k + m_i) \; \frac{\bar{r}_{ik}}{r_{ik}^3} + k^2 \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} \left[\frac{\bar{r}_{jk} - \bar{r}_{ik}}{r_{ij}^3} - \frac{\bar{r}_{jk}}{r_{jk}^3} \right]$$

The oblateness correction remains the same as in that Report. The change to include a greater number of bodies does not affect either the method of solution or the integration method employed. The heart of the extended program - the planetary tables - was described in the Scientific Report No. 1.

The physical data appropriate for the added bodies are given in Table I of this Appendix. It is to be noted that much of the data is included simply for interest and is not used in computations.

The present Appendix also includes the operational procedure for the "n-body interplanetary trajectory program" as written for the IBM 7090 computer. The information contained in this procedure is that required by an engineer and a machine operator to set up problems on the computer.

TABLE I - ASTRONOMICAL CONSTANTS

Body	Code Digit	Mass (1/solar mass units)	Mass (solar mass units)	Equatorial radius (visible)* (10^{-4} A. U.)	Oblateness (Apparent)*	Oblateness constant (J)	Equatorial rotational angular velocity (visible)* (radian / hr)	Mean planetary distance (A. U.)	Mean daily motion(epoch Jan. 0, 1959) (degrees)	Inclination of orbit to ecliptic (epoch Jan. 0, 1959)(degrees)	Declination of North pole of planet (epoch Jan. 0, 1959) (degrees)
Earth	0	333,432	2.9991123×10^{-6}	.426636	.0034	1.6232×10^{-3}	.262516	1.0	3,548.193	0	90.0
Sun	1	1.0	1.0	46.55	0.0	0.0	.01044	0	0	0	-
Moon	2	27,158,036.4	$.36821513 \times 10^{-7}$	.11625	**	$.3761 \times 10^{-3}$	.00958212	1.002571	47,434.890	5.15	-
Mercury	3	6,000,000	$.16666667 \times 10^{-6}$	.162	0.0	0.0	.00297604	.387099	14,732.420	7.00397	unknown
Venus	4	408,000	2.4509804×10^{-6}	.414	0.0	0.0	unknown	.723332	5,767.670	3.39422	unknown
Mars	5	3,093,500	$.32325845 \times 10^{-6}$	.226	.0052	3.017×10^{-3}	.255176	1.523691	1,886.519	1.84994	54.689
Jupiter	6	1,047.355	954.78610×10^{-6}	4.78	.062	2.22×10^{-2}	.6384	5.202803	299.128	1.30542	64.551
Saturn	7	3,501.6	285.58373×10^{-6}	4.04	.096	2.501×10^{-2}	.6140	9.538843	120.455	2.48997	86.724
Uranus	8	22,869	43.727316×10^{-6}	1.60	.06	-	.5809	19.181945	42.235	.77306	14.162
Neptune	9	19,314	51.775914×10^{-6}	1.49	.02	7.4×10^{-3}	.4010	30.057767	21.532	1.77383	41.488

*For some bodies this data is for the atmosphere of the body
**Tri-axial ellipsoid

In addition, a description is given of the binary table tapes used to obtain planetary positions and a flow diagram showing the general sequence of computations and logic of the program. Finally a complete listing of the n-body trajectory program is provided.

A. COMPUTER INPUT

This section lists the computer input required for the n-body program. This covers the computation of the trajectory from an initial velocity and position.

All decimal input is read on line by a modified DBC Fortran subroutine which accepts variable length fields. A copy of the write-up for this subroutine is incorporated in Section F of this Appendix. Note that the number of fields per card is arbitrary except that the first field of each Read statement must start a new card. Also note that the first field of each Read statement must be preceded by the character identifying the type of conversion. There are eight Read statements in the program and the quantities starting these statements will be designated as such.

B. N-BODY TRAJECTORY PROGRAM (Operational Directory)

Following is the list of the input quantities and then the input for a sample problem. All fields are floating point numbers except for starred fields which are integers. The units used are:

Mass - Solar mass units.

Time - hours (Table time is time in hours from the beginning of the input table tape. It is directly related to calendar time.)

Distance - Astronomical units.

Velocity - Astronomical units/hour.

Read Statement 1

Field 1 = k^2 (gravitational constant)

Field 2 = M_e (Mass of Earth)

Field 3 = R_e (Radius of Earth)

Field 4 = M_v (Mass of Vehicle)

Field 5 = J' (Constant used for oblateness = $J . M_e . R_e$)

Read Statement 2

Field 1 = D_{emax} (Maximum distance of vehicle from Earth)

Field 2 = D_{tmax} (Maximum distance of vehicle from Target)

Field 3 = D_{smax} (Maximum distance of vehicle from Sun)

Field 4 = T_{end} (Maximum trip time of run)

Field 5 = T_{start} (Starting trip time of run)

Read Statement 3

*Field 1 = 1 if initial scheme is Encke
0 if initial scheme is Cowell

*Field 2 = 0 if Δ T is determined by ϵ test
1 if 3 fixed Δ T's are used

*Field 3 = 1 if switching computing scheme
0 if computing scheme is fixed

*Field 4 = M print output every M intervals

*Field 5 = 1 switch origins
0 do not switch origins

Read Statement 4 (used only if Field 2 of Read Statement 3 is a 0.)

Field 1 = ΔT_i (Initial Δ T)

Field 2 = ΔT_{max} (Maximum ΔT)

Field 3 = ϵ value to test minimum accuracy of integration

Read Statement 5 (used only if Field 2 of Read Statement 3 is a 1.)

Field 1 = ΔT_1 (ΔT to be used when the vehicle is within 3 radii of origin).

Field 2 = ΔT_2 (ΔT to be used when the vehicle is within 100 radii of origin).

Field 3 = ΔT_3 (ΔT to be used when the vehicle is further than 100 radii from origin).

Read Statement 6

*Field 1 = N (The number of bodies to be used other than the earth which is always included).

*Field 2 = Code (Code digit indicating the body used as the initial origin. The code is listed below).

*Field 3 = Code (Code digit of the target body).

Read Statement 7 (This statement is read once for each body to be used, except the earth. The number is obtained from Field 1 of Read Statement 6).

*Field 1 = Code (Code digit of a body to be used).

Field 2 = M_m (Mass of this body)

Field 3 = R_m (Radius of this body)

Read Statement 8

Field 1 = X (X distance of vehicle from origin)

Field 2 = Y (Y distance of vehicle from origin)

Field 3 = Z (Z distance of vehicle from origin)

Field 4 = X (X velocity WRT origin)

Field 5 = Y (Y velocity WRT origin)

Field 6 = Z (Z velocity WRT origin)

Code Digits of Bodies

0 = Earth

1 = Sun

2 = Moon

3 = Mercury

4 = Venus

5 = Mars

6 = Jupiter

7 = Saturn

8 = Uranus

9 = Neptune

SAMPLE TRAJECTORY

Description:

Cowell method only.

Earth is origin and remains as origin.

Other bodies included are -

Sun

Moon (target)

Initial Time is 6:00 A. M., January 5, 1960

Maximum Flight time is 30 days (720 hrs.)

Starting time of table tape is 0.00, January 1, 1960.

$X = 4.82537 \times 10^{-5}$

$Y = -4.18 \times 10^{-5}$

$Z = 4.22 \times 10^{-6}$

$\dot{X} = -1.44092 \times 10^{-4}$

$\dot{Y} = 1.967513 \times 10^{-4}$

$\dot{Z} = 9.23 \times 10^{-6}$

The ΔT is to be determined by the program using an ϵ of 4.0×10^{-10} and an initial ΔT of 1 hr. and a maximum ΔT of 8 hrs.

Input Cards:

1. F5.1373647E-7, 2.99911226E-6, 4.263E-5, 0, -8.846148E-18*
2. F.5, .5, 1, 822, 102*
3. X0, 0, 0, 1, 0*
4. F1, 8, 4E-10*
5. X2, 0, 2*
6. X1, F1.0, 4.646E-3*
7. X2, F3.68215133E-8, 1.1637E-5*
8. F4.82537E-5, -4.18E-5, 4.22E-6*
9. -1.44092E-4, -1.967513E-4, -9.23E-6*

C. COMPUTER OUTPUT

The following information for the n-body integration program is printed-out after every n integration steps, where n is an input-control parameter.

1. Flight time, hours from start of trajectory
2. Table time, hours from beginning of tape
3. Time increment of integration step, hours
4. Planetary code digit of body at the origin
5. Acceleration components of the vehicle with respect to the origin, A.U./hr^2
6. Velocity components of the vehicle with respect to the origin, A.U./hr
7. Position coordinates of the vehicle with respect to the origin, A.U.

8. Position coordinates of the vehicle with respect to the Earth, A. U.

9. Position coordinates of the vehicle with respect to the target, A. U.

10. Position coordinates of the vehicle with respect to the Sun, A. U.

D. OPERATING NOTES FOR THE N-BODY TRAJECTORY PROGRAM

1. Tapes Used

   All tapes used in normal Fortran System. Tape B-6, Planet Position Tables Tape.

2. Sense Switches

   Sense switch I is the only switch tested. In the down position, output will be printed on-line as well as on tape A-3. The sense switch may be repositioned at any time. Printing on line uses the Share No. 2 printer board.

3. Card Deck

   Normal Fortran system deck set-up with input cases following. Operation is initiated by the Fortran system.

4. Stops

   a. All Fortran stops

   b. Stops of form HTR*. These are double precision subroutine stops caused by overflow. Experience has shown these to be due to machine error or input error.

   c. HPR 77777 - Error in two-body solution.

* To begin processing another case when machine has stopped, manually transfer to 171_8.

E. DESCRIPTION OF BINARY-TABLE TAPES

The binary-table tapes give the "table time" and position coordinates of the Sun, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and the Moon at intervals of 12 hours. The positions are specified in terms of rectangular, geocentric equatorial coordinates referenced to the mean equator and equinox of 1950.0. Table Time is time in hours from the beginning of the tape; i. e., the first time on the tape is zero. Zero time may correspond to any given calendar time; however, it should be chosen to anticipate future requirements for the following reasons:

1. It is desirable to minimize the time used by the computer in searching the tape. This time is minimized if the starting time is near "table time" zero.
2. The writing of tapes should be kept to a minimum due to the relatively long time required to write a tape.

Table Time is directly related, and easily converted, to Calendar Time or Julian Time. Table Time was used because it provides certain advantages which Julian Time and Calendar Time do not provide. Calendar Time is awkward to use because the number of days per month and per year is not constant. While Julian Time does not present this problem, the numbers are of a large magnitude with respect to the units used in the program; i. e., hours. In order to reduce round-off error, it is necessary to measure the time with numbers of a smaller magnitude than those provided by Julian Time.

Zero time for the tape is January 1, 1960.0 Ephemeris Time, and the corresponding Julian day is 2436934.5. A table relating table day to Julian date and Calendar date may be found in Section G of this Appendix. The time of day for each listing is 0^h E. T., and therefore the Table Time in hours from the beginning of the tape for any given day may be found by multiplying the table-day by 24 hours. The table in Section G covers the period from January 1, 1960.0 to December 31, 1964.0.

The binary tape is composed of one tape with the actual arrangement of the binary information as follows:

WORD	1	TIME
WORD	2	X for body 1
WORD	3	Y for body 1
WORD	4	Z for body 1
WORD	5	X for body 2
WORD	6	Y for body 2
WORD	7	Z for body 2
.		. . . .
.		. . . .
.		. . . .
WORD	26	X for body 9
WORD	27	Y for body 9
WORD	28	Z for body 9
WORD	29	Time
WORD	30	X for body 1
WORD	31	Y for body 1
WORD	32	Z for body 1
WORD	33	X for body 2

etc.

This information is grouped by taking 60 sets of data totaling 1680 words (a set is composed of a time and all the associated positions) and writing one physical tape record, except for the first record which has only 5 sets of data, totaling 140 words.

F. DESCRIPTION OF MODIFIED DBC FORTRAN INPUT ROUTINE

This section describes a Fortran II BCD input-routine capable of accepting variable-length fields.

1. Replace the DBC subprogram and its control card with deck input 00-.

2. The lists for "READ" and "READ INPUT TAPE" remain the same. Since format statements are ignored these may be anything which permits legal compilation. (Exception: see Hollerith No. 6 below).

3. a. Fields must be separated by commas.

   b. Blanks are treated as zeros and blank cards are ignored.

   c. The number of fields per card is completely arbitrary and may vary from run to run. The first field of each read statement must start a new card.

   d. The last field of every card must be terminated by an asterisk.

   e. The type of conversion is determined by the first character of a field. Omission of this character results in continuing the previous type conversion. (The first field for each read statement must contain a type conversion entry.)

   f. Signs are optional.

4. Floating Point Conversion

   a. Identifying character F

   b. Powers of 10 may be represented by the exponent preceded by an E.

   c. Decimal points are optional. When not included it is assumed to be following the least significant digit of the number.

   d. Decimal points are illegal for exponents.

Examples:

F + 5.8E.1, F58, F.58E2, F58., F580.E-1,	Acceptable formats of the same number

5. Integer Conversion

   a. Identifying Character X.

   b. The converted number modulo 32767 is stored in the decrement.

   c. Decimal points should not be punched.

   Examples:

X150, X15-, X--150,	Acceptable formats of the same number

6. Hollerith Conversion (Used only to replace format statements)

   a. Format statements may be replaced (but not the concluding 7's) by the Hollerith characters if the associated "READ" statement has no list.

   b. Identifying Character H

   c. The number of BCD words and a comma must immediately follow the H with no blanks.

   d. Maximum number of BCD words 99.

   e. Column 1 of any continuation card follows column 72 of the previous card.

Examples:

H4, ---(312, 7H1TEST- = E12.4-)

7. Examples:

a. The following is an example of a read statement, and cards that could be used. (Format 5 is ignored.)

READ 5, N, L, J. (A(I), I = 1, J), (B(M), M = 1, L)

Deck 1

Card 1 X5, 2, ---4, F3.24E-08, -8.2E.4, 1, F9.2*

Card 2 3.2, 5.8E + 4*

Deck 2

Card 1 X5, X2, X4, F.324E-7, 82---, 1.0*

Card 2 92E-1, 320E-02*

Card 3 58.E3*

Deck 3

Card 1 X-------5, ------2*

Card 2

Card 3 F.00324E-05, -.82E + 5, .1E1, 9.2*

Card 4 3.2*

Card 5 5800-*

Error Stops:

Stop	Error	Action to be taken
HPR-1, 1	Card does not end with *or number of BCD words undefined.	Correct card, ready reader, and start.

Stop	Error	Action to be taken
HPR-1, 2	No comma between fields.	Start to treat unidentified character as a comma.
HPR-1, 3	Field undefined.	Start to treat as floating point conversion.

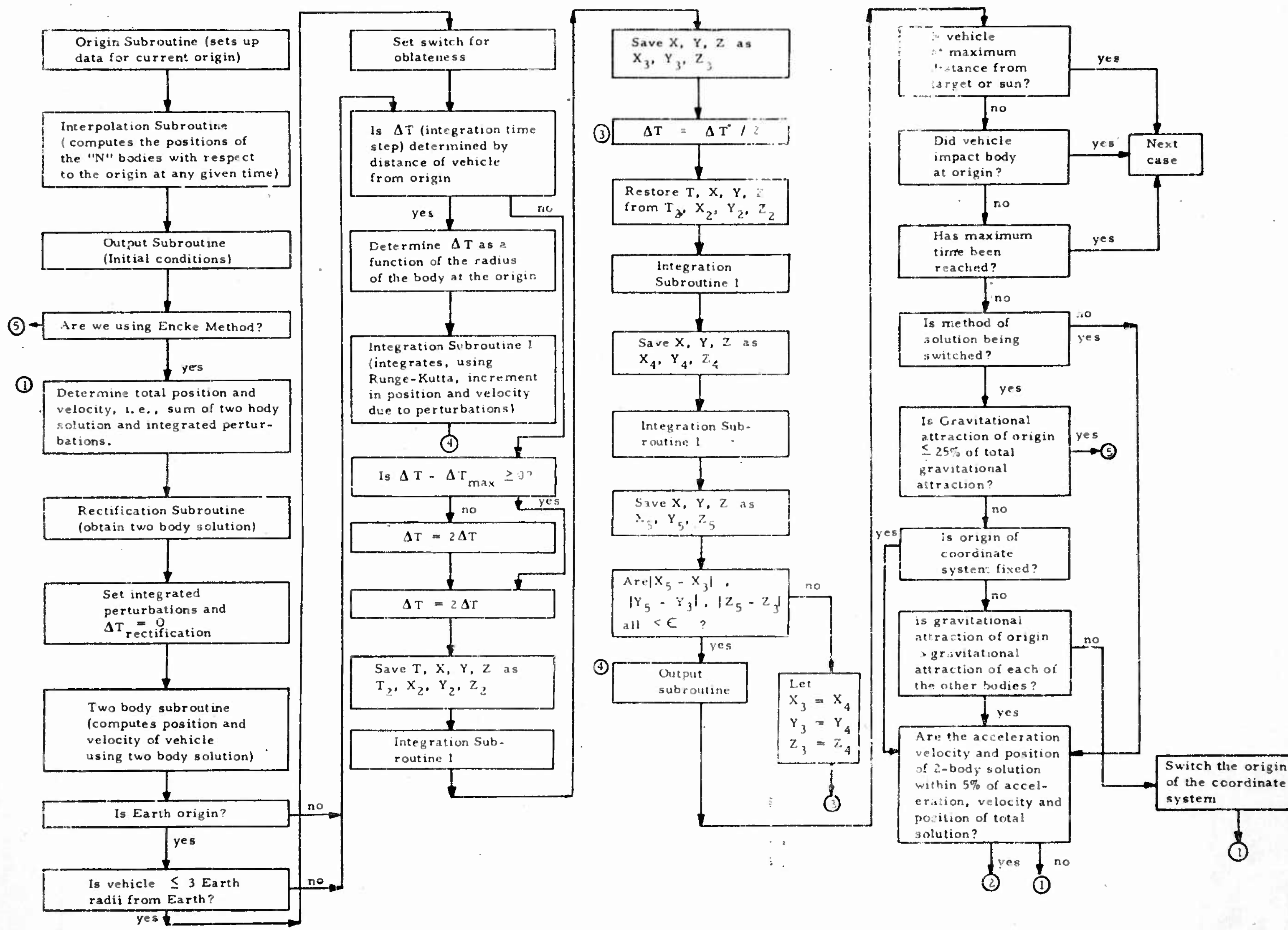


Figure A-1a. Flow Chart of the N-Body Program

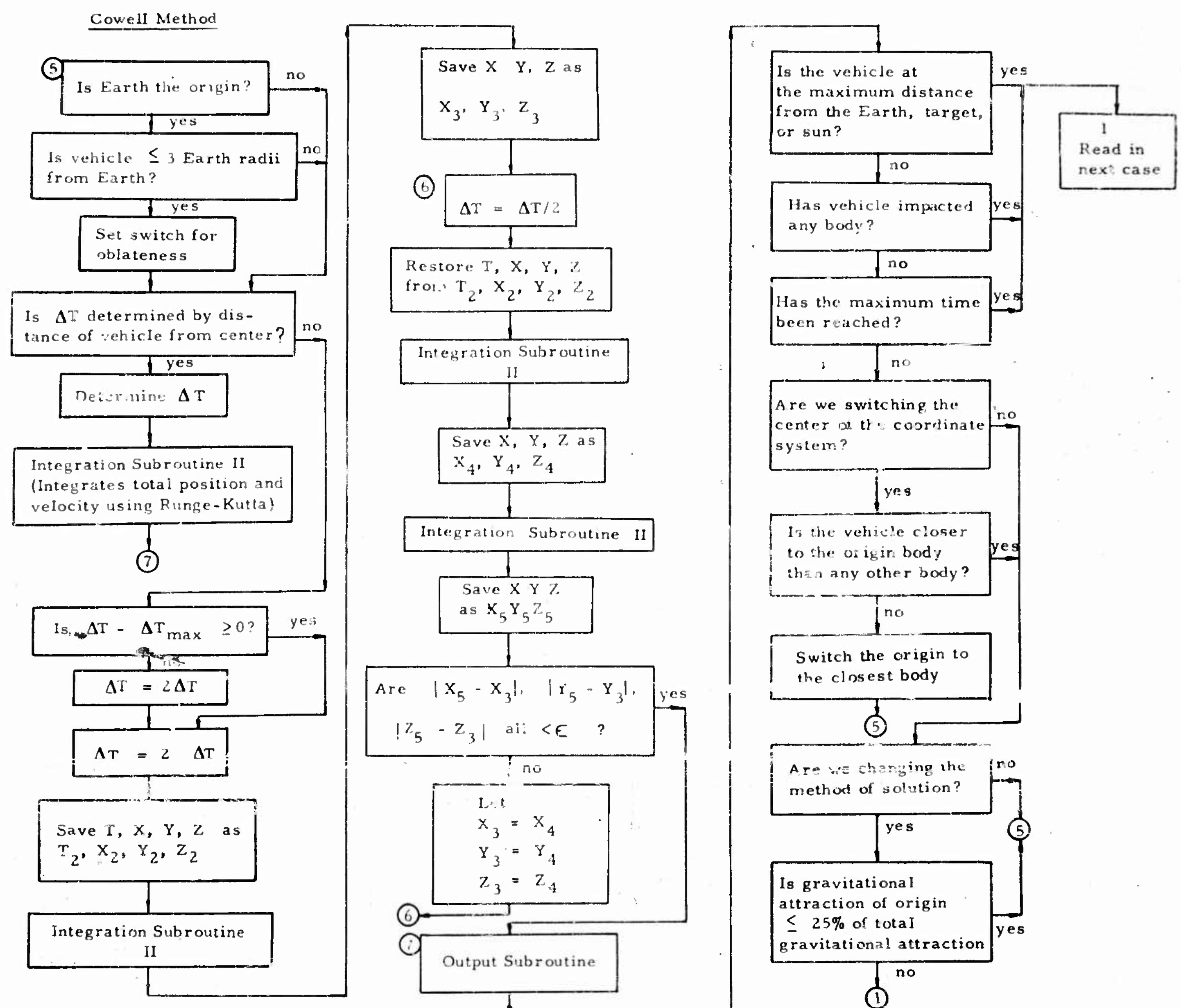


Figure A-1b. Flow Chart of the N-Body Program

G. JULIAN DATE AND CALENDAR DATE VS. TABLE-DAY

Julian Day	Calendar Day (O^h E. T.)	Table Day	Julian Day	Calendar Day (O^h E. T.)	Table Day
2436934.5	JAN 1, 1960		2436993.5	FEB 29, 1960	59
2436935.5	JAN 2, 1960	1	2436994.5	MAR 1, 1960	60
2436936.5	JAN 3, 1960	2	2436995.5	MAR 2, 1960	61
2436937.5	JAN 4, 1960	3	2436996.5	MAR 3, 1960	62
2436938.5	JAN 5, 1960	4	2436997.5	MAR 4, 1960	63
2436939.5	JAN 6, 1960	5	2436998.5	MAR 5, 1960	64
2436940.5	JAN 7, 1960	6	2436999.5	MAR 6, 1960	65
2436941.5	JAN 8, 1960	7	2437000.5	MAR 7, 1960	66
2436942.5	JAN 9, 1960	8	2437001.5	MAR 8, 1960	67
2436943.5	JAN 10, 1960	9	2437002.5	MAR 9, 1960	68
2436944.5	JAN 11, 1960	10	2437003.5	MAR 10, 1960	69
2436945.5	JAN 12, 1960	11	2437004.5	MAR 11, 1960	70
2436946.5	JAN 13, 1960	12	2437005.5	MAR 12, 1960	71
2436947.5	JAN 14, 1960	13	2437006.5	MAR 13, 1960	72
2436948.5	JAN 15, 1960	14	2437007.5	MAR 14, 1960	73
2436949.5	JAN 16, 1960	15	2437008.5	MAR 15, 1960	74
2436950.5	JAN 17, 1960	16	2437009.5	MAR 16, 1960	75
2436951.5	JAN 18, 1960	17	2437010.5	MAR 17, 1960	76
2436952.5	JAN 19, 1960	18	2437011.5	MAR 18, 1960	77
2436953.5	JAN 20, 1960	19	2437012.5	MAR 19, 1960	78
2436954.5	JAN 21, 1960	20	2437013.5	MAR 20, 1960	79
2436955.5	JAN 22, 1960	21	2437014.5	MAR 21, 1960	80
2436956.5	JAN 23, 1960	22	2437015.5	MAR 22, 1960	81
2436957.5	JAN 24, 1960	23	2437016.5	MAR 23, 1960	82
2436958.5	JAN 25, 1960	24	2437017.5	MAR 24, 1960	83
2436959.5	JAN 26, 1960	25	2437018.5	MAR 25, 1960	84
2436960.5	JAN 27, 1960	26	2437019.5	MAR 26, 1960	85
2436961.5	JAN 28, 1960	27	2437020.5	MAR 27, 1960	86
2436962.5	JAN 29, 1960	28	2437021.5	MAR 28, 1960	87
2436963.5	JAN 30, 1960	29	2437022.5	MAR 29, 1960	88
2436964.5	JAN 31, 1960	30	2437023.5	MAR 30, 1960	89
2436965.5	FEB 1, 1960	31	2437024.5	MAR 31, 1960	90
2436966.5	FEB 2, 1960	32	2437025.5	APR 1, 1960	91
2436967.5	FEB 3, 1960	33	2437026.5	APR 2, 1960	92
2436968.5	FEB 4, 1960	34	2437027.5	APR 3, 1960	93
2436969.5	FEB 5, 1960	35	2437028.5	APR 4, 1960	94
2436970.5	FEB 6, 1960	36	2437029.5	APR 5, 1960	95
2436971.5	FEB 7, 1960	37	2437030.5	APR 6, 1960	96
2436972.5	FEB 8, 1960	38	2437031.5	APR 7, 1960	97
2436973.5	FEB 9, 1960	39	2437032.5	APR 8, 1960	98
2436974.5	FEB 10, 1960	40	2437033.5	APR 9, 1960	99
2436975.5	FEB 11, 1960	41	2437034.5	APR 10, 1960	100
2436976.5	FEB 12, 1960	42	2437035.5	APR 11, 1960	101
2436977.5	FEB 13, 1960	43	2437036.5	APR 12, 1960	102
2436978.5	FEB 14, 1960	44	2437037.5	APR 13, 1960	103
2436979.5	FEB 15, 1960	45	2437038.5	APR 14, 1960	104
2436980.5	FEB 16, 1960	46	2437039.5	APR 15, 1960	105
2436981.5	FEB 17, 1960	47	2437040.5	APR 16, 1960	106
2436982.5	FEB 18, 1960	48	2437041.5	APR 17, 1960	107
2436983.5	FEB 19, 1960	49	2437042.5	APR 18, 1960	108
2436984.5	FEB 20, 1960	50	2437043.5	APR 19, 1960	109
2436985.5	FEB 21, 1960	51	2437044.5	APR 20, 1960	110
2436986.5	FEB 22, 1960	52	2437045.5	APR 21, 1960	111
2436987.5	FEB 23, 1960	53	2437046.5	APR 22, 1960	112
2436988.5	FEB 24, 1960	54	2437047.5	APR 23, 1960	113
2436989.5	FEB 25, 1960	55	2437048.5	APR 24, 1960	114
2436990.5	FEB 26, 1960	56	2437049.5	APR 25, 1960	115
2436991.5	FEB 27, 1960	57	2437050.5	APR 26, 1960	116
2436992.5	FEB 28, 1960	58	2437051.5	APR 27, 1960	117

2437052.5	APR 28, 1960	118	2437111.5	JUN 26, 1960	177
2437053.5	APR 29, 1960	119	2437112.5	JUN 27, 1960	178
2437054.5	APR 30, 1960	120	2437113.5	JUN 28, 1960	179
2437055.5	MAY 1, 1960	121	2437114.5	JUN 29, 1960	180
2437056.5	MAY 2, 1960	122	2437115.5	JUN 30, 1960	181
2437057.5	MAY 3, 1960	123	2437116.5	JUL 1, 1960	182
2437058.5	MAY 4, 1960	124	2437117.5	JUL 2, 1960	183
2437059.5	MAY 5, 1960	125	2437118.5	JUL 3, 1960	184
2437060.5	MAY 6, 1960	126	2437119.5	JUL 4, 1960	185
2437061.5	MAY 7, 1960	127	2437120.5	JUL 5, 1960	186
2437062.5	MAY 8, 1960	128	2437121.5	JUL 6, 1960	187
2437063.5	MAY 9, 1960	129	2437122.5	JUL 7, 1960	188
2437064.5	MAY 10, 1960	130	2437123.5	JUL 8, 1960	189
2437065.5	MAY 11, 1960	131	2437124.5	JUL 9, 1960	190
2437066.5	MAY 12, 1960	132	2437125.5	JUL 10, 1960	191
2437067.5	MAY 13, 1960	133	2437126.5	JUL 11, 1960	192
2437068.5	MAY 14, 1960	134	2437127.5	JUL 12, 1960	193
2437069.5	MAY 15, 1960	135	2437128.5	JUL 13, 1960	194
2437070.5	MAY 16, 1960	136	2437129.5	JUL 14, 1960	195
2437071.5	MAY 17, 1960	137	2437130.5	JUL 15, 1960	196
2437072.5	MAY 18, 1960	138	2437131.5	JUL 16, 1960	197
2437073.5	MAY 19, 1960	139	2437132.5	JUL 17, 1960	198
2437074.5	MAY 20, 1960	140	2437133.5	JUL 18, 1960	199
2437075.5	MAY 21, 1960	141	2437134.5	JUL 19, 1960	200
2437076.5	MAY 22, 1960	142	2437135.5	JUL 20, 1960	201
2437077.5	MAY 23, 1960	143	2437136.5	JUL 21, 1960	202
2437078.5	MAY 24, 1960	144	2437137.5	JUL 22, 1960	203
2437079.5	MAY 25, 1960	145	2437138.5	JUL 23, 1960	204
2437080.5	MAY 26, 1960	146	2437139.5	JUL 24, 1960	205
2437081.5	MAY 27, 1960	147	2437140.5	JUL 25, 1960	206
2437082.5	MAY 28, 1960	148	2437141.5	JUL 26, 1960	207
2437083.5	MAY 29, 1960	149	2437142.5	JUL 27, 1960	208
2437084.5	MAY 30, 1960	150	2437143.5	JUL 28, 1960	209
2437085.5	MAY 31, 1960	151	2437144.5	JUL 29, 1960	210
2437086.5	JUN 1, 1960	152	2437145.5	JUL 30, 1960	211
2437087.5	JUN 2, 1960	153	2437146.5	JUL 31, 1960	212
2437088.5	JUN 3, 1960	154	2437147.5	AUG 1, 1960	213
2437089.5	JUN 4, 1960	155	2437148.5	AUG 2, 1960	214
2437090.5	JUN 5, 1960	156	2437149.5	AUG 3, 1960	215
2437091.5	JUN 6, 1960	157	2437150.5	AUG 4, 1960	216
2437092.5	JUN 7, 1960	158	2437151.5	AUG 5, 1960	217
2437093.5	JUN 8, 1960	159	2437152.5	AUG 6, 1960	218
2437094.5	JUN 9, 1960	160	2437153.5	AUG 7, 1960	219
2437095.5	JUN 10, 1960	161	2437154.5	AUG 8, 1960	220
2437096.5	JUN 11, 1960	162	2437155.5	AUG 9, 1960	221
2437097.5	JUN 12, 1960	163	2437156.5	AUG 10, 1960	222
2437098.5	JUN 13, 1960	164	2437157.5	AUG 11, 1960	223
2437099.5	JUN 14, 1960	165	2437158.5	AUG 12, 1960	224
2437100.5	JUN 15, 1960	166	2437159.5	AUG 13, 1960	225
2437101.5	JUN 16, 1960	167	2437160.5	AUG 14, 1960	226
2437102.5	JUN 17, 1960	168	2437161.5	AUG 15, 1960	227
2437103.5	JUN 18, 1960	169	2437162.5	AUG 16, 1960	228
2437104.5	JUN 19, 1960	170	2437163.5	AUG 17, 1960	229
2437105.5	JUN 20, 1960	171	2437164.5	AUG 18, 1960	230
2437106.5	JUN 21, 1960	172	2437165.5	AUG 19, 1960	231
2437107.5	JUN 22, 1960	173	2437166.5	AUG 20, 1960	232
2437108.5	JUN 23, 1960	174	2437167.5	AUG 21, 1960	233
2437109.5	JUN 24, 1960	175	2437168.5	AUG 22, 1960	234
2437110.5	JUN 25, 1960	176	2437169.5	AUG 23, 1960	235

2437170.5	AUG 24, 1960	236	2437229.5	OCT 22, 1960	295
2437171.5	AUG 25, 1960	237	2437230.5	OCT 23, 1960	296
2437172.5	AUG 26, 1960	238	2437231.5	OCT 24, 1960	297
2437173.5	AUG 27, 1960	239	2437232.5	OCT 25, 1960	298
2437174.5	AUG 28, 1960	240	2437233.5	OCT 26, 1960	299
2437175.5	AUG 29, 1960	241	2437234.5	OCT 27, 1960	300
2437176.5	AUG 30, 1960	242	2437235.5	OCT 28, 1960	301
2437177.5	AUG 31, 1960	243	2437236.5	OCT 29, 1960	302
2437178.5	SEP 1, 1960	244	2437237.5	OCT 30, 1960	303
2437179.5	SEP 2, 1960	245	2437238.5	OCT 31, 1960	304
2437180.5	SEP 3, 1960	246	2437239.5	NOV 1, 1960	305
2437181.5	SEP 4, 1960	247	2437240.5	NOV 2, 1960	306
2437182.5	SEP 5, 1960	248	2437241.5	NOV 3, 1960	307
2437183.5	SEP 6, 1960	249	2437242.5	NOV 4, 1960	308
2437184.5	SEP 7, 1960	250	2437243.5	NOV 5, 1960	309
2437185.5	SEP 8, 1960	251	2437244.5	NOV 6, 1960	310
2437186.5	SEP 9, 1960	252	2437245.5	NOV 7, 1960	311
2437187.5	SEP 10, 1960	253	2437246.5	NOV 8, 1960	312
2437188.5	SEP 11, 1960	254	2437247.5	NOV 9, 1960	313
2437189.5	SEP 12, 1960	255	2437248.5	NOV 10, 1960	314
2437190.5	SEP 13, 1960	256	2437249.5	NOV 11, 1960	315
2437191.5	SEP 14, 1960	257	2437250.5	NOV 12, 1960	316
2437192.5	SEP 15, 1960	258	2437251.5	NOV 13, 1960	317
2437193.5	SEP 16, 1960	259	2437252.5	NOV 14, 1960	318
2437194.5	SEP 17, 1960	260	2437253.5	NOV 15, 1960	319
2437195.5	SEP 18, 1960	261	2437254.5	NOV 16, 1960	320
2437196.5	SEP 19, 1960	262	2437255.5	NOV 17, 1960	321
2437197.5	SEP 20, 1960	263	2437256.5	NOV 18, 1960	322
2437198.5	SEP 21, 1960	264	2437257.5	NOV 19, 1960	323
2437199.5	SEP 22, 1960	265	2437258.5	NOV 20, 1960	324
2437200.5	SEP 23, 1960	266	2437259.5	NOV 21, 1960	325
2437201.5	SEP 24, 1960	267	2437260.5	NOV 22, 1960	326
2437202.5	SEP 25, 1960	268	2437261.5	NOV 23, 1960	327
2437203.5	SEP 26, 1960	269	2437262.5	NOV 24, 1960	328
2437204.5	SEP 27, 1960	270	2437263.5	NOV 25, 1960	329
2437205.5	SEP 28, 1960	271	2437264.5	NOV 26, 1960	330
2437206.5	SEP 29, 1960	272	2437265.5	NOV 27, 1960	331
2437207.5	SEP 30, 1960	273	2437266.5	NOV 28, 1960	332
2437208.5	OCT 1, 1960	274	2437267.5	NOV 29, 1960	333
2437209.5	OCT 2, 1960	275	2437268.5	NOV 30, 1960	334
2437210.5	OCT 3, 1960	276	2437269.5	DEC 1, 1960	335
2437211.5	OCT 4, 1960	277	2437270.5	DEC 2, 1960	336
2437212.5	OCT 5, 1960	278	2437271.5	DEC 3, 1960	337
2437213.5	OCT 6, 1960	279	2437272.5	DEC 4, 1960	338
2437214.5	OCT 7, 1960	280	2437273.5	DEC 5, 1960	339
2437215.5	OCT 8, 1960	281	2437274.5	DEC 6, 1960	340
2437216.5	OCT 9, 1960	282	2437275.5	DEC 7, 1960	341
2437217.5	OCT 10, 1960	283	2437276.5	DEC 8, 1960	342
2437218.5	OCT 11, 1960	284	2437277.5	DEC 9, 1960	343
2437219.5	OCT 12, 1960	285	2437278.5	DEC 10, 1960	344
2437220.5	OCT 13, 1960	286	2437279.5	DEC 11, 1960	345
2437221.5	OCT 14, 1960	287	2437280.5	DEC 12, 1960	346
2437222.5	OCT 15, 1960	288	2437281.5	DEC 13, 1960	347
2437223.5	OCT 16, 1960	289	2437282.5	DEC 14, 1960	348
2437224.5	OCT 17, 1960	290	2437283.5	DEC 15, 1960	349
2437225.5	OCT 18, 1960	291	2437284.5	DEC 16, 1960	350
2437226.5	OCT 19, 1960	292	2437285.5	DEC 17, 1960	351
2437227.5	OCT 20, 1960	293	2437286.5	DEC 18, 1960	352
2437228.5	OCT 21, 1960	294	2437287.5	DEC 19, 1960	353

2437288.5	DEC 20, 1960	354	2437347.5	FEB 17, 1961	413
2437289.5	DEC 21, 1960	355	2437348.5	FEB 18, 1961	414
2437290.5	DEC 22, 1960	356	2437349.5	FEB 19, 1961	415
2437291.5	DEC 23, 1960	357	2437350.5	FEB 20, 1961	416
2437292.5	DEC 24, 1960	358	2437351.5	FEB 21, 1961	417
2437293.5	DEC 25, 1960	359	2437352.5	FEB 22, 1961	418
2437294.5	DEC 26, 1960	360	2437353.5	FEB 23, 1961	419
2437295.5	DEC 27, 1960	361	2437354.5	FEB 24, 1961	420
2437296.5	DEC 28, 1960	362	2437355.5	FEB 25, 1961	421
2437297.5	DEC 29, 1960	363	2437356.5	FEB 26, 1961	422
2437298.5	DEC 30, 1960	364	2437357.5	FEB 27, 1961	423
2437299.5	DEC 31, 1960	365	2437358.5	FEB 28, 1961	424
2437300.5	JAN 1, 1961	366	2437359.5	MAR 1, 1961	425
2437301.5	JAN 2, 1961	367	2437360.5	MAR 2, 1961	426
2437302.5	JAN 3, 1961	368	2437361.5	MAR 3, 1961	427
2437303.5	JAN 4, 1961	369	2437362.5	MAR 4, 1961	428
2437304.5	JAN 5, 1961	370	2437363.5	MAR 5, 1961	429
2437305.5	JAN 6, 1961	371	2437364.5	MAR 6, 1961	430
2437306.5	JAN 7, 1961	372	2437365.5	MAR 7, 1961	431
2437307.5	JAN 8, 1961	373	2437366.5	MAR 8, 1961	432
2437308.5	JAN 9, 1961	374	2437367.5	MAR 9, 1961	433
2437309.5	JAN 10, 1961	375	2437368.5	MAR 10, 1961	434
2437310.5	JAN 11, 1961	376	2437369.5	MAR 11, 1961	435
2437311.5	JAN 12, 1961	377	2437370.5	MAR 12, 1961	436
2437312.5	JAN 13, 1961	378	2437371.5	MAR 13, 1961	437
2437313.5	JAN 14, 1961	379	2437372.5	MAR 14, 1961	438
2437314.5	JAN 15, 1961	380	2437373.5	MAR 15, 1961	439
2437315.5	JAN 16, 1961	381	2437374.5	MAR 15, 1961	440
2437316.5	JAN 17, 1961	382	2437375.5	MAR 17, 1961	441
2437317.5	JAN 18, 1961	383	2437376.5	MAR 18, 1961	442
2437318.5	JAN 19, 1961	384	2437377.5	MAR 19, 1961	443
2437319.5	JAN 20, 1961	385	2437378.5	MAR 20, 1961	444
2437320.5	JAN 21, 1961	386	2437379.5	MAR 21, 1961	445
2437321.5	JAN 22, 1961	387	2437380.5	MAR 22, 1961	446
2437322.5	JAN 23, 1961	388	2437381.5	MAR 23, 1961	447
2437323.5	JAN 24, 1961	389	2437382.5	MAR 24, 1961	448
2437324.5	JAN 25, 1961	390	2437383.5	MAR 25, 1961	449
2437325.5	JAN 26, 1961	391	2437384.5	MAR 26, 1961	450
2437326.5	JAN 27, 1961	392	2437385.5	MAR 27, 1961	451
2437327.5	JAN 28, 1961	393	2437386.5	MAR 28, 1961	452
2437328.5	JAN 29, 1961	394	2437387.5	MAR 29, 1961	453
2437329.5	JAN 30, 1961	395	2437388.5	MAR 30, 1961	454
2437330.5	JAN 31, 1961	396	2437389.5	MAR 31, 1961	455
2437331.5	FEB 1, 1961	397	2437390.5	APR 1, 1961	456
2437332.5	FEB 2, 1961	398	2437391.5	APR 2, 1961	457
2437333.5	FEB 3, 1961	399	2437392.5	APR 3, 1961	458
2437334.5	FEB 4, 1961	400	2437393.5	APR 4, 1961	459
2437335.5	FEB 5, 1961	401	2437394.5	APR 5, 1961	460
2437336.5	FEB 6, 1961	402	2437395.5	APR 6, 1961	461
2437337.5	FEB 7, 1961	403	2437396.5	APR 7, 1961	462
2437338.5	FEB 8, 1961	404	2437397.5	APR 8, 1961	463
2437339.5	FEB 9, 1961	405	2437399.5	APR 10, 1961	464
2437340.5	FEB 10, 1961	406	2437398.5	APR 9, 1961	465
2437341.5	FEB 11, 1961	407	2437400.5	APR 11, 1961	466
2437342.5	FEB 12, 1961	408	2437401.5	APR 12, 1961	467
2437343.5	FEB 13, 1961	409	2437402.5	APR 13, 1961	468
2437344.5	FEB 14, 1961	410	2437403.5	APR 14, 1961	469
2437345.5	FEB 15, 1961	411	2437404.5	APR 15, 1961	470
2437346.5	FEB 16, 1961	412	2437405.5	APR 16, 1961	471

2437406.5	APR 17, 1961	472	2437465.5	JUN 15, 1961	531
2437407.5	APR 18, 1961	473	2437466.5	JUN 16, 1961	532
2437408.5	APR 19, 1961	474	2437467.5	JUN 17, 1961	533
2437409.5	APR 20, 1961	475	2437468.5	JUN 18, 1961	534
2437410.5	APR 21, 1961	476	2437469.5	JUN 19, 1961	535
2437411.5	APR 22, 1961	477	2437470.5	JUN 20, 1961	536
2437412.5	APR 23, 1961	478	2437471.5	JUN 21, 1961	537
2437413.5	APR 24, 1961	479	2437472.5	JUN 22, 1961	538
2437414.5	APR 25, 1961	480	2437473.5	JUN 23, 1961	539
2437415.5	APR 26, 1961	481	2437474.5	JUN 24, 1961	540
2437416.5	APR 27, 1961	482	2437475.5	JUN 25, 1961	541
2437417.5	APR 28, 1961	483	2437476.5	JUN 26, 1961	542
2437418.5	APR 29, 1961	484	2437477.5	JUN 27, 1961	543
2437419.5	APR 30, 1961	485	2437478.5	JUN 28, 1961	544
2437420.5	MAY 1, 1961	486	2437479.5	JUN 29, 1961	545
2437421.5	MAY 2, 1961	487	2437480.5	JUN 30, 1961	546
2437422.5	MAY 3, 1961	488	2437481.5	JUL 1, 1961	547
2437423.5	MAY 4, 1961	489	2437482.5	JUL 2, 1961	548
2437424.5	MAY 5, 1961	490	2437483.5	JUL 3, 1961	549
2437425.5	MAY 6, 1961	491	2437484.5	JUL 4, 1961	550
2437426.5	MAY 7, 1961	492	2437485.5	JUL 5, 1961	551
2437427.5	MAY 8, 1961	493	2437486.5	JUL 6, 1961	552
2437428.5	MAY 9, 1961	494	2437487.5	JUL 7, 1961	553
2437429.5	MAY 10, 1961	495	2437488.5	JUL 8, 1961	554
2437430.5	MAY 11, 1961	496	2437489.5	JUL 9, 1961	555
2437431.5	MAY 12, 1961	497	2437490.5	JUL 10, 1961	556
2437432.5	MAY 13, 1961	498	2437491.5	JUL 11, 1961	557
2437433.5	MAY 14, 1961	499	2437492.5	JUL 12, 1961	558
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2438000.5	DEC 2, 1962	1066	2438059.5	JAN 30, 1963	1125
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2438011.5	DEC 13, 1962	1077	2438070.5	FEB 10, 1963	1136
2438012.5	DEC 14, 1962	1078	2438071.5	FEB 11, 1963	1137
2438013.5	DEC 15, 1962	1079	2438072.5	FEB 12, 1963	1138
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2438017.5	DEC 19, 1962	1083	2438076.5	FEB 16, 1963	1142
2438018.5	DEC 20, 1962	1084	2438077.5	FEB 17, 1963	1143
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2438020.5	DEC 22, 1962	1086	2438079.5	FEB 19, 1963	1145
2438021.5	DEC 23, 1962	1087	2438080.5	FEB 20, 1963	1146
2438022.5	DEC 24, 1962	1088	2438081.5	FEB 21, 1963	1147
2438023.5	DEC 25, 1962	1089	2438082.5	FEB 22, 1963	1148
2438024.5	DEC 26, 1962	1090	2438083.5	FEB 23, 1963	1149
2438025.5	DEC 27, 1962	1091	2438084.5	FEB 24, 1963	1150
2438026.5	DEC 28, 1962	1092	2438085.5	FEB 25, 1963	1151
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2438028.5	DEC 30, 1962	1094	2438087.5	FEB 27, 1963	1153
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2438500.5	APR 15, 1964	1566	2438559.5	JUN 13, 1964	1625
2438501.5	APR 16, 1964	1567	2438560.5	JUN 14, 1964	1626
2438502.5	APR 17, 1964	1568	2438561.5	JUN 15, 1964	1627
2438503.5	APR 18, 1964	1569	2438562.5	JUN 16, 1964	1628
2438504.5	APR 19, 1964	1570	2438563.5	JUN 17, 1964	1629
2438505.5	APR 20, 1964	1571	2438564.5	JUN 18, 1964	1630
2438506.5	APR 21, 1964	1572	2438565.5	JUN 19, 1964	1631
2438507.5	APR 22, 1964	1573	2438566.5	JUN 20, 1964	1632
2438508.5	APR 23, 1964	1574	2438567.5	JUN 21, 1964	1633
2438509.5	APR 24, 1964	1575	2438568.5	JUN 22, 1964	1634
2438510.5	APR 25, 1964	1576	2438569.5	JUN 23, 1964	1635
2438511.5	APR 26, 1964	1577	2438570.5	JUN 24, 1964	1636
2438512.5	APR 27, 1964	1578	2438571.5	JUN 25, 1964	1637
2438513.5	APR 28, 1964	1579	2438572.5	JUN 26, 1964	1638
2438514.5	APR 29, 1964	1580	2438573.5	JUN 27, 1964	1639
2438515.5	APR 30, 1964	1581	2438574.5	JUN 28, 1964	1640
2438516.5	MAY 1, 1964	1582	2438575.5	JUN 29, 1964	1641
2438517.5	MAY 2, 1964	1583	2438576.5	JUN 30, 1964	1642
2438518.5	MAY 3, 1964	1584	2438577.5	JUL 1, 1964	1643
2438519.5	MAY 4, 1964	1585	2438578.5	JUL 2, 1964	1644
2438520.5	MAY 5, 1964	1586	2438579.5	JUL 3, 1964	1645
2438521.5	MAY 6, 1964	1587	2438580.5	JUL 4, 1964	1646
2438522.5	MAY 7, 1964	1588	2438581.5	JUL 5, 1964	1647
2438523.5	MAY 8, 1964	1589	2438582.5	JUL 6, 1964	1648
2438524.5	MAY 9, 1964	1590	2438583.5	JUL 7, 1964	1649
2438525.5	MAY 10, 1964	1591	2438584.5	JUL 8, 1964	1650
2438526.5	MAY 11, 1964	1592	2438585.5	JUL 9, 1964	1651

2438586.5	JUL 10, 1964	1652
2438587.5	JUL 11, 1964	1653
2438588.5	JUL 12, 1964	1654
2438589.5	JUL 13, 1964	1655
2438590.5	JUL 14, 1964	1656
2438591.5	JUL 15, 1964	1657
2438592.5	JUL 16, 1964	1658
2438593.5	JUL 17, 1964	1659
2438594.5	JUL 18, 1964	1660
2438595.5	JUL 19, 1964	1661
2438596.5	JUL 20, 1964	1662
2438597.5	JUL 21, 1964	1663
2438598.5	JUL 22, 1964	1664
2438599.5	JUL 23, 1964	1665
2438600.5	JUL 24, 1964	1666
2438601.5	JUL 25, 1964	1667
2438602.5	JUL 26, 1964	1668
2438603.5	JUL 27, 1964	1669
2438604.5	JUL 28, 1964	1670
2438605.5	JUL 29, 1964	1671
2438606.5	JUL 30, 1964	1672
2438607.5	JUL 31, 1964	1673
2438608.5	AUG 1, 1964	1674
2438609.5	AUG 2, 1964	1675
2438610.5	AUG 3, 1964	1676
2438611.5	AUG 4, 1964	1677
2438612.5	AUG 5, 1964	1678
2438613.5	AUG 6, 1964	1679
2438614.5	AUG 7, 1964	1680
2438615.5	AUG 8, 1964	1681
2438616.5	AUG 9, 1964	1682
2438617.5	AUG 10, 1964	1683
2438618.5	AUG 11, 1964	1684
2438619.5	AUG 12, 1964	1685
2438620.5	AUG 13, 1964	1686
2438621.5	AUG 14, 1964	1687
2438622.5	AUG 15, 1964	1688
2438623.5	AUG 16, 1964	1689
2438624.5	AUG 17, 1964	1690
2438625.5	AUG 18, 1964	1691
2438626.5	AUG 19, 1964	1692
2438627.5	AUG 20, 1964	1693
2438628.5	AUG 21, 1964	1694
2438629.5	AUG 22, 1964	1695
2438630.5	AUG 23, 1964	1696
2438631.5	AUG 24, 1964	1697
2438632.5	AUG 25, 1964	1698
2438633.5	AUG 26, 1964	1699
2438634.5	AUG 27, 1964	1700
2438635.5	AUG 28, 1964	1701
2438636.5	AUG 29, 1964	1702
2438637.5	AUG 30, 1964	1703
2438638.5	AUG 31, 1964	1704
2438639.5	SEP 1, 1964	1705
2438640.5	SEP 2, 1964	1706
2438641.5	SEP 3, 1964	1707
2438642.5	SEP 4, 1964	1708
2438643.5	SEP 5, 1964	1709
2438644.5	SEP 6, 1964	1710
2438645.5	SEP 7, 1964	1711
2438646.5	SEP 8, 1964	1712
2438647.5	SEP 9, 1964	1713
2438648.5	SEP 10, 1964	1714
2438649.5	SEP 11, 1964	1715
2438650.5	SEP 12, 1964	1716
2438651.5	SEP 13, 1964	1717
2438652.5	SEP 14, 1964	1718
2438653.5	SEP 15, 1964	1719
2438654.5	SEP 16, 1964	1720
2438655.5	SEP 17, 1964	1721
2438656.5	SEP 18, 1964	1722
2438657.5	SEP 19, 1964	1723
2438658.5	SEP 20, 1964	1724
2438659.5	SEP 21, 1964	1725
2438660.5	SEP 22, 1964	1726
2438661.5	SEP 23, 1964	1727
2438662.5	SEP 24, 1964	1728
2438663.5	SEP 25, 1964	1729
2438664.5	SEP 26, 1964	1730
2438665.5	SEP 27, 1964	1731
2438666.5	SEP 28, 1964	1732
2438667.5	SEP 29, 1964	1733
2438668.5	SEP 30, 1964	1734
2438669.5	OCT 1, 1964	1735
2438670.5	OCT 2, 1964	1736
2438671.5	OCT 3, 1964	1737
2438672.5	OCT 4, 1964	1738
2438673.5	OCT 5, 1964	1739
2438674.5	OCT 6, 1964	1740
2438675.5	OCT 7, 1964	1741
2438676.5	OCT 8, 1964	1742
2438677.5	OCT 9, 1964	1743
2438678.5	OCT 10, 1964	1744
2438679.5	OCT 11, 1964	1745
2438680.5	OCT 12, 1964	1746
2438681.5	OCT 13, 1964	1747
2438682.5	OCT 14, 1964	1748
2438683.5	OCT 15, 1964	1749
2438684.5	OCT 16, 1964	1750
2438685.5	OCT 17, 1964	1751
2438686.5	OCT 18, 1964	1752
2438687.5	OCT 19, 1964	1753
2438688.5	OCT 20, 1964	1754
2438689.5	OCT 21, 1964	1755
2438690.5	OCT 22, 1964	1756
2438691.5	OCT 23, 1964	1757
2438692.5	OCT 24, 1964	1758
2438693.5	OCT 25, 1964	1759
2438694.5	OCT 26, 1964	1760
2438695.5	OCT 27, 1964	1761
2438696.5	OCT 28, 1964	1762
2438697.5	OCT 29, 1964	1763
2438698.5	OCT 30, 1964	1764
2438699.5	OCT 31, 1964	1765
2438700.5	NOV 1, 1964	1766
2438701.5	NOV 2, 1964	1767
2438702.5	NOV 3, 1964	1768
2438703.5	NOV 4, 1964	1769

2438704.5	NOV 5, 1964	1770
2438705.5	NOV 6, 1964	1771
2438706.5	NOV 7, 1964	1772
2438707.5	NOV 8, 1964	1773
2438708.5	NOV 9, 1964	1774
2438709.5	NOV 10, 1964	1775
2438710.5	NOV 11, 1964	1776
2438711.5	NOV 12, 1964	1777
2438712.5	NOV 13, 1964	1778
2438713.5	NOV 14, 1964	1779
2438714.5	NOV 15, 1964	1780
2438715.5	NOV 16, 1964	1781
2438716.5	NOV 17, 1964	1782
2438717.5	NOV 18, 1964	1783
2438718.5	NOV 19, 1964	1784
2438719.5	NOV 20, 1964	1785
2438720.5	NOV 21, 1964	1786
2438721.5	NOV 22, 1964	1787
2438722.5	NOV 23, 1964	1788
2438723.5	NOV 24, 1964	1789
2438724.5	NOV 25, 1964	1790
2438725.5	NOV 26, 1964	1791
2438726.5	NOV 27, 1964	1792
2438727.5	NOV 28, 1964	1793
2438728.5	NOV 29, 1964	1794
2438729.5	NOV 30, 1964	1795
2438730.5	DEC 1, 1964	1796
2438731.5	DEC 2, 1964	1797
2438732.5	DEC 3, 1964	1798
2438733.5	DEC 4, 1964	1799
2438734.5	DEC 5, 1964	1800
2438735.5	DEC 6, 1964	1801
2438736.5	DEC 7, 1964	1802
2438737.5	DEC 8, 1964	1803
2438738.5	DEC 9, 1964	1804
2438739.5	DEC 10, 1964	1805
2438740.5	DEC 11, 1964	1806
2438741.5	DEC 12, 1964	1807
2438742.5	DEC 13, 1964	1808
2438743.5	DEC 14, 1964	1809
2438744.5	DEC 15, 1964	1810
2438745.5	DEC 16, 1964	1811
2438746.5	DEC 17, 1964	1812
2438747.5	DEC 18, 1964	1813
2438748.5	DEC 19, 1964	1814
2438749.5	DEC 20, 1964	1815
2438750.5	DEC 21, 1964	1816
2438751.5	DEC 22, 1964	1817
2438752.5	DEC 23, 1964	1818
2438753.5	DEC 24, 1964	1819
2438754.5	DEC 25, 1964	1820
2438755.5	DEC 26, 1964	1821
2438756.5	DEC 27, 1964	1822
2438757.5	DEC 28, 1964	1823
2438758.5	DEC 29, 1964	1824
2438759.5	DEC 30, 1964	1825
2438760.5	DEC 31, 1964	1826

H. LISTING OF THE N-BODY PROGRAM

```
C           N BODY TRAJECTORY       SHERVIN
      EQUIVALENCE(WM(1),XIM),(WM(2),YIM),(WM(3),ZIM),(WM(4),XIDM),
     1 (WM(6),ZIDM),(WM(5),YIDM),(WL(1),XIL),(WL(2),YIL),(WL(3),ZIL),
     2 (WL(4),XIDL),(WL(5),YIDL),(WL(6),ZIDL)
      DAC PX(9,67),PY(9,67),PZ(9,67),TL(67),RKM(4,6),RKL(4,6),NB(9),
     1 WT(9),RAD(9),X(9),Y(9),Z(9),DIS(9),DIST(9),PM(6),PL(6),PPM(6),
     2 PPL(6),QM(6),QL(6),WM(6),WL(6),XX(9),YY(9),ZZ(9),TI(6),CL(6)
      COMMON NEWORG,ZER,FMX,FLX,FMY,FLY,FMZ,FLZ,AM,AL,BM,BL,CM,CL
      COMMON NORG,NTARG,NSUN,N,NN,NNN,T,TO,TMAX,DTMAX,HAFDT,WTE,WTV,MGM,
     1 XTARG,YTARG,ZTARG,SCEN,VCEN,SMAX,GMAX,EMAX,RAD1,RAD2,RADE,WTA,
     2 NBE,NBT,NBS,RADORG,SAM,SBM,SCM, SAL,SBL,SCL,XIM,XIL,YIM,YIL,ZIM,
     3 ZIL,XIDM,XIDL,YIDM,YIDL,ZIDM,ZIDL,RM,RL,PP,PQ,SS,RPM,RPL,TEM,TEL,
     4 GMX,GLX,GMY,GLY,GMZ,GLZ,SUMX,SULX,SUMY,SULY,SUMZ,SULZ,FM,FL,GM,GL,
     5 HM,HL,RRR,SSS,XM,YM,ZM,A,BX,BY,BZ,WRM,WRL,RCM,RCL,ROM,ROL,RPPM,
     6 XPM,XPL,YPM,YPL,ZPM,ZPL,
     7 KOB,VNZ,MEDT,MSDT,MDT,NOT,MET,NOUT,NCKE,TSAV,DTA,DTB,DTC,EP1
      DAC G(2),WTS(2)
      DAC       SOGOOD(6),XDOT(2),YDOT(2),ZDOT(2),WRONG(6),OX(2)
     1 ,OY(2),OZ(2),XP(2),YP(2),ZP(2),XDOTP(2),YDOTP(2),ZDOTP(2),PI(2),
     2 PI2(2),E(2),XMJ(2),CCON(2),XN(2)
      COMMON A1,A2,FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,S1,S2,EPS1
      COMMON DT,DTME,RPM
      DAC SM(6),SL(6),XMJP(2)
      COMMON OBJ,TSAVE,REA,RTA,RSN,HPI,THPI,KK,NOSW,PQM(6),PQL(6)
      MGM = 512
      EPS1 = 1.0E-11
B     PI = 202622077325
B     PI(2) = 147042055061
B     PI2 = 203622077325
B     PI2(2) = 150042055061
B     HPI = 201622077325
B     THPI = 203455457437
    2 REWIND 26
      RIT 2,3,G(2),WTE,RADE,WTV,OBJ
    3 FORMAT (E9.5)
    4 RIT 2,3,EMAX,GMAX,SMAX,TMAX,TO
    5 RIT 2,3,NCKE,MSDT,MET,NOUT,NOSW
      IF (MSDT) 6, 7, 6
    6 RIT 2,3,DTA,DTB,DTC
      ERASE DT
      GO TO 8
    7 RIT 2,3,DT,DTMAX,EP1
      HAFDT = DT/2.
    8 RIT 2,3,N,NEWORG,NTARG
      RIT 2,3,(NB(I),WT(I),RAD(I),I = 1,N)
      RIT 2,3,(PM(I),I = 1,6)
      READ TAPE26, (TL(J),(PX(I,J),PY(I,J),PZ(I,J),I = 1,9),J = 3,7)
      READ TAPE26, (TL(J),(PX(I,J),PY(I,J),PZ(I,J),I = 1,9),J = 8,67)
    9 NNN = 6
      T = TO
      TMAX = TO+TMAX
      G = -G(2)
      ERASE NOT,NORG,PL,KOB
```

```
      CALL TITLE
11    CALL ORGN
      IF (TEM) 12,2,12
12    CALL TERP
      WOT 10,10
10    FORMAT (2H1 )
      CALL OUT
      IF (NCKE) 26,100,26
26    ERASE WM,WL
      XDOTP = PM(4)
      YDOTP = PM(5)
      ZDOTP = PM(6)
      XDOTP(2) = PL(4)
      YDOTP(2) = PL(5)
      ZDOTP(2) = PL(6)
27    SOGOOD (1) = PM (1)
      SOGOOD (2) = PL (1)
      SOGOOD (3) = PM (2)
      SOGOOD (4) = PL (2)
      SOGOOD (5) = PM (3)
      SOGOOD (6) = PL (3)
      CALL FAD (WM(4),WL(4),XDOTP(1),XDOTP(2),XDOT(1),XDOT(2))
      CALL FAD (WM(5),WL(5),YDOTP(1),YDOTP(2),YDOT(1),YDOT(2))
      CALL FAD (WM(6),WL(6),ZDOTP(1),ZDOTP(2),ZDOT(1),ZDOT(2))
28    CALL RECT
      WOT 10,29,E(1),A1,XN(1)
29    FORMAT (1P35H0                RECTIFICATION     E=E15.7,6H     A=E15.
     1 7,6H    N=E15.7/1H0 )
30    ERASE WM, WL, DTME, VNZ
      CALL POSN
32    IF(NORG) 34,31,34
31    IF(SCEN-1.5E-4) 41,33,33
41    KOB = 1
      GO TO 34
33    ERASE KOB
34    IF (MSDT) 35, 50, 35
35    IF (SCEN-RAD2) 36, 38, 39
36    IF (SCEN-RAD1) 37, 37, 38
37    DT = DTA
      GO TO 40
38    DT = DTB
      GO TO 40
39    DT = DTC
40    CONTINUE
H       SUB  MGM
H       STO  HAFDT
42    CALL INTN
      GO TO 72
50    TSAV = DTME
      TSAVE = T
      NSAV = NNN
      IF (DTMAX-DT) 99,99,98
98    DT = DT + DT
```

```
      99   DT = DT + DT
H           SUB MGM
H           SLW HAFDT
           DO 55 I = 1, N
           XX (I) = X (I)
           YY (I) = Y (I)
      55   ZZ (I) = Z (I)
           QM(4) = XPM
           QM(5) = YPM
           QM(6) = ZPM
           QL(4) = XPL
           QL(5) = YPL
           QL(6) = ZPL
           DO 60 I = 1, 6
           PPM (I) = WM (I)
      60   PPL (I) = WL (I)
           DO 61 I = 1,3
           PQM(I) = PM(I)
      61   PQL(I) = PL(I)
           SM(4) = XP(1)
           SM(5) = YP(1)
           SM(6) = ZP(1)
           SL(4) = XP(2)
           SL(5) = YP(2)
           SL(6) = ZP(2)
           CALL INTN
           SM (1) = WM (1)
           SL (1) = WL (1)
           SM (2) = WM (2)
           SL (2) = WL (2)
           SM (3) = WM (3)
           SL (3) = WL (3)
      66   DT = HAFDT
H           SUB MGM
H           SLW HAFDT
           DTME = TSAV
           T = TSAVE
           IF (NNN-NSAV) 63,65,64
      63   NSAV = 4
      64   NNN = NSAV
      65   DO 68 I = 1,N
           X (I) = XX (I)
           Y (I) = YY( I)
      68   Z (I) = ZZ (I)
           DO 67 I=1,3
           PM(I) = PQM(I)
      67   PL(I) = PQL(I)
           DO 69 I = 1, 6
           WM (I) = PPM (I)
      69   WL (I) = PPL (I)
           XP(1) = SM(4)
           YP(1) = SM(5)
           ZP(1) = SM(6)
```

```
      XP(2) = SL(4)
      YP(2) = SL(5)
      ZP(2) = SL(6)
      XPM = QM(4)
      YPM = QM(5)
      ZPM = QM(6)
      XPL = QL(4)
      YPL = QL(5)
      ZPL = QL(6)
      CALL INTN
      DO 70 I = 1, 3
      OM (I) = WM (I)
   70 OL (I) = WL (I)
      CALL FAD (XP(1),XP(2),WM(1),WL(1),PM(1),PL(1))
      CALL FAD (YP(1),YP(2),WM(2),WL(2),PM(2),PL(2))
      CALL FAD (ZP(1),ZP(2),WM(3),WL(3),PM(3),PL(3))
      CALL INTN
      CALL FSB (SM(1),SL(1),WM(1),WL(1),TEM,TEL)
      IF (ABSF(TEM)-EP1) 73, 73, 75
   73 CALL FSB (SM(2),SL(2),WM(2),WL(2),TEM,TEL)
      IF (ABSF(TEM)-EP1) 74, 74, 75
   74 CALL FSB (SM(3),SL(3),WM(3),WL(3),TEM,TEL)
      IF (ABSF(TEM)-EP1) 72, 72, 75
   75 DO 71 I = 1, 3
      SM (I) = OM (I)
   71 SL (I) = OL (I)
      GO TO 66
   72 NOT = NOT + 1
      CALL FAD (XP(1),XP(2),WM(1),WL(1),PM(1),PL(1))
      CALL FAD (YP(1),YP(2),WM(2),WL(2),PM(2),PL(2))
      CALL FAD (ZP(1),ZP(2),WM(3),WL(3),PM(3),PL(3))
      IF (NOT-NOUT) 83,84,83
   83 SCEN = SQRTF(PM(1)*PM(1)+PM(2)*PM(2)+PM(3)*PM(3))
      GO TO 76
   84 PM(4) = WM(4)+XDOTP
      PM(5) = WM(5)+YDOTP
      PM(6) = WM(6)+ZDOTP
      SL(4) = RKM(4,4)/DT
      SL(5) = RKM(4,5)/DT
      SL(6) = RKM(4,6)/DT
      CALL OUT
      ERASE NOT
      IF (EMAX-REA)333,333,430
  430 IF (GMAX-RTA)333,333,431
  431 IF (SMAX-RSN)333,333,76
   76 IF (SCEN-RADORG) 277,277,86
  277 IF (NOT) 84,279,84
  279 IF (NTARG-NORG) 169,165,169
   86 IF (T-TMAX) 202,278,278
  278 PM(4) = WM(4)+XDOTP
      PM(5) = WM(5)+YDOTP
      PM(6) = WM(6)+ZDOTP
      SL(4) = RKM(4,4)/DT
```

```
          SL(5) = RKM(4,5)/DT
          SL(6) = RKM(4,6)/DT
          GO TO 150
    202   IF (MET) 203,210,203
    203   TEM = WTS/RCM - WTS/RPM
          TEL = SUMX*SUMX+SUMY*SUMY+SUMZ*SUMZ
          IF (TEM-(4.*TEL)) 207,207,210
    207   ERASE NCKE
          CALL FAD (WM(4),WL(4),XDOTP(1),XDOTP(2),PM(4),PL(4))
          CALL FAD (WM(5),WL(5),YDDTP(1),YDOTP(2),PM(5),PL(5))
          CALL FAD (WM(6),WL(6),ZDOTP(1),ZDDTP(2),PM(6),PL(6))
          WOT 10,208
    208   FORMAT (12H0       COWELL/2H0 )
          GO TO 100
    210   IF (NOSW) 211,88,211
    211   IF (NBT) 212,88,212
    212   IF (TEM-(WT(NBT)/DIS(NBT))) 214,212,212
    214   IF (NTARG) 216,218,216
    216   NEWORG = NTARG
    217   CALL FAD (WM(4),WL(4),XDOTP(1),XDDTP(2),PM(4),PL(4))
          CALL FAD (WM(5),WL(5),YDOTP(1),YDOTP(2),PM(5),PL(5))
          CALL FAD (WM(6),WL(6),ZDOTP(1),ZDOTP(2),PM(6),PL(6))
          CALL SWITCH
          GO TO 26
    218   NEWORG = 0
          GO TO 217
     88   IF (ABSF(WM(1)/XP(1))-.05) 89,27,27
     89   IF (ABSF(WM(2)/YP(1))-.05) 90,27,27
     90   IF (ABSF(WM(3)/ZP(1))-.05) 91,27,27
     91   IF (ABSF(WM(4)/XDOTP(1))-.05) 92,27,27
     92   IF (ABSF(WM(5)/YDOTP(1))-.05) 93,27,27
     93   IF (ABSF(WM(6)/ZDOTP(1))-.05) 94,27,27
     94   IF (ABSF(GMX/XPM)-.05) 95,27,27
     95   IF (ABSF(GMY/YPM)-.05) 96,27,27
     96   IF (ABSF(GMZ/ZPM)-.05) 32,27,27
    100   IF (NDRG) 1030,101,1030
    101   IF(SCEN-1.5E-4) 102,103,103
    102   KOB = 1
          GO TO 1030
    103   ERASE KDB
   1030   IF (MSDT) 104,112,104
    104   IF (SCEN-RAD2) 105, 107, 108
    105   IF (SCEN-RAD1) 106, 106, 107
    106   DT = DTA
          GO TO 109
    107   DT = DTB
          GD TO 109
    108   DT = DTC
    109   CONTINUE
H          SUB   MGM
H          STO   HAFDT
          CALL INT
          GD TO 142
```

```
      112   TSAV = T
            NSAV = NNN
      139   IF (DTMAX-DT) 141,141,140
      140   DT = DT + DT
      141   DT = DT + DT
H            SUB MGM
H            SLW HAFDT
            DO 118 I = 1, N
            XX (I) = X (I)
            YY (I) = Y (I)
      118   ZZ (I) = Z (I)
            DO 119 I = 1, 6
            PPM (I) = PM (I)
      119   PPL (I) = PL (I)
            CALL INT
            DO 120 I = 1, 3
            SM (I) = PM (I)
      120   SL (I) = PL (I)
      121   DT = HAFDT
H            SUB MGM
H            SLW HAFDT
            T = TSAV
            IF (NNN-NSAV) 122,125,123
      122   NSAV = 4
      123   NNN = NSAV
      125   DO 124 I = 1,N
            X (I) = XX (I)
            Y (I) = YY (I)
      124   Z (I) = ZZ (I)
            DO 126 I = 1, 6
            PM (I) = PPM (I)
      126   PL (I) = PPL (I)
            CALL INT
            DO 127 I = 1, 3
            QM (I) = PM (I)
      127   QL (I) = PL (I)
            CALL INT
      128   CALL FSB (SM(1),SL(1),PM(1),PL(1),TEM,TEL)
            IF (ABSF(TEM)-EP1) 130, 130, 134
      130   CALL FSB (SM(2),SL(2),PM(2),PL(2),TEM,TEL)
            IF (ABSF(TEM)-EP1) 132, 132, 134
      132   CALL FSB (SM(3),SL(3),PM(3),PL(3),TEM,TEL)
            IF (ABSF(TEM)-EP1) 142,142,134
      134   DO 136 I = 1, 3
            SM (I) = QM (I)
      136   SL (I) = QL (I)
            DO 138 I = 1, 6
            PM (I) = PPM (I)
      138   PL (I) = PPL (I)
            GO TO 121
      142   NOT = NOT + 1
            IF (NOUT-NOT) 146, 145, 146
      145   SL(4) = RKM(4,4)/DT
```

```
      SL(5) = RKM(4,5)/DT
      SL(6) = RKM(4,6)/DT
      CALL OUT
      ERASE NOT
      IF (EMAX-REA)333,333,330
330   IF (TMAX-RTA)333,333,331
331   IF (SMAX-RSN)333,333, 146
333   WOT 10,334
334   FORMAT (24H0        MAXIMUM DISTANCE)
      GO TO 2
146   IF(RR-RADORG) 168,168,147
168   IF (NBT) 169,165,169
169   WOT 10,144
144   FORMAT (1H012X,13HIMPACT ORIGIN)
      GO TO 2
147   DO 148 I = 1,N
      IF (DIST(I)-RAD(I)) 77,77,148
148   CONTINUE
149   IF(T-TMAX) 400,250,250
400   IF (NOSW) 155,170,155
155   IF (NBT) 157,170,157
157   IF (DIST(NBT)-RR) 178,170,170
178   IF (NTARG) 179,181,179
179   NEWORG = NTARG
180   CALL SWITCH
      GO TO 100
181   NEWORG = 0
      GO TO 180
170   IF (MET) 175, 100, 175
175   TEL = SUMX*SUMX+SUMY*SUMY+SUMZ*SUMZ
      TEM = GMX*GMX+GMY*GMY+GMZ+GMZ
      IF (TEM-(4.*TEL)) 100,100,176
176   NCKE = 1
      GO TO 26
250   SL(4) = RKM(4,4)/DT
      SL(5) = RKM(4,5)/DT
      SL(6) = RKM(4,6)/DT
150   IF (NOT) 151,152,151
151   CALL OUT
152   WOT 10,153
153   FORMAT (25H0          MAXIMUM TIME
      GO TO 2
 77   IF (NOT) 78, 162, 78
 78   CALL OUT
162   IF(I-1) 79,163,79
163   IF(NORG) 280, 79, 280
280   WOT 10, 281
281   FORMAT (1H012X,12HIMPACT EARTH)
      GO TO 2
 79   IF (I-NBT) 80,165,80
 80   WOT 10, 82, NB(I)
 82   FORMAT (1H012X,12HIMPACT BODY I2)
      GO TO 2
```

```
165   WOT 10, 167
167   FORMAT (1H012X,13HIMPACT TARGET)
      GO TO 2
      END (1,1,0,0,0,1)
      SUBROUTINE OUT
 3    SCEN = SQRTF (PM(1)*PM(1)+PM(2)*PM(2)+PM(3)*PM(3))
      VCEN = SQRTF (PM(4)*PM(4)+PM(5)*PM(5)+PM(6)*PM(6))
 5    IF (NORG) 8, 9, 8
 8    XEA = PM (1) - X
      YEA = PM (2) - Y
      ZEA = PM (3) - Z
      REA = SQRTF (XEA*XEA+YEA*YEA+ZEA*ZEA)
      GO TO 10
 9    XEA = PM (1)
      YEA = PM (2)
      ZEA = PM (3)
      REA = SCEN
 10   IF (NORG-NTARG) 11, 12, 11
 11.  XTARG = PM (1) - X (NBT)
      YTARG = PM (2) - Y (NBT)
      ZTARG = PM (3) - Z (NBT)
      RTA = SQRTF (XTARG*XTARG+YTARG*YTARG+ZTARG*ZTARG)
      GO TO 14
 12   XTARG = PM (1)
      YTARG = PM (2)
      ZTARG = PM (3)
      RTA = SCEN
 14   IF (NORG-1) 15, 16, 15
 15   XSN = PM (1) - X (NBS)
      YSN = PM (2) - Y (NBS)
      ZSN = PM (3) - Z (NBS)
      RSN = SQRTF (XSN*XSN+YSN*YSN+ZSN*ZSN)
      GO TO 17
 16   XSN = PM (1)
      YSN = PM (2)
      ZSN = PM (3)
      RSN = SCEN
 17   TOUT = T - TO
      CALL MLO
      WOT 10, 19, TOUT, T, DT, NORG
 19   FORMAT (11HO      TIME=F9.4,14H     TAPE TIME=F10.3,12H     DELTA T=F
    1 8.3,17H     CENTER IS NO.I3/112HO    POSIT FROM    EARTH           TA
    2 RGET              SUN                 CENTER              VELOCITY        AC
    3 CELERATION       )
 20   WOT 10, 22, XEA, XTARG, XSN, PM (1), PM (4), SL (4)
 22   FORMAT (8H          X1P6E17.7)
      WOT 10, 24, YEA, YTARG, YSN, PM (2), PM (5), SL (5)
 24   FORMAT (8H          Y1P6E17.7)
      WOT 10, 26, ZEA, ZTARG, ZSN, PM (3), PM (6), SL (6)
 26   FORMAT (8H          Z1P6E17.7)
      WOT 10, 28, REA, RTA, RSN, SCEN, VCEN
 28   FORMAT (8H          R1P6E17.7)
      CALL WML
```

```
36    IF (SENSE SWITCH 1) 37, 42
37    PRINT 19, TOUT, T, DT, NORG
      PRINT 22, XEA, XTARG, XSN, PM (1), PM (4), SL (4)
39    PRINT 24, YEA, YTARG, YSN, PM (2), PM (5), SL (5)
      PRINT 26, ZEA, ZTARG, ZSN, PM (3), PM (6), SL (6)
      PRINT 28, REA, RTA, RSN, SCEN, VCEN
42    RETURN
      END (1,1,0,0,0,1)
      SUBROUTINE ORGN
 4    IF (NORG) 6, 10, 6
 6    WT (1) = WTS - WTV
      RAD(1) = RADORG
10    NORG = NEWORG
      IF (NORG) 12, 20, 12
12    IF (NORG-NB(1)) 13, 17, 13
13    DO 14 I = 2,N
      IF (NORG-NB(I))14, 15, 14
14    CONTINUE
      WOT 10,50
50    FORMAT (1H010X,22HORIGIN BODY IS MISSING )
      ERASE TEM
      GO TO 42
15    J = I
      K = NB (1)
      NB (1) = NB (J)
      NB (J) = K
      TEM = WT (1)
      WT (1) = WT (J)
      WT (J) = TEM
      TEL = RAD (1)
      RAD (1) = RAD (J)
      RAD (J) = TEL
17    RAD1 = 3. * RAD (1)
      RAD2 = 100. * RAD (1)
      RADORG = RAD (1)
      RAD(1) = RADE
      WTS = WT (1) + WTV
      WT(1) = WTE
      GO TO 25
20    WTS = WTE + WTV
      RADORG = RADE
      RAD1 = 3. * RADE
      RAD2 = 100. * RADE
25    IF (NTARG) 29, 26, 29
26    IF (NORG) 28, 27, 28
27    NBT = 0
      GO TO 35
28    NBT = 1
      GO TO 35
29    IF (NTARG-NORG) 31, 30, 31
30    NBT = 0
      GO TO 35
31    DO 32 I = 1, N
```

```
      IF (NTARG-NB(I))32, 33, 32
32    CONTINUE
      WOT 10,55
55    FORMAT (1H010X,22HTARGET BODY IS MISSING )
      ERASE TEM
      GO TO 42
33    NBT = I
35    IF (NORG-I) 36, 39, 36
36    DO 38 I = 1, N
      IF (NB(I)-1) 38, 40, 38
38    CONTINUE
39    NBS = 0
      GO TO 41
40    NBS = I
41    TEM = 1.
42    RETURN
      END (1,1,0,0,0,0)
      SUBROUTINE TITLE
 2    TEM = 1.
      WOT 10,3
 3    FORMAT (1H15X,17HN BODY TRAJECTORY/1H010X,20HGENERAL ELECTRIC CO./
     1 1H010X,8HM.S.V.D./1H0)
      WOT 10,5,TO
 5    FORMAT (1H020X,22HSTARTING TABLE TIME  = F10.3)
      IF (NEWORG) 10,7,10
 7    ERASE TEM
      WOT 10,8,NEWORG,WTE
 8    FORMAT (1H020X,14HORIGIN IS BODYI2,8H  MASS = 1PE14.6)
      GO TO 11
10    DO 12I = 1,N
      IF (NEWORG-NB(I)) 12,9,12
 9    WOT 10,8,NB(I),WT(I)
      GO TO 11
12    CONTINUE
11    IF (NTARG) 14,17,14
17    ERASE TEM
      WOT 10,13,NTARG,WTE
13    FORMAT (1H020X,19HDESTINATION IS BODYI2,8H  MASS = 1PE14.6)
      GO TO 20
14    DO 16I = 1,N
      IF (NTARG-NB(I))16,15,16
15    WOT 10,13,NB(I),WT(I)
      GO TO 20
16    CONTINUE
19    FORMAT (1H020X,17HMASS OF VEHICLE = 1PE14.6)
20    WOT 10,19,WTV
      IF (N-1) 33,33,21
21    WOT 10,22
22    FORMAT (1H020X,17HOTHER BODIES ARE-)
      IF (TEM) 24,26,24
24    WOT 10,25,NOT,WTE
25    FORMAT (1H025X,6HBODY  I1,8H  MASS = 1PE14.6)
26    DO 32I = 1,N
```

```
      IF (NEWORG-NB(I)) 27,32,27
27    IF (NTARG-NB(I)) 28,32,28
28    WOT 10,25,NB(I),WT(I)
32    CONTINUE
33    WOT 10,29,G(2)
29    FORMAT (1HO20X,24HGRAVITATIONAL CONSTANT =1PE14.6)
      IF (MSDT) 34,36,34
34    WOT 10,35,DTA,DTB,DTC
35    FORMAT (1HO20X,6HDT 1 =F7.3,3X,6HDT 2 =F7.3,3X,6HDT 3 =F7.3)
      GO TO 39
36    WOT 10,37,EPI,DTMAX
37    FORMAT (1HO20X,24HEPSILON OF INTEGRATION = 1PE9.2,3X,18HMAXIMUM DE
     1 LTA T = 0PF5.1)
39    IF (MET) 40,42,40
40    WOT 10,41
41    FORMAT (1HO20X,33HCOWELL AND ENCKE METHODS ARE USED)
      GO TO 50
42    IF (NCKE) 43,46,43
43    WOT 10,44
44    FORMAT (1HO20X,20HENCKE METHOD IS USED)
      GO TO 50
46    WOT 10,47
47    FORMAT (1HO20X,21HCOWELL METHOD IS USED)
50    IF (NOSW) 52,54,52
52    WOT 10,53
53    FORMAT (1HO20X,21HTHE ORIGIN MAY CHANGE)
      GO TO 58
54    WOT 10,55
55    FORMAT (1HO20X,19HTHE ORIGIN IS FIXED)
58    RETURN
      END (1,1,0,0,0,0)
      SUBROUTINE TERP
10    DO 25 NN =NNN,65
20    IF (TL(NN)-T) 25, 30, 30
25    CONTINUE
26    DO 27 IT = 1, 7
      TL(IT) = TL(IT+60)
      DO 27 J = 1,9
      PX (J,IT) = PX (J,IT+60)
      PY (J,IT) = PY (J,IT+60)
27    PZ (J,IT) = PZ (J,IT+60)
      READ TAPE26, (TL(J),(PX(I,J),PY(I,J),PZ(I,J),I = 1,9),J = 8,67)
      NNN= 6
      GO TO 10
30    NNN = NN
      TI (1) = T - TL (NN-3)
      TI (2) = T - TL (NN-2)
      TI (3) = T - TL (NN-1)
      TI (4) = T - TL (NN)
      TI (5) = T - TL (NN+1)
      TI (6) = T - TL (NN+2)
32    TT = TI (4) * TI (5) * TI (6)
      TTT = TI(3)*TT
```

```
      CL(1) = TI(2)*TTT/(-29859840.)
      CL(2) = TI(1)*TTT/5971968.
      CL (3) = TI (1) * TI (2) * TT /(- 2985984.)
      TT = TI (1) * TI (2) * TI (3)
      TTT = TI(4)*TT
      CL (4) = TT * TI (5) * TI (6) / 2985984.
      CL(5) = TTT*T+(6)/(-5971968.)
      CL(6) = TTT*TI(5)/29859840.
   33 DO 36 K = 1, N
      NA = NB (K)
   34 X ( K) = PX (NA,NN-3) * CL (1) + PX (NA,NN-2) * CL (2) + PX (NA,NN
     1 -1) * CL (3) + PX (NA,NN) * CL (4) + PX (NA,NN+1) * CL (5) + PX (N
     2 A,NN+2) * CL (6)
   35 Y ( K) = PY (NA,NN-3) * CL (1) + PY (NA,NN-2) * CL (2) + PY (NA,NN
     1 -1) * CL (3) + PY (NA,NN) * CL (4) + PY (NA,NN+1) * CL (5) + PY (N
     2 A,NN+2) * CL (6)
   36 Z ( K) = PZ (NA,NN-3) * CL (1) + PZ (NA,NN-2) * CL (2) + PZ (NA,NN
     1 -1) * CL (3) + PZ (NA,NN) * CL (4) + PZ (NA,NN+1) * CL (5) + PZ (N
     2 A,NN+2) * CL (6)
      IF (NORG) 40,43,40
   40 DO 42 K = 2,N
      X(K) = X(K)-X
      Y(K) = Y(K)-Y
   42 Z(K) = Z(K)-Z
      X = -X
      Y = -Y
      Z = -Z
   43 RETURN
      END (1,1,0,0,0,0)
      SUBROUTINE INT
    6 CALL FMPS(PM(4),PL(4),DT,RKM(1,1),RKL(1,1))
      CALL FMPS(PM(5),PL(5),DT,RKM(1,2),RKL(1,2))
    7 CALL FMPS(PM(6),PL(6),DT,RKM(1,3),RKL(1,3))
      SAM = PM(1)
      SAL = PL(1)
      SBM = PM(2)
      SBL = PL(2)
      SCM = PM(3)
      SCL = PL(3)
      CALL ACC
    8 CALL FMPS(FMX,FLX,DT,RKM(1,4),RKL(1,4))
      CALL FMPS(FMY,FLY,DT,RKM(1,5),RKL(1,5))
      CALL FMPS(FMZ,FLZ,DT,RKM(1,6),RKL(1,6))
H       CAL RKM-12
H       SUB MGM
H       SLW FM
H       CAL RKL-12
H       SUB MGM
H       SLW FL
H       CAL RKM-16
H       SUB MGM
H       SLW GM
H       CAL RKL-16
```

```
H        SUB MGM
H        SLW GL
H        CAL RKM-20
H        SUB MGM
H        SLW HM
H        CAL RKL-20
H        SUB MGM
H        SLW HL
  10    CALL FAD(PM(4),PL(4),FM,FL,AM,AL)
        CALL FAD(PM(5),PL(5),GM,GL,BM,BL)
        CALL FAD(PM(6),PL(6),HM,HL,CM,CL)
        CALL FMPS(AM,AL,DT,RKM(2,1),RKL(2,1))
        CALLFMPS(BM,BL,DT,RKM(2,2),RKL(2,2))
        CALL FMPS(CM,CL,DT,RKM(2,3),RKL(2,3))
H        CAL RKM
H        SUB MGM
H        SLW FM
H        CAL RKL
H        SUB MGM
H        SLW FL
H        CAL RKM-4
H        SUB MGM
H        SLW GM
H        CAL RKL -4
H        SUB MGM
H        SLW GL
H        CAL RKM-8
H        SUB MGM
H        SLW HM
H        CAL RKL-8
H        SUB MGM
H        SLW HL
  14    CALL FAD (PM(1),PL(1),FM,FL,SAM,SAL)
        CALL FAD (PM(2),PL(2),GM,GL,SBM,SBL)
        CALL FAD (PM(3),PL(3),HM,HL,SCM,SCL)
        T = T+HAFDT
        CALL TERP
        CALL ACC
  16    CALL FMPS(FMX,FLX,DT,RKM(2,4),RKL(2,4))
        CALL FMPS(FMY,FLY,DT,RKM(2,5),RKL(2,5))
        CALL FMPS(FMZ,FLZ,DT,RKM(2,6),RKL(2,6))
H        CAL RKM-13
H        SUB MGM
H        SLW FM
H        CAL RKL-13
H        SUB MGM
H        SLW FL
H        CAL RKM-17
H        SUB MGM
H        SLW GM
H        CAL RKL-17
H        SUB MGM
H        SLW GL
```

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H          CAL RKM-21
H          SUB MGM
H          SLW HM
H          CAL RKL-21
H          SUB MGM
H          SLW HL
     20   CALL FAD(PM(4),PL(4),FM,FL,AM,AL)
          CALL FAD(PM(5),PL(5),GM,GL,BM,BL)
          CALL FAD(PM(6),PL(6),HM,HL,CM,CL)
          CALL FMPS(AM,AL,DT,RKM(3,1),RKL(3,1))
          CALL FMPS(BM,BL,DT,RKM(3,2),RKL(3,2))
          CALL FMPS(CM,CL,DT,RKM(3,3),RKL(3,3))
H          CAL RKM-1
H          SUB MGM
H          SLW FM
H          CAL RKL-1
H          SUB MGM
H          SLW FL
H          CAL RKM-5
H          SUB MGM
H          SLW GM
H          CAL RKL-5
H          SUB MGM
H          SLW GL
H          CAL RKM-9
H          SUB MGM
H          SLW HM
H          CAL RKL-9
H          SUB MGM
H          SLW HL
     24   CALL FAD (PM(1),PL(1),FM,FL,SAM,SAL)
          CALL FAD (PM(2),PL(2),GM,GL,SBM,SBL)
          CALL FAD (PM(3),PL(3),HM,HL,SCM,SCL)
          CALL ACC
          CALL FMPS(FMX,FLX,DT,RKM(3,4),RKL(3,4))
          CALL FMPS(FMY,FLY,DT,RKM(3,5),RKL(3,5))
          CALL FMPS(FMZ,FLZ,DT,RKM(3,6),RKL(3,6))
     28   CALL FAD (PM(4),PL(4),RKM(3,4),RKL(3,4),AM,AL)
          CALL FAD (PM(5),PL(5),RKM(3,5),RKL(3,5),BM,BL)
          CALL FAD (PM(6),PL(6),RKM(3,6),RKL(3,6),CM,CL)
          CALL FMPS(AM,AL,DT,RKM(4,1),RKL(4,1))
          CALL FMPS(BM,BL,DT,RKM(4,2),RKL(4,2))
          CALL FMPS(CM,CL,DT,RKM(4,3),RKL(4,3))
     32   CALL FAD (PM(1),PL(1),RKM(3,1),RKL(3,1),SAM,SAL)
          CALL FAD (PM(2),PL(2),RKM(3,2),RKL(3,2),SBM,SBL)
          CALL FAD (PM(3),PL(3),RKM(3,3),RKL(3,3),SCM,SCL)
          T = T+HAFDT
          CALL TERP
          VNZ = 1.
          CALL ACC
          VNZ = 0.
          CALL FMPS(FMX,FLX,DT,RKM(4,4),RKL(4,4))
          CALL FMPS(FMY,FLY,DT,RKM(4,5),RKL(4,5))
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```
      CALL FMPS(FMZ,FLZ,DT,RKM(4,6),RKL(4,6))
34    DO 42 NA = 1,6
      SDM = RKM(1,NA)
      SDL = RKL(1,NA)
      SEM = RKM(2,NA)
      SEL = RKL(2,NA)
      SFM = RKM(3,NA)
      SFL = RKL(3,NA)
      SGM = RKM(4,NA)
      SGL = RKL(4,NA)
36    CALL FAD (SEM,SEL,SFM,SFL,SEM,SEL)
      CALL FAD (SEM,SEL,SEM,SEL,SEM,SEL)
38    CALL FAD (SDM,SDL,SEM,SEL,SEM,SEL)
      CALL FAD (SGM,SGL,SEM,SEL,SEM,SEL)
      CALL FDYS (SEM,SEL,6.0,SEM,SEL)
      SFM = PM(NA)
      SFL = PL(NA)
40    CALL FAD (SFM,SFL,SEM,SEL,SFM,SFL)
      PM(NA) = SFM
42    PL(NA) = SFL
      RETURN
      END (1,1,0,0,0,0)
      SUBROUTINE ACC
 2    CALL SQ (SAM,SAL,AM,AL)
      CALL SQ (SBM,SBL,BM,BL)
      CALL SQ (SCM,SCL,CM,CL)
 3    CALL FAD (AM,AL,BM,BL,RM,RL)
      CALL FAD (RM,RL,CM,CL,RO,RL)
 4    CALL DPSQRT (RO,RL,RR,SS)
 5    CALL FMP (RR,SS,RO,RL,RRM,RRL)
 6    CALL FDY (WTS,ZER,RRM,RRL,TEM,TEL)
 7    CALL FMP ( SAM , SAL ,TEM,TEL,GMX,GLX)
      CALL FMP ( SBM ,SBL , TEM,TEL,GMY,GLY)
      CALL FMP ( SCM , SCL ,TEM,TEL,GMZ,GLZ)
70    IF (KOB) 71,8,71
71    CALL FMP (RO,RL,RRM,RRL,TI(1),TI(2))
      CALL FMPS (CM,CL,5.0,TEM,TEL)
      CALL FDY (TEM,TEL,RO,RL,TEM,TEL)
      CALL FSBS (TEM,TEL,1.0,TEM,TEL)
72    CALL FDY (TEM,TEL,TI(1),TI(2),TEM,TEL)
      CALL FMPS(TEM,TEL,OBJ,TEM,TEL)
      CALL FMP (TEM,TEL,SAM,SAL,XM,XL)
      CALL FMP (TEM,TEL,SBM,SBL,YM,YL)
73    CALL FDY (2.0,ZER,TI(1),TI(2),ZM,ZL)
      CALL FMPS (ZM,ZL,OBJ,ZM,ZL)
      CALL FSB (TEM,TEL,ZM,ZL,ZM,ZL)
      CALL FMP (ZM,ZL,SCM,SCL,ZM,ZL)
74    CALL FSB (GMX,GLX,XM,XL,GMX,GLX)
      CALL FSB (GMY,GLY,YM,YL,GMY,GLY)
      CALL FSB (GMZ,GLZ,ZM,ZL,GMZ,GLZ)
 8    ERASE SUMX,SULX,SUMY,SULY,SUMZ,SULZ
 9    DO 29 J = 1, N
      XM = X (J)
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      YM = Y (J)
      ZM = Z (J)
      WTA = WT (J)
12    CALL FSBS (SAM,SAL,XM,XM,XL)
      CALL FSBS (SBM,SBL,YM,YM,YL)
      CALL FSBS (SCM,SCL,ZM,ZM,ZL)
13    CALL SQ (XM,XL,AM,AL)
      CALL SQ (YM,YL,BM,BL)
      CALL SQ (ZM,ZL,CM,CL)
14    CALL FAD (AM,AL,BM,BL,RM,RL)
      CALL FAD (RM,RL,CM,CL,RM,RL)
15    CALL DPSQRT (RM,RL,RRR,SSS)
      IF (VNZ) 16,17,16
16    DIS(J) = RM
      DIST(J) = RRR
17    CALL FMP (RM,RL,RRR,SSS,RM,RL)
      CALL FDY (XM,XL,RM,RL,AM,AL)
      CALL FDY (YM,YL,RM,RL,BM,BL)
      CALL FDY (ZM,ZL,RM,RL,CM,CL)
      A  =   (X(J)*X(J)+Y(J)*Y(J)+Z(J)*Z(J))**1.5
      BX = X(J)/A
      BY = Y(J)/A
      BZ = Z(J)/A
26    CALL FADS (AM,AL,BX,AM,AL)
      CALL FADS (BM,BL,BY,BM,BL)
      CALL FADS (CM,CL,BZ,CM,CL)
22    CALL FMPS (AM,AL,WTA,AM,AL)
      CALL FMPS (BM,BL,WTA,BM,BL)
      CALL FMPS (CM,CL,WTA,CM,CL)
23    CALL FAD (AM,AL,SUMX,SULX,SUMX,SULX)
      CALL FAD (BM,BL,SUMY,SULY,SUMY,SULY)
25    CALL FAD (CM,CL,SUMZ,SULZ,SUMZ,SULZ)
      CALL FAD (GMX,GLX,SUMX,SULX,FMX,FLX)
      CALL FAD (GMY,GLY,SUMY,SULY,FMY,FLY)
32    CALL FAD (GMZ,GLZ,SUMZ,SULZ,FMZ,FLZ)
      CALL FMPS (FMX,FLX,G,FMX,FLX)
      CALL FMPS (FMY,FLY,G,FMY,FLY)
      CALL FMPS (FMZ,FLZ,G,FMZ,FLZ)
      RETURN
      END (1,1,0,0,0,0)
      SUBROUTINE SWITCH
1     WOT 10,2
2     FORMAT (8HO SWITCH )
      IF (DIST(NBT)-20.*RAD(NBT)) 5,5,8
5     DT = DT/8.
      GO TO 10
8     DT = DT/2.
10    HAFDT = DT/2.
25    QM(1) = 137.
      QM(2) = -300.
      QM(3) = 300.
      QM(4) = -200.
      QM(5) = 75.
```

```
      QM(6) = -12.
      TSAVE = T
      IF (NNN-5) 14,14,15
14    NSAVE = 4
      GO TO 16
15    NSAVE = NNN-2
16    XPM = X(NBT)
      YPM = Y(NBT)
      ZPM = Z(NBT)
26    CALL FSBS (PM(1),PL(1),XPM,PPM(1),PPL(1))
      CALL FSBS (PM(2),PL(2),YPM,PPM(2),PPL(2))
      CALL FSBS (PM(3),PL(3),ZPM,PPM(3),PPL(3))
27    CALL FMPS (PPM(1),PPL(1),QM(1),WM(1),WL(1))
      CALL FMPS (PPM(2),PPL(2),QM(1),WM(2),WL(2))
      CALL FMPS (PPM(3),PPL(3),QM(1),WM(3),WL(3))
      DO 35 I = 2,6
28    CALL INT
29    SL(4) = RKM(4,4)/DT
      SL(5) = RKM(4,5)/DT
      SL(6) = RKM(4,6)/DT
      CALL OUT
      XM = X(NBT)
      YM = Y(NBT)
      ZM = Z(NBT)
      FM = QM(I)
      CALL FSBS(PM(1),PL(1),XM,QL(1),QL(2))
      CALL FSBS(PM(2),PL(2),YM,QL(3),QL(4))
      CALL FSBS(PM(3),PL(3),ZM,QL(5),QL(6))
30    CALL FMPS (QL(1),QL(2),FM,FMX,FLX)
      CALL FMPS (QL(3),QL(4),FM,FMY,FLY)
      CALL FMPS (QL(5),QL(6),FM,FMZ,FLZ)
      CALL FAD (WM(1),WL(1),FMX,FLX,WM(1),WL(1))
      CALL FAD (WM(2),WL(2),FMY,FLY,WM(2),WL(2))
35    CALL FAD (WM(3),WL(3),FMZ,FLZ,WM(3),WL(3))
      GM = -60.*DT
      CALL FDYS (WM(1),WL(1),GM,PPM(4),PPL(4))
      CALL FDYS (WM(2),WL(2),GM,PPM(5),PPL(5))
      CALL FDYS (WM(3),WL(3),GM,PPM(6),PPL(6))
      EPSL = SQRTF (PPM(1)**2+PPM(2)**2+PPM(3)**2)*4.0E-10
37    CALL ORGN
38    DO 40 I = 1,6
      PM(I) = PPM(I)
40    PL(I) = PPL(I)
      T = TSAVE
      IF (NNN-NSAVE) 41,43,42
41    BACKSPACE 26
      BACKSPACE 26
      READ TAPE26, (TL(J),(PX(I,J),PY(I,J),PZ(I,J),I = 1,9),J = 8,67)
42    NNN = NSAVE
43    CALL TERP
      DO 44 I = 2,6
      CALL INT
44    WOT 10,45,T,PM(1),PM(2),PM(3)
```

```
45     FORMAT (1H0F12.4,1P3E17.7)
50     CALL FSB (QL(1),QL(2),PM(1),PL(1),SM(1),SL(1))
       CALL FSB (QL(3),OL(4),PM(2),PL(2),SM(2),SL(2))
       CALL FSB (QL(5),QL(6),PM(3),PL(3),SM(3),SL(3))
       DO 53 I = 1,3
52     IF (ABSF(SM(I))-EPSL) 53,53,58
53     CONTINUE
55     RETURN
58     HM = T-TSAVE
       CALL FDYS (SM(1),SL(1),HM,PM(1),PL(1))
       CALL FDYS (SM(2),SL(2),HM,PM(2),PL(2))
       CALL FDYS (SM(3),SL(3),HM,PM(3),PL(3))
       CALL FAD (PPM(4),PPL(4),PM(1),PL(1),PPM(4),PPL(4))
       CALL FAD (PPM(5),PPL(5),PM(2),PL(2),PPM(5),PPL(5))
       CALL FAD (PPM(6),PPL(6),PM(3),PL(3),PPM(6),PPL(6))
       GO TO 38
       END (1,1,0,0,0,0)
       RECTIFICATION   BY   MARY ELLEN
       SUBROUTINE RECT
       EQUIVALENCE(WM(1),XIM),(WM(2),YIM),(WM(3),ZIM),(WM(4),XIDM),
     1 (WM(6),ZIDM),(WM(5),YIDM),(WL(1),XIL),(WL(2),YIL),(WL(3),ZIL),
     2 (WL(4),XIDL),(WL(5),YIDL),(WL(6),ZIDL)
       DAC IX(9,67),IY(9,67),IZ(9,67),TL(67),RKM(4,6),RKL(4,6),NB(9),
     1 WT(9),RAD(9),    DUMMY(27)   ,DIS(9),DIST(9),PM(6),PL(6),PPM(6),
     2 PPL(6),QM(6),QL(6),WM(6),WL(6),XX(9),YY(9),ZZ(9),TI(6),CL(6)
       COMMON NEWORG,ZER,FMX,FLX,FMY,FLY,FMZ,FLZ,AM,AL,BM,BL,CM,CL
       COMMON NORG,NTARG,NSUN,N,NN,NNN,T,TO,TMAX,DTMAX,HAFDT,WTE,WTV,MGM,
     1 XTARG,YTARG,ZTARG,SCEN,VCEN,SMAX,GMAX,EMAX,RAD1,RAD2,RADE,WTA,
     2 NBE,NBT,NBS,RADORG,SAM,SBM,SCM, SAL,SBL,SCL,XIM,XIL,YIM,YIL,ZIM,
     3 ZIL,XIDM,XIDL,YIDM,YIDL,ZIDM,ZIDL,RM,RL,RR,RO,SS,RRM,RRL,TEM,TEL,
     4 GMX,GLX,GMY,GLY,GMZ,GLZ,SUMX,SULX,SUMY,SULY,SUMZ,SULZ,FM,FL,GM,GL,
     5 HM,HL,RRR,SSS,XM,YM,ZM,A,BX,BY,BZ,WRM,WRL,RCM,RCL,ROM,ROL,RPPM,
     6 XPM,XPL,YPM,YPL,ZPM,ZPL,
     7 KOB,VNZ,MEDT,MSDT,MDT,NOT,MET,NOUT,NCKE,TSAV,DTA,DTB,DTC,EP1
       DAC XKSQ(2), XMI(2)
       DAC X(2),Y(2),Z(2),XDOT(2),YDOT(2),ZDOT(2),PX(2),PY(2),PZ(2),QX(2)
     1 ,QY(2),OZ(2),XP(2),YP(2),ZP(2),XDOTP(2),YDOTP(2),ZDOTP(2),PI(2),
     2 PI2(2),E(2),XMJ(2),CCON(2),XN(2)
       COMMON A1,A2,FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,S1,S2,EPS1
       COMMON DT,DTME,RPM
       DAC SM(6),SL(6),XMJP(2)
       COMMON OBJ,TSAVE,REA,RTA,RSN,HPI,THPI,KK,NOSW
100    CALL FMP (X(1), X(2), XDOT(1), XDOT(2), RR1, RR2)
101    CALL FMP (Y(1), Y(2), YDOT(1), YDOT(2), T1, T2)
102    CALL FAD (T1, T2, RR1, RR2, RR1, RR2)
103    CALL FMP (Z(1), Z(2), ZDOT(1), ZDOT(2), T1, T2)
104    CALL FAD (T1, T2, RR1, RR2, RR1, RR2)
              VECTOR DOT PRODUCT OF R,RDOT IN LOCS. RR1,RR2
105    CALL SQ (XDOT(1), XDOT(2), SD1, SD2)
106    CALL SQ (YDOT(1), YDOT(2), T1, T2)
107    CALL FAD (SD1, SD2, T1, T2, SD1, SD2)
108    CALL SQ (ZDOT(1), ZDOT(2), T1, T2)
109    CALL FAD (T1, T2, SD1, SD2, SD1, SD2)
```

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C                   VECTOR DOT PRODUCT OF RDOT WITH ITSELF (SDOT**2) IN SD1,2
  110     CALL SQ (X(1), X(2), AR1, AR2)
  111     CALL SQ (Y(1), Y(2), T1, T2)
  112     CALL FAD (T1, T2, AR1, AR2, AR1, AR2)
  113     CALL SQ (Z(1), Z(2), T1, T2)
  114     CALL FAD (T1, T2, AR1, AR2, AR1, AR2)
  115     CALL DPSQRT (AR1, AR2, AR1, AR2)
C                   MAGNITUDE OF VECTOR R IN LOCS. AR1,AR2
  120     CALL FMPS (XMI(1),XMI(2),XKSQ(2),C1,C2)
  121     CALL FDY (2.0, 0.0, AR1, AR2, T1, T2)
  122     CALL FDY (SD1, SD2, C1, C2, TT1, TT2)
  123     CALL FSB (T1, T2, TT1, TT2, T1, T2)
  124     CALL FDY (1.0, 0.0, T1, T2, A1, A2)
C                   SECTION 120-124 CALCULATES VALUE A, STORED IN LOCS. A1,A2
  125     CALL FDY (AR1, AR2, A1, A2, T1, T2)
  126     CALL FSB (1.0, 0.0, T1, T2, CON1, CON2)
  130     IF (A1) 133, 131, 137
C               QUANTITY A = 0 INDICATES PARABOLIC ORBIT.  SET A = -.1E-8 AND
C               USE EQUATIONS FOR HYPERBOLIC ORBIT.
  131     A1 = -1.0 E-8
  132     ERASE A2
  133     CALL FMPS(A1,A2,-1.0,A1,A2)
C                   SET A = ABSF(A) FOR COMPUTING EASE IN THIS SUBROUTINE
  135     M = 1
C                   QUANTITY A NEGATIVE INDICATES HYPERBOLIC ORBIT - SET M = 1
  136     GO TO 138
C                   QUANTITY A POSITIVE INDICATES ELLIPTIC ORBIT - SET M = 2
  137     M = 2
  138     CALL FMP(C1,C2,A1,A2,S1,S2)
  139     CALL SQ (RR1,RR2,TT1,TT2)
  140     CALL FDY (TT1,TT2,S1,S2,T1,T2)
  141     CALL DPSQRT(S1,S2,S1,S2)
  143     CALL FDY (RR1,RR2,S1,S2,SV1,SV2)
  144     CALL CUBE(A1,A2,SAV1,SAV2)
C                   EXTRA CALCULATIONS TO OBTAIN MAXIMUM ACCURACY
  145     CALL SQ (CON1, CON2, TT1, TT2)
  146     GO TO (147, 150), M
  147     CALL FSB (TT1, TT2, T1, T2, ESQ1, ESQ2)
  148     GO TO 151
  150     CALL FAD (TT1, TT2, T1, T2, ESQ1, ESQ2)
  151     CALL DPSQRT (ESQ1, ESQ2, E(1), E(2))
C                   E CALCULATED IN SECTION 143-151
  153     IF (E(1)) 900,910,154
C         E = 0 INDICATES CIRCULAR ORBIT - ERROR RETURN - NERR = 3
  154     CALL FDY (SV1, SV2, E(1), E(2), T1, T2)
  155     CALL FDY (CON1, CON2, E(1), E(2), TT1, TT2)
  156     GO TO (160, 180), M
C               SECTION 160 - 172 USED WHEN A IS NEG., M=1, HYPERBOLIC ORBIT
  160     CALL FAD (T1, T2, TT1, TT2, T1, T2)
  161     CALL DPLOG (T1, T2, FJ1, FJ2, I)
C                   SECTION 160-161 CALCULATES FJ  IN LOCS. FJ1,FJ2
  162     GO TO (163, 900), I
  163     CALL DPSINH (FJ1, FJ2, SFJ1, SFJ2, I)
```

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  164   GO TO (165, 900), I
  165   CALL FMP (E(1), E(2), SFJ1, SFJ2, T1, T2)
  166   CALL FSB (T1, T2, FJ1, FJ2, XMJ(1), XMJ(2))
C               M SUB J COMPUTED IN SECTION 162-166,STORED XMJ(2)
  167   CALL FSBS (ESQ1, ESQ2, 1.0, T1, T2)
  168   CALL DPSQRT (T1,T2,CCON(1),CCON(2))
C       ERROR RETURN - NERR = 4 - IF E SQUARED MINUS 1 IS NEGATIVE
  170   CALL FDY(-1.0,0.0,CCON(1),CCON(2),CON1,CON2)
  171   CALL DPCOSH (FJ1, FJ2, CFJ1, CFJ2, I)
  172   GO TO (200, 900), I
C           SECTION 180 - 195 USED WHEN A IS POSITIVE, M= 2,ELLIPTIC ORBIT
  180   IF (T1)  181,194,184
  181   IF (TT1) 182,183,183
  182   NOP = 3
        GO TO 187
  183   NOP = 4
        GO TO 187
  184   IF (TT1) 185,186,186
  185   NOP = 2
        GO TO 187
  186   NOP = 1
  187   IF (ABSF(T1) - ABSF(TT1)) 188,189,189
  188   CALL SQ (TT1,TT2,TT1,TT2)
        CALL FSB (1.0,0.,TT1,TT2,TT1,TT2)
        CALL DPSQRT (TT1,TT2,T1,T2)
C       ERROR RETURN  NERR = 6 IF UNABLE TO CALC. COS EJ OR SIN EJ
  189   CALL DPASIN (T1,T2,FJ1,FJ2,I)
        IF (I) 920,920,1890
 1890   GO TO (1899,1892,1893,1893), NOP
 1892   CALL FSB (PI(1),PI(2),FJ1,FJ2,FJ1,FJ2)
        GO TO 1899
 1893   XNUM = FJ1 / ABSF(FJ1)
 1894   CALL FMPS(FJ1,FJ2,XNUM,FJ1,FJ2)
 1895   GO TO (1899,1899,1896,1897), NOP
 1896   CALL FAD (PI(1), PI(2), FJ1,FJ2,FJ1,FJ2)
        GO TO 1899
 1897   CALL FSB (PI2(1),PI2(2),FJ1,FJ2,FJ1,FJ2)
 1899   CALL DPSC (FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,1)
  190   CALL FMP (E(1), E(2), SFJ1, SFJ2, T1, T2)
  191   CALL FSB (FJ1, FJ2, T1, T2, XMJ(1), XMJ(2))
C               SECTION 186-191 COMPUTES M SUB J -STORED IN LOCS. XMJ(2)
  192   CALL FSB (1.0, 0.0, ESQ1, ESQ2, T1, T2)
C             SECTION 192-195 FOR ELLIPTIC CASE PARALLELS SECTION 167-171
C             FOR HYPERBOLIC CASE
  193   CALL DPSQRT (T1,T2,CCON(1),CCON(2))
  195   CALL FDY (1.0,0.0,CCON(1),CCON(2), CON1,CON2)
C               SECTION 200 USED FOR BOTH TYPES OF ORBITS
  200   CALL FDY (CFJ1, CFJ2, AR1, AR2, ESQ1,ESQ2)
  201   CALL FDY (A1,A2,C1,C2,T1,T2)
        CALL DPSQRT (T1,T2,T1,T2)
  202   CALL FMP (T1, T2, SFJ1, SFJ2, RR1, RR2)
  203   CALL FDY (SFJ1, SFJ2, AR1, AR2, SV1, SV2)
  204   CALL FSB (CFJ1, CFJ2, E(1), E(2), TT1, TT2)
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```
      205   CALL FMP (TT1, TT2, T1, T2, SD1, SD2)
      206   GO TO (207, 21D), M
      207   CALL FMPS(SV1,SV2,-1.0,SV1,SV2)
      208   CALL FMPS(A1,A2,-1.0,A1,A2)
C                 CHANGE SIGN DF A TD MINUS AGAIN
C                 SECTIDN 210-218 CALCULATES 3 CDMPDNENTS DF VECTDR P FOR
C                 BOTH TYPES OF ORBITS
      210   CALL FMP (X(1), X(2), ESD1, ESQ2, TT1, TT2)
      211   CALL FMP (XDOT(1), XDOT(2), RR1, RR2, T1, T2)
      212   CALL FSB (TT1, TT2, T1, T2, PX(1), PX(2))
      213   CALL FMP (Y(1), Y(2), ESD1, ESQ2, TT1, TT2)
      214   CALL FMP (YDOT(1), YDOT(2), RR1, RR2, T1, T2)
      215   CALL FSB (TT1, TT2, T1, T2, PY(1), PY(2))
      216   CALL FMP (Z(1), Z(2), ESQI, ESQ2, TT1, TT2)
      217   CALL FMP (ZDDT(1), ZDOT(2), RR1, RR2, T1, T2)
      218   CALL FSB (TT1, TT2, T1, T2, PZ(1), PZ(2))
C                 SECTIDN 220-242 CALCULATES 3 COMPDNENTS OF VECTOR Q FDR
C                 BOTH TYPES OF DRBITS
      220   CALL FMP (X(1), X(2), SV1, SV2, T1, T2)
      221   CALL FMP (XDOT(1), XDOT(2), SD1, SD2, TT1, TT2)
      222   CALL FAD (T1, T2, TT1, TT2, QX(1), QX(2))
      223   CALL FMP (Y(1), Y(2), SV1, SV2, T1, T2)
      224   CALL FMP (YDOT(1), YDDT(2), SD1, SD2, TT1, TT2)
      225   CALL FAD (T1, T2, TT1, TT2, DY(1), DY(2))
      226   CALL FMP (Z(1), Z(2), SV1, SV2, T1, T2)
      227   CALL FMP (ZDOT(1), ZDDT(2), SD1, SD2, TT1, TT2)
      228   CALL FAD (T1, T2, TT1, TT2, DZ(1), DZ(2))
      24D   CALL FMP (QX(1), QX(2), CON1, CON2, DX(1), QX(2))
      241   CALL FMP (QY(1), DY(2), CON1, CON2, DY(1), QY(2))
      242   CALL FMP (QZ(1), QZ(2), CON1, CON2, DZ(1), DZ(2))
      245   CALL FDY (C1,C2,SAV1,SAV2,T1,T2)
      246   CALL DPSORT (T1,T2,XN(1),XN(2))
      250   RETURN
      900   STOP 77777
      910   STOP 77777
      920   STOP 77777
            END (1,1,D,D,0,0)
C           POSITIDN          BY  MARY ELLEN
            SUBRDUTINE POSN
            EQUIVALENCE(WM(1),XIM),(WM(2),YIM),(WM(3),ZIM),(WM(4),XIDM),
          1 (WM(6),ZIDM),(WM(5),YIDM),(WL(1),XIL),(WL(2),YIL),(WL(3),ZIL),
          2 (WL(4),XIDL),(WL(5),YIDL),(WL(6),ZIDL)
            DAC IX(9,67),IY(9,67),IZ(9,67),TL(67),RKM(4,6),RKL(4,6),NB(9),
          1 WT(9),RAD(9),    DUMMY(27)   ,DIS(9),DIST(9),PM(6),PL(6),PPM(6),
          2 PPL(6),DM(6),DL(6),WM(6),WL(6),XX(9),YY(9),ZZ(9),TI(6),CL(6)
            COMMON NEWDRG,ZER,FMX,FLX,FMY,FLY,FMZ,FLZ,AM,AL,BM,BL,CM,CL
            COMMDN NORG,NTARG,NSUN,N,NN,NNN,T,TD,TMAX,DTMAX,HAFDT,WTE,WTV,MGM,
          1 XTARG,YTARG,ZTARG,SCEN,VCEN,SMAX,GMAX,EMAX,RAD1,RAD2,RADE,WTA,
          2 NBE,NBT,NBS,RADDRG,SAM,SBM,SCM, SAL,SBL,SCL,XIM,XIL,YIM,YIL,ZIM,
          3 ZIL,XIDM,XIDL,YIDM,YIDL,ZIDM,ZIDL,RM,RL,RR,RO,SS,RRM,RRL,TEM,TEL,
          4 GMX,GLX,GMY,GLY,GMZ,GLZ,SUMX,SULX,SUMY,SULY,SUMZ,SULZ,FM,FL,GM,GL,
          5 HM,HL,RRR,SSS,XM,YM,ZM,A,BX,BY,BZ,WRM,WRL,RCM,RCL,RDM,ROL,RPPM,
          6 XPM,XPL,YPM,YPL,ZPM,ZPL,
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      7 KOB,VNZ,MEDT,MSDT,MDT,NOT,MET,NOUT,NCKE,TSAV,DTA,DTB,DTC,EP1
        DAC XKSQ(2), XMI(2)
        DAC X(2),Y(2),Z(2),XDOT(2),YDOT(2),ZDOT(2),PX(2),PY(2),PZ(2),QX(2)
      1 ,QY(2),QZ(2),XP(2),YP(2),ZP(2),XDOTP(2),YDOTP(2),ZDOTP(2),PI(2),
      2 PI2(2),E(2),XMJ(2),CCON(2),XN(2)
        COMMON A1,A2,FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,S1,S2,EPS1
        COMMON DTME,DT,RPM
        DAC SM(6),SL(6),XMJP(2)
        COMMON OBJ,TSAVE,REA,RTA,RSN,HPI,THPI,KK,NOSW
   90   CALL FMP (A1,A2,CCON(1),CCON(2),TC1,TC2)
   91   IF (A1) 92,900,99
   92   CALL FMPS(TC1,TC2,-1.0,TC1,TC2)
C               QUANTITY A NEGATIVE INDICATES HYPERBOLIC ORBIT - SET M=1
   93   M= 1
   94   GO TO 102
C               QUANTITY A POSITIVE INDICATES ELLIPTIC ORBIT - SET M = 2
   99   M = 2
  102   CALL FMPS(XN(1),XN(2),DT,SD1,SD2)
  103   CALL FAD (XMJ(1),XMJ(2),SD1,SD2,XMJP(1),XMJP(2))
  104   EJ1 = FJ1
  105   EJ2 = FJ2
C           USE VALUE EJ OR FJ AT T=[illegible] FOR FIRST GUESS AT T= TJ +DELTA T
  106   SEJ1 = SFJ1
  107   SEJ2 = SFJ2
  108   CEJ1 = CFJ1
  109   CEJ2 = CFJ2
C               SECTION 110 - 160 DOES NEWTON-RAPHSON ITERATION FOR E(J+1)
  110   DO 160 K = 1, 99
  111   GO TO (120,112),M
  112   CALL FMP (E(1),E(2),SEJ1,SEJ2,T1,T2)
  113   CALL FSB (EJ1,EJ2,T1,T2,T1,T2)
  114   CALL FMP (E(1),E(2),CEJ1,CEJ2,TT1,TT2)
  115   CALL FSB (1.0,0.0,TT1,TT2,TT1,TT2)
  116   GO TO 130
C           SECTION 112-116 INITIALIZES DELTA F CALC. FOR ELLIPTIC CASE
  120   CALL FMP (E(1),E(2),SEJ1,SEJ2,T1,T2)
  121   CALL FSB (T1,T2,EJ1,EJ2,T1,T2)
  122   CALL FMP (E(1),E(2),CEJ1,CEJ2,TT1,TT2)
  123   CALL FSBS (TT1,TT2,1.0,TT1,TT2)
C           SECTION 120-123 INITIALIZES DELTA F CALC. FOR HYPERBOLIC CASE
  130   CALL FSB (XMJP(1),XMJP(2),T1,T2,XNJ1,XNJ2)
  131   CALL FDY (XNJ1,XNJ2,TT1,TT2,DEJ1,DEJ2)
  132   IF (ABSF(DEJ1)-EPS1) 170, 170, 140
  140   CALL FAD (EJ1,EJ2,DEJ1,DEJ2,EJ1,EJ2)
C               REPLACE OLD GUESS WITH NEW ONE
  148   GO TO (155,150),M
  150   CALL DPSC (EJ1,EJ2,SEJ1,SEJ2,CEJ1,CEJ2,1)
  151   GO TO 160
  155   CALL DPSINH (EJ1,EJ2,SEJ1,SEJ2,I)
  156   GO TO (157,900), I
  157   CALL DPCOSH (EJ1,EJ2,CEJ1,CEJ2,I)
  158   GO TO (160,900), I
  160   CONTINUE
```

```
C               GIVE NEWTON-RAPHSON 99 CHANCES TO CONVERGE
  161     GO TO 900
C            MAKE ALL SORTS OF TESTS ON SIN,COS EJ TO DETERMINE FINAL VALUE
C            EJ AT T+DELTA T  - SECTION 171-193 FOR ELLIPTIC CASE
  170     GO TO (200,171),M
  171     IF (SEJ1) 178,178,172
  172     IF (CEJ1) 176,176,173
  173     XMAX = HPI
  174     ERASE XMIN
  175     GO TO 183
  176     XMAX = PI
  177     XMIN = HPI
          GO TO 183
  178     IF (CEJ1) 179,179,181
  179     XMAX = THPI
  180     XMIN = PI
          GO TO 183
  181     XMAX = PI2(1)
  182     XMIN = THPI
  183     IF (EJ1 -XMAX) 184,200,185
  184     IF (EJ1 -XMIN) 186,200,200
  185     CALL FSB(EJ1,EJ2,PI2(1),PI2(2),EJ1,EJ2)
          GO TO  187
  186     CALL FAD (EJ1,EJ2,PI2(1),PI2(2),EJ1,EJ2)
  187     IF (EJ1 -XMAX) 188,110,185
  188     IF (EJ1 -XMIN) 186,110,110
  200     CALL FSB (CEJ1,CEJ2,E(1),E(2),T1,T2)
  201     CALL FMP (T1,T2,A1,A2,XW1,XW2)
  202     CALL FMP(TC1,TC2,SEJ1,SEJ2,YW1,YW2)
C               SECTION 200-202 COMPUTES XW AND YW AT J+1
  204     CALL SQ (XW1,XW2,T1,T2)
  205     CALL SQ (YW1,YW2,TT1,TT2)
  206     CALL FAD (T1,T2,TT1,TT2,T1,T2)
  207     CALL DPSQRT (T1,T2,AR1,AR2)
C               SECTION 210-215 COMPUTES XW DOT, YW DOT AT J+1
  210     CALL FMP (S1,S2,SEJ1,SEJ2,T1,T2)
  211     CALL FDY (T1,T2,AR1,AR2,XWD1,XWD2)
  212     CALL FMPS (XWD1,XWD2,-1.0,XWD1,XWD2)
  213     CALL FMP(S1,S2,CCON(1),CCON(2),T1,T2)
  214     CALL FMP (T1,T2,CEJ1,CEJ2,T1,T2)
  215     CALL FDY (T1,T2,AR1,AR2,YWD1,YWD2)
C               SECTION 220-238 COMPUTES 3COMPONENTS OF VECTORS R,RDOT AT
C               T = T +DELTA T     (J+1)
  220     CALL FMP (XW1,XW2,PX(1),PX(2),T1,T2)
  221     CALL FMP (YW1,YW2,QX(1),QX(2),TT1,TT2)
  222     CALL FAD (T1,T2,TT1,TT2,XP(1),XP(2))
  223     CALL FMP (XW1,XW2,PY(1),PY(2),T1,T2)
  224     CALL FMP (YW1,YW2,QY(1),QY(2),TT1,TT2)
  225     CALL FAD (T1,T2,TT1,TT2,YP(1),YP(2) )
  226     CALL FMP (XW1,XW2,PZ(1),PZ(2),T1,T2)
  227     CALL FMP (YW1,YW2,QZ(1),QZ(2),TT1,TT2)
  228     CALL FAD (T1,T2,TT1,TT2,ZP(1),ZP(2))
          CALL SQ (XP(1),XP(2),AM,AL)
```

```
      CALL SQ (YP(1),YP(2),BM,BL)
      CALL SQ (ZP(1),ZP(2),CM,CL)
      CALL FAD (AM,AL,BM,BL,RCM,RCL)
      CALL FAD (CM,CL,RCM,RCL,RPM,RPL)
      CALL DPSQRT (RPM,RPL,RCM,RCL)
      CALL FMP (RCM,RCL,RPM,RPL,RCM,RCL)
      CALL FDY (XMI,ZER,RCM,RCL,RWM,RWL)
      CALL FMP (XP(1),XP(2),RWM,RWL,XPM,XPL)
      CALL FMP (YP(1),YP(2),RWM,RWL,YPM,YPL)
      CALL FMP (ZP(1),ZP(2),RWM,RWL,ZPM,ZPL)
      IF (VNZ) 230,243,230
230   CALL FMP (XWD1,XWD2,PX(1),PX(2),T1,T2)
231   CALL FMP (YWD1,YWD2,QX(1),QX(2),TT1,TT2)
232   CALL FAD (T1,T2,TT1,TT2,XDOTP(1),XDOTP(2))
233   CALL FMP (XWD1,XWD2,PY(1),PY(2),T1,T2)
234   CALL FMP (YWD1,YWD2,QY(1),QY(2),TT1,TT2)
235   CALL FAD (T1,T2,TT1,TT2,YDOTP(1),YDOTP(2))
236   CALL FMP (XWD1,XWD2,PZ(1),PZ(2),T1,T2)
237   CALL FMP (YWD1,YWD2,QZ(1),QZ(2),TT1,TT2)
238   CALL FAD (T1,T2,TT1,TT2,ZDOTP(1),ZDOTP(2))
243   FJ1 = EJ1
      FJ2 = EJ2
      RETURN
900   STOP 77777
      END (1,1,0,0,0,0)
      SUBROUTINE INTN
 6    CALL FMPS (XIDM,XIDL,DT,RKM(1,1),RKL(1,1))
      CALL FMPS (YIDM,YIDL,DT,RKM(1,2),RKL(1,2))
      CALL FMPS (ZIDM,ZIDL,DT,RKM(1,3),RKL(1,3))
 8    SAM = PM (1)
      SAL = PL (1)
      SBM = PM (2)
      SBL = PL (2)
      SCM = PM (3)
      SCL = PL (3)
      CALL ACCN
12    CALL FMPS (FMX,FLX,DT,RKM(1,4),RKL(1,4))
      CALL FMPS (FMY,FLY,DT,RKM(1,5),RKL(1,5))
      CALL FMPS (FMZ,FLZ,DT,RKM(1,6),RKL(1,6))
H       CAL RKM-12
H       SUB MGM
H       SLW FM
H       CAL RKL-12
H       SUB MGM
H       SLW FL
H       CAL RKM-16
H       SUB MGM
H       SLW GM
H       CAL RKL-16
H       SUB MGM
H       SLW GL
H       CAL RKM-20
H       SUB MGM
```

```
H          SLW HM
H          CAL RKL-20
H          SUB MGM
H          SLW HL
     14    CALL FAD (XIDM,XIDL,FM,FL,AM,AL)
           CALL FAD (YIDM,YIDL,GM,GL,BM,BL)
           CALL FAD (ZIDM,ZIDL,HM,HL,CM,CL)
           CALL FMPS (AM,AL,DT,RKM(2,1),RKL(2,1))
           CALL FMPS (BM,BL,DT,RKM(2,2),RKL(2,2))
           CALL FMPS (CM,CL,DT,RKM(2,3),RKL(2,3))
H          CAL RKM
H          SUB MGM
H          SLW FM
H          CAL RKL
H          SUB MGM
H          SLW FL
H          CAL RKM-4
H          SUB MGM
H          SLW GM
H          CAL RKL-4
H          SUB MGM
H          SLW GL
H          CAL RKM-8
H          SUB MGM
H          SLW HM
H          CAL RKL-8
H          SUB MGM
H          SLW HL
     18    DTME = DTME + HAFDT
           T = T + HAFDT
           CALL TERP
           CALL POSN
     20    CALL FAD (XIM,XIL,XP(1),XP(2),XAM,XAL)
           CALL FAD (YIM,YIL,YP(1),YP(2),YAM,YAL)
           CALL FAD (ZIM,ZIL,ZP(1),ZP(2),ZAM,ZAL)
           CALL FAD (XAM,XAL,FM,FL,SAM,SAL)
           CALL FAD (YAM,YAL,GM,GL,SBM,SBL)
           CALL FAD (ZAM,ZAL,HM,HL,SCM,SCL)
     22    CALL ACCN
           CALL FMPS (FMX,FLX,DT,RKM(2,4),RKL(2,4))
           CALL FMPS (FMY,FLY,DT,RKM(2,5),RKL(2,5))
           CALL FMPS (FMZ,FLZ,DT,RKM(2,6),RKL(2,6))
H          CAL RKM-13
H          SUB MGM
H          SLW FM
H          CAL RKL-13
H          SUB MGM
H          SLW FL
H          CAL RKM-17
H          SUB MGM
H          SLW GM
H          CAL RKL-17
H          SUB MGM
```

```
H           SLW GL
H           CAL RKM-21
H           SUB MGM
H           SLW HM
H           CAL RKL-21
H           SUB MGM
H           SLW HL
      24    CALL FAD (XIDM,XIDL,FM,FL,AM,AL)
            CALL FAD (YIDM,YIDL,GM,GL,BM,BL)
            CALL FAD (ZIDM,ZIDL,HM,HL,CM,CL)
            CALL FMPS (AM,AL,DT,RKM(3,1),RKL(3,1))
            CALL FMPS (BM,BL,DT,RKM(3,2),RKL(3,2))
            CALL FMPS (CM,CL,DT,RKM(3,3),RKL(3,3))
H           CAL RKM-1
H           SUB MGM
H           SLW FM
H           CAL RKL-1
H           SUB MGM
H           SLW FL
H           CAL RKM-5
H           SUB MGM
H           SLW GM
H           CAL RKL-5
H           SUB MGM
H           SLW GL
H           CAL RKM-9
H           SUB MGM
H           SLW HM
H           CAL RKL-9
H           SUB MGM
H           SLW HL
      28    CALL FAD (XAM,XAL,FM,FL,SAM,SAL)
            CALL FAD (YAM,YAL,GM,GL,SBM,SBL)
            CALL FAD (ZAM,ZAL,HM,HL,SCM,SCL)
            CALL ACCN
            CALL FMPS (FMX,FLX,DT,RKM(3,4),RKL(3,4))
            CALL FMPS (FMY,FLY,DT,RKM(3,5),RKL(3,5))
            CALL FMPS (FMZ,FLZ,DT,RKM(3,6),RKL(3,6))
      32    CALL FAD (XIDM,XIDL,RKM(3,4),RKL(3,4),AM,AL)
            CALL FAD (YIDM,YIDL,RKM(3,5),RKL(3,5),BM,BL)
            CALL FAD (ZIDM,ZIDL,RKM(3,6),RKL(3,6),CM,CL)
            CALL FMPS (AM,AL,DT,RKM(4,1),RKL(4,1))
            CALL FMPS (BM,BL,DT,RKM(4,2),RKL(4,2))
            CALL FMPS (CM,CL,DT,RKM(4,3),RKL(4,3))
      36    DTME = DTME + HAFDT
            T = T + HAFDT
            CALL TERP
            VNZ = 1.
            CALL POSN
      40    CALL FAD (XIM,XIL,RKM(3,1),RKL(3,1),AM,AL)
            CALL FAD (YIM,YIL,RKM(3,2),RKL(3,2),BM,BL)
            CALL FAD (ZIM,ZIL,RKM(3,3),RKL(3,3),CM,CL)
            CALL FAD (AM,AL,XP(1),XP(2),SAM,SAL)
```

```
      CALL FAD (BM,BL,YP(1),YP(2),SBM,SBL)
      CALL FAD (CM,CL,ZP(1),ZP(2),SCM,SCL)
      CALL ACCN
      ERASE VNZ
      CALL FMPS (FMX,FLX,DT,RKM(4,4),RKL(4,4))
      CALL FMPS (FMY,FLY,DT,RKM(4,5),RKL(4,5))
      CALL FMPS (FMZ,FLZ,DT,RKM(4,6),RKL(4,6))
44    DO 52 NA = 1,6
      SDM = RKM(1,NA)
      SDL = RKL(1,NA)
      SEM = RKM(2,NA)
      SEL = RKL(2,NA)
      SFM = RKM(3,NA)
      SFL = RKL(3,NA)
      SGM = RKM(4,NA)
      SGL = RKL(4,NA)
46    CALL FAD (SEM,SEL,SFM,SFL,SEM,SEL)
      CALL FAD (SEM,SEL,SEM,SEL,SEM,SEL)
48    CALL FAD (SDM,SDL,SEM,SEL,SEM,SEL)
      CALL FAD (SGM,SGL,SEM,SEL,SEM,SEL)
      CALL FDYS (SEM,SEL,6.0,SEM,SEL)
      SFM = WM(NA)
      SFL = WL(NA)
50    CALL FAD (SFM,SFL,SEM,SEL,SFM,SFL)
      WM(NA) = SFM
52    WL(NA) = SFL
      RETURN
      END (1,1,0,0,0,0)
      SUBROUTINE ACCN
 2    CALL SQ (SAM,SAL,AM,AL)
      CALL SQ (SBM,SBL,BM,BL)
      CALL SQ (SCM,SCL,CM,CL)
 3    CALL FAD (AM,AL,BM,BL,RM,RL)
      CALL FAD (RM,RL,CM,CL,RO,RL)
 4    CALL DPSQRT (RO,RL,RR,SS)
 5    CALL FMP (RR,SS,RO,RL,RRM,RRL)
 6    CALL FDY (WTS,ZER,RRM,RRL,TEM,TEL)
 7    CALL FMP ( SAM , SAL ,TEM,TEL,GMX,GLX)
      CALL FMP ( SBM ,SBL , TEM,TEL,GMY,GLY)
      CALL FMP ( SCM , SCL ,TEM,TEL,GMZ,GLZ)
      CALL FSB (GMX,GLX,XPM,XPL,GMX,GLX)
      CALL FSB (GMY,GLY,YPM,YPL,GMY,GLY)
      CALL FSB (GMZ,GLZ,ZPM,ZPL,GMZ,GLZ)
70    IF (KOB) 71,8,71
71    CALL FMP (RO,RL,RRM,RRL,TI(1),TI(2))
      CALL FMPS (CM,CL,5.0,TEM,TEL)
      CALL FDY (TEM,TEL,RO,RL,TEM,TEL)
      CALL FSBS (TEM,TEL,1.0,TEM,TEL)
72    CALL FDY (TEM,TEL,TI(1),TI(2),TEM,TEL)
      CALL FMPS(TEM,TEL,OBJ,TEM,TEL)
      CALL FMP (TEM,TEL,SAM,SAL,XM,XL)
      CALL FMP (TEM,TEL,SBM,SBL,YM,YL)
73    CALL FDY (2.0,ZER,TI(1),TI(2),ZM,ZL)
```

```
      CALL FMPS (ZM,ZL,OBJ,ZM,ZL)
      CALL FSB (TEM,TEL,ZM,ZL,ZM,ZL)
      CALL FMP (ZM,ZL,SCM,SCL,ZM,ZL)
   74 CALL FSB (GMX,GLX,XM,XL,GMX,GLX)
      CALL FSB (GMY,GLY,YM,YL,GMY,GLY)
      CALL FSB (GMZ,GLZ,ZM,ZL,GMZ,GLZ)
    8 ERASE SUMX,SULX,SUMY,SULY,SUMZ,SULZ
    9 DO 29 J = 1, N
      XM = X (J)
      YM = Y (J)
      ZM = Z (J)
      WTA = WT (J)
   12 CALL FSBS (SAM,SAL,XM,XM,XL)
      CALL FSBS (SBM,SBL,YM,YM,YL)
      CALL FSBS (SCM,SCL,ZM,ZM,ZL)
   13 CALL SQ (XM,XL,AM,AL)
      CALL SQ (YM,YL,BM,BL)
      CALL SQ (ZM,ZL,CM,CL)
   14 CALL FAD (AM,AL,BM,BL,RM,RL)
      CALL FAD (RM,RL,CM,CL,RM,RL)
   15 CALL DPSQRT (RM,RL,RRR,SSS)
      IF (VNZ) 16,17,16
   16 DIS(J) = RM
      DIST(J) = RRR
   17 CALL FMP (RM,RL,RRR,SSS,RM,RL)
      CALL FDY (XM,XL,RM,RL,AM,AL)
      CALL FDY (YM,YL,RM,RL,BM,BL)
      CALL FDY (ZM,ZL,RM,RL,CM,CL)
      A   =   (X(J)*X(J)+Y(J)*Y(J)+Z(J)*Z(J))**1.5
      BX = X(J)/A
      BY = Y(J)/A
      BZ = Z(J)/A
   26 CALL FADS (AM,AL,BX,AM,AL)
      CALL FADS (BM,BL,BY,BM,BL)
      CALL FADS (CM,CL,BZ,CM,CL)
   22 CALL FMPS (AM,AL,WTA,AM,AL)
      CALL FMPS (BM,BL,WTA,BM,BL)
      CALL FMPS (CM,CL,WTA,CM,CL)
   23 CALL FAD (AM,AL,SUMX,SULX,SUMX,SULX)
      CALL FAD (BM,BL,SUMY,SULY,SUMY,SULY)
   29 CALL FAD (CM,CL,SUMZ,SULZ,SUMZ,SULZ)
      CALL FAD (GMX,GLX,SUMX,SULX,FMX,FLX)
      CALL FAD (GMY,GLY,SUMY,SULY,FMY,FLY)
   32 CALL FAD (GMZ,GLZ,SUMZ,SULZ,FMZ,FLZ)
      CALL FMPS (FMX,FLX,G,FMX,FLX)
      CALL FMPS (FMY,FLY,G,FMY,FLY)
      CALL FMPS (FMZ,FLZ,G,FMZ,FLZ)
      RETURN
      END (1,1,0,0,0,0)
```

APPENDIX B

OPERATIONAL DIRECTORIES, LISTING AND CHECK PROBLEMS FOR SECTIONS II, III AND IV

A. TWO POSITION VECTOR PROGRAM

a. Operational Directory

Purpose

To find the position and velocity vector of an observed body at time T_o from two-range vectors.

Usage

Input - All decimal input is read by a modified DBC FORTRAN sub-routine which accepts variable length fields. All fields are floating point numbers except for starred fields which are integers. Following is the list of input quantities in the order in which they are read into the program. All range measurements must be converted to units of earth's equatorial radius (6378.388 km. = equatorial radius of earth). All angles are to be expressed in degrees and decimals of a degree. Time is to be in days and decimals of a day.

Field 1 -	ρ_o	Magnitude of range measurement at time, T_o
Field 2 -	ρ_1	Magnitude of range measurement at time, T_1
Field 3 -	X_o,	Geocentric, equatorial coordinates
Field 4 -	Y_o,	of observer's position on earth
Field 5 -	Z_o,	at time, T_o.
Field 6 -	X_1,	Geocentric, equatorial coordinates
Field 7 -	Y_1,	of observer's position on earth

Field 8 - Z_1, at time, T.

Field 9 - δ_o Declination of observed body at time, T_o.

Field 10 - δ_1 Declination of observed body at time, T_1

Field 11 - α_o Right ascension of observed body at time, T_o

Field 12 - α_1 Right ascension of observed body at time, T_1

Field 13 - T_o, Time

Field 14 - T_1, Time

Field 15 - Control parameter, KO. If KO ≤ 0, the unit of time will be equal to 806.996813 sec.. If KO ≥ 1, the unit of time will be 58.132441 days.

Sample Input

Card 1

F 1.41889, 1.47513, 1.00082, .05063, .02196*

Card 2

F 2.3587, -.27808, -.19578, -9.34, -10.51*

Card 3

F 342.34, 341.465, 72.5, 80.5, XO*

Output - The X, Y, Z components of the position vector are printed in units of Earth's equatorial radius, kilometers, and astronomical units. The $\dot{X}$, $\dot{Y}$, $\dot{Z}$ components of the velocity vector are printed in units of Earth's equatorial radius per 806.996813 sec., km. per sec. and A.U. per hr., if KO $\leq$ O. If KO ≥ 1, the velocity will be printed out in units of Earth's equatorial radius per 58.132441 days, km. per sec. and A.U. per hr.

b. Two Position Vector Program

```
      DIMENSION PM(6), PL(6), PZM(6), PZL(6)
    1 RIT2, 16, RHO, RHOI, XO, YO, ZO, XI, YI, ZI, PHIO, PHII, ALPO,
    1 ALPI, TO, TI, KO
   16 FORMAT (515)
      ERASE K, KNT
      NO = 1
    2 CALL DPSC(PHIO, O. , SPOM, SPOL, CPOM, CPOL, O)
      CALL DPSC(PHII, O. , SPIM, SPIL, CPIM, CPIL, O)
      CALL DPSC(ALPO, O, SAOM, SAOL, CAOM, CAOL, O)
      CALL DPSC(ALPI, O. , SAIM, SAIL, CAIM, CAIL, O)
      CALL FMP(CPOM, CPOL, CAOM, CAOL, CCM, CCL)
      CALL FMPS(CCM, CCL, RHO, CCM, CCL)
      CALL FADS(CCM, CCL, XO, PM(1), PL(1) )
      CALL FMP(CPOM, CPOL, SAOM, SAOL, CCM, CCL)
      CALL FMPS(CCM, CCL, RHO, CCM, CCL)
      CALL FADS(CCM, CCL, YO, PM(2), PL(2) )
      CALL FMPS(SPOM, SPOL, RHO, CCM, CCL)
      CALL FADS(CCM, CCL, ZO, PM(3), PL(3) )
      CALL FMP(CPIM, CPIL, CAIM, CAIL, CCM, CCL)
      CALL FMPS(CCM, CCL, RHOI, CCM, CCL)
      CALL FADS(CCM, CCL, XI, XIM, XIL)
      CALL FMP(CPIM, CPIL, SAIM, SAIL, CCM, CCL)
      CALL FMPS(CCM, CCL, RHOI, CCM, CCL)
      CALL FADS(CCM, CCL, YI, YIM, YIL)
      CALL FMPS(SPIM, SPIL, RHOI; CCM, CCL)
      CALL FADS(CCM, CCL, ZI, ZIM, ZIL)
    3 CALL SQ(PM(1), PL(1), XSQOM, XSQOL)
      CALLSQ(PM(2), PL(2), YSQOM, YSQOL)
      CALL SQ(PM(3), PL(3), ZSQOM, ZSQOL)
```

```
      CALL FAD(XSQOM, XSQOL, YSQOM, YSQOL, CCM,CCL)
      CALL FAD(ZSQOM, ZSQOL, CCM, CCL, RO2M, RO2L)
      CALL DPSQRT(RO2M, RO2L, ROM, ROL)
      CALL CUBE(ROM, ROL, ROCM, ROCL)
      CALL FDY(1., O., ROCM, ROCL, MUM, MUL)
      CALL FSBS(TI, O., TO, CCM, CCL)
      IF (KO) 30, 30, 31
30    TU = 1.7072813E+2
      GO TO 32
31    TU = .017202098
32    CALL FMPS(CCM, CCL, TU, TAUM, TAUL)
      CALL SQ(TAUM, TAUL, TAU2M, TAU2L)
      CALL CUBE(TAUM, TAUL, TAU3M, TAU3L)
      CALL SQ(TAU2M, TAU2L, TAU4M, TAU4L)
      CALL FMP(TAU2M, TAU2L, TAU3M, TAU3L, TAU5M, TAU5L)
      CALL FMP(MUM, MUL, TAU3M, TAU3L, MTM, MTL)
      CALL FDYS(MTM, MTL, 6., GIM, GIL)
      CALL FSB(TAUM, TAUL, GIM, GIL, GIM, GIL)
      CALL FMP(MUM, MUL, TAU2M, TAU2L, FIM, FIL)
      CALL FMPS(FIM, FIL, .5, FIM, FIL)
      CALL FSB(1., 0., FIM, FIL, FIM, FIL)
      CALL FMPS(MTM, MTL, .5, AM, AL)
      CALL FMP(MUM, MUL, TAU4M, TAU4L, MCM, MCL)
      CALL FDYS(MCM, MCL, 24., CM, CL)
      CALL FMPS(MCM, MCL, .25, HM, HL)
      CALL FMP(MUM, MUL, TAU5M, TAU5L, MDM, MDL)
      CALL FDYS(MDM, MDL, -8., EM, EL)
      CALL FDYS(MDM, MDL, 120., OM, OL)
      CALL FAD(MUM, MUL, MUM, MUL, DM, DL)
      CALL FMPS(DM, DL, 4., QM, QL)
      GM = GIM
```

```
      GL = GIL
      FM = FIM
      FL = FIL
    4 CALL FMP(PM(1), PL(1), FM, FL, CCM, CCL)
      CALL FSB(XIM, XIL, CCM, CCL, CCM, CCL)
      CALL FDY(CCM, CCL, GM, GL, DXOM, DXOL)
      CALL FMP(PM(2), PL(2), FM, FL, CCM, CCL)
      CALL FSB(YIM, YIL, CCM, CCL, CCM, CCL)
      CALL FDY(CCM, CCL, GM, GL, DYOM, DYOL)
      CALL FMP(PM(3), PL(3), FM, FL, CCM, CCL)
      CALL FSB(ZIM, ZIL, CCM, CCL, CCM, CCL)
      CALL FDY(CCM, CCL, GM, GL, DZOM, DZOL)
      GO TO (9, 10), NO
    9 NO = 2
      GO TO 6
   10 CALL FSB(DXOM, DXOL, DXM, DXL, DIXM, DIXL)
      IF (ABSF(DIXM) -4E-8) 11, 11, 5
   11 CALL FSB(DYOM, DYOL, DYM, DYL, DIYM, DIYL)
      IF (ABSF(DIYM) 04E-8) 12, 12, 5
   12 CALL FSB(DZOM, DZOL, DZM, DZL, DIZM, DIZL)
      IF (ABSF(DIZM) -4E-8) 13, 13, 5
   13 K = K+1
      IF (K-2) 6, 14, 14
    5 ERASE K
    6 KNT = KNT+1
      CALL SQ(DXOM, DXOL, DSQXM, DSQXL)
      CALL SQ(DYOM, DYOL, DSQYM, DSQYL)
      CALL SQ(DZOM, DZOL, DSQZM, DSQZL)
      CALL FAD(DSQXM, DSQXL, DSQYM, DSQYL, DSM, DSL)
      CALL FAD(DSM, DSL, DSQZM, DSQZL, DRO2M, DRO2L)
      CALL FMP(PM(1), PL(1), DXOM, DXOL, XDXM, XDXL)
```

```
      CALL FMP(PM(2), PL(2), DYOM, DYOL, YDYM, YDYL)
      CALL FMP(PM(3), PL(3), DZOM, DZOL, ZDZM, ZDZL)
      CALL FAD(XDXM, XDXL, YDYM, YDYL, CCM, CCL)
      CALL FAD(CCM, CCL, ZDZM, ZDZL, RDRM, RDRL)
      CALL FDY(RDRM, RDRL, RO2M, RO2L, SIGMAM, SIGMAL)
      CALL FDY(DRO2M, DRO2L, RO2M, RO2L, OMEGAM, OMEGAL)
      CALL SQ(SIGMAM, SIGMAL, SIGMSM, SIGMSL)
      CALL FMPS(SIGMSM, SIGMSL, 7. , RM, RL)
      CALL FMPS(SIGMSM, SIGMSL, 15. , SM, SL)
      CALL FMPS(SM, SL, 3. , TM, TL)
      CALL FMPS(OMEGAM, OMEGAL, 3. , UM, UL)
      CALL FMPS(UM, UL, 3. , VM, VL)
      CALL FMP(AM, AL, SIGMAM, SIGMAL, TAM, TAL)
      CALL FSB(UM, UL, DM, DL, CCM, CCL)
      CALL FSB(CCM, CCL, SM, SL, TSM, TSL)
      CALL FMP(CM, CL, TSM, TSL, TSM, TSL)
      CALL FSB(CCM, CCL, RM, RL, CCM, CCL)
      CALL FMP(EM, EL, CCM, CCL, CDM, CDL)
      CALL FMP(CDM, CDL, SIGMAM, SIGMAL, TRM, TRL)
C     COMPUTE F
      CALL FAD(FIM, FIL, TAM, TAL, FM, FL)
      CALL FAD(FM, FL, TSM, TSL, FM, FL)
      CALL FAD(FM, FL, TRM, TRL, FM, FL)
      CALL FMP(HM, HL, SIGMAM, SIGMAL, HTM, HTL)
      CALL FSB(VM, VL, QM, QL, TOM, TOL)
      CALL FSB(TOM, TOL, TM, TL, TOM, TOL)
      CALL FMP(TOM, TOL, OM, OL, TOM, TOL)
      CALL FAD(TOM, TOL, HTM, HTL, HTM, HTL)
      CALL FAD(HTM, HTL, GIM, GIL, GM, GL)
      DXM = DXOM
      DXL = DXOL
```

```
      DYM=DYOM
      DYL = DYOL
      DZM = DZOM
      DZL = DZOL
      IF (KNT-100) 4, 4, 22
14    PM(4) = DXOM
      PL(4) = DXOL
      PM(5) = DYOM
      PL(5) = DYOL
      PM(6) = DZOM
      PL(6) = DZOL
      IF (KO) 17, 17, 18
17    WOT 10, 15, TO, (PM(I), I=1, 6)
      GO TO 20
18    WOT 10, 19, TO, (PM(I), I=1, 6)
15    FORMAT(1H13X, 6HTIME = F9. 2, 39X, 1HX, 24X, 1HY, 24X 1HZ/
     1 1HO3X, 8HPOSITION 9X, 1PE40. 7, 9X, E16. 7, 11X, E14. 7/1H 6X
     2 20HUNITS OF EARTH RADII/ 1HO3X, 8HVELOCITY35X, E14. 7,
     3 11X, E14. 7, 11X, E14. 7/1H 6X, 33HUNITS OF EARTH RADII/
     4 806. 928 SEC. ///)
19    FORMAT(1H13X, 6HTIME = F9. 2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/1
     1 HO3X, 8HPOSITION 9X, 1PE40. 7, 9X, E16. 7, 11X, E14. 7/1H 6X,
     2 20HUNITS OF EARTH RADII/ 1HO3X, 8HVELOCITY35X, E14. 7,
     3 11X, E14. 7, 11X, E14. 7/1H 6X, 33HUNITS OF EARTH RADII/
     4 58. 13244 DA. ///)
20    DO 50 I = 1, 3
      PS = PM(I)
      PSL = PL(I)
      CALL FMPS(PS, PSL, 6378. 388, PAM, PAL)
      PZM(I) = PAM
      PZL(I) = PAL
      CALL FDYS(PAM, PAL, 149. 5042132E+6, PAM, PAL)
```

```
      PM(I) = PAM
50    PL(I) = PAL
      DO 51 I = 4, 6
      PS = PM(I)
      PSL = PL(I)
      IF (KO) 60. 60, 61
60    CALL FMPS(PS, PSL, 7.90453641, PAM, PAL)
      GO TO 62
61    CALL FMPS(PS, PSL, .126992648E-2. PAM. PAL)
62    PZM(I) = PAM
      PZL(I) = PAL
      CALL FMPS(PAM, PAL, .24079588E-4, PAM, PAL)
      PM(I) = PAM
51    PL(I) = PAL
      WOT 10, 24, TO, (PZM(I), I = 1, 6)
      WOT 10, 27, TO, (PM(I), I = 1, 6)
24    FORMAT(1HO3X, 6HTIME = F9.2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/
    1 1HO3X, 8HPOSITION9X, 1PE40.7, 5H KM  E20.7, 3H KM8X.
    2 E14.7, 3H KM/1HO3X, 8HVELOCITY35X, E14.7, 7H KM/SEC4X,
    3 E14.7, 7H KM/SEC4X, E14.7, 7H KM/SEC///)
27    FORMAT(1HO3X, 6HTIME = F9.2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/
    1 1HO3X, 8HPOSITION 9X, 1PE40.7, 7H A.U.  E18.7, 5H A.U.
    2 6X, E14.7, 5H A.U./1HO3X, 8HVELOCITY 35X, E14.7, 8H A.U./
    3 HR3X, E14.7, 8H A.U./HR 3X, E14.7, 8H A.U./HR)
      GO TO 1
22    WOT 10, 23
23    FORMAT(1H15X, 9XGIVING UP)
      GO TO 1
      END (1, 1, 0, 0, 0, 1)
```

c. Two Position Vector Program

Check Problem

<u>Output:</u>

Time = 72.50

Position - Units of Earth Radii

X = 2.3358772E 00 Y = -3.7095247E-01 Z = -2.0855989E-01

Velocity - Units of Earth Radii/58.13244 DAY

X = 1.7789361E-01 Y = 6.7356692E-01 Z = 9.2517561E-02

Time = 72.50

Position

X = 1.4899131E 04KM Y = -2.3660788E 03KM Z = 1.3302759E 03KM

Velocity

X = 2.2591180E-04KM/SEC Y = 8.5538046E-04KM/SEC

Z = 1.1749050E-04KM/SEC

Time = 72.50

Position

X = 9.9656929E-05 A. U. Y = -1.5826168E-05 A. U.

Z = -8.8979160E-06 A. U.

Velocity

X = 5.4398631E-09 A. U./HR Y = 2.0597209E-08 A. U./HR

Z = 2.8291228E-09 A. U. HR

<u>Input:</u>

Card 1: F 1.41889, 1.47513, 1.00082, .05063, .02196*

Card 2: F .98156, .175430, .07609, -9.35, -10.6166667*

Card 3: F 342.475, 341.775, 72.5, 80.5*

Card 4: X1*

B. THREE ANGULAR POSITION PROGRAM

a. Operational Directory

Purpose

To find the position and velocity of an observed body at Time, T_o, from measurements of the right ascension and declination or azimuth and elevation at three different Times, T_1, T_o, T_3.

Usage

Input - All decimal input is read by a modified DBC FORTRAN subroutine which accepts variable length fields. All fields are floating point numbers except starred fields which are integers. The list of input quantities follows in the order in which they must be read into the program and then the input for a sample problem. All angle inputs must be in degrees and decimals of a degree. Range measurements must be in units of Earth's equatorial radius, (6378.388 km. = equatorial radius of Earth).

Card 1

*Field 1 - Control Parameter, KO If KO ≤ 0, the unit of time will be equal to 58.132441 days. If KO ≥ 1, the unit of time will equal to 806.996813 sec.

Card 2

Field 1 - T_1, Time T_1 in days and decimals of a day.

Field 2 - α_1, right ascension in degrees and decimals of a degree at T_1.

Field 3 - δ_1, declination in degrees and decimals of a degree at T_1.

Field 4 - X_1
Field 5 - Y_1
Field 6 - Z_1
Geocentric, equatorial coordinates of observer's position on Earth at time T_1 in units of earth's equatorial radius.

Card 3 - T_o, α_o, δ_o, x_o, y_o, z_o ; Same format as Card 2.

Card 4 - T_3, α_3, δ_3, x_3, y_3, z_3 : Same format as Card 2.

Sample Input

Card 1

X1*

Card 2

F 64.5, 343.279, - 7.98, 1.0012, - .0712, - .03258*

Card 3

F 72.5, 342.280, - 9.34, 1.00082, .05063, .02196*

Card 4

F 80.5, 341.469, - 10.56, .98125, .17543, .07609*

Output - The X, Y, Z components of the position vector are printed out in units of Earth's equatorial radius, kilometers, and astronomical units; the X, Y, Z components of the velocity vector are printed out in units of Earth's equatorial radius per 806.996813 sec., km. per sec. and A. U. per hr., If KO $\geq$ 1. If KO $\leq$ 0, the velocity will be printed out in units of Earth's equatorial radius per 58.132441 days, km. per sec. and A. U. per hr.

b. Three Angular Position Program

```
      DIMENSION T(3), ALPHA(3), PHI(3), X(3), Y(3), Z(3), YOUM (3),
     1 YOUL(3), VEEM(3), VEEL(3), PIM(3), PIL(3), QM(3), QL(3), PM(6)
     2 PL(6)
      DIMENSION PZM(6), PZL(6)
    6 RIT2, 1, KO,  (T(I), ALPHA(I), PHI(I), X(I), Y(I), Z(I), I = 1, 3)
    1 FORMAT (5I5)
      ERASE KNT
      DO 5 I = 1, 3
      ALPM = ALPHA (I)
```

```
      PHIM = PHI (I)
      CALL DPSC (ALPM, O. , SINAM, SINAL, COSAM, COSAL, O. )
      CALL DPSC (PHIM, O. , SINPM, SINPL, COSPM, COSPL, O. )
      CALL FDY (SINAM, SINAL, COSAM, COSAL, TANAM, TANAL)
      CALL FDY (SINPM, SINPL, COSPM, COSPL, TANPM, TANPL)
      YOUM (I) = TANAM
      YOUL (I) = TANAL
      CALL FDY (TANPM, TANPL, COSAM, COSAL, TASEM, TASEL)
      VEEM (I) = TASEM
      VEEL (I) = TASEL
      XM = X (I)
      CALL FMPS (TANAM, TANAL, XM, UXM, UXL)
      YM = Y (I)
      CALL FSB (YM, O. , UXM, UXL, YUXM, YUXL)
      PIM (I) = YUXM
      PIL (I) = YUXL
      CALL FMPS (TASEM, TASEL, XM, VXM, VXL)
      ZM = Z (I)
      CALL FSB (ZM, O. , VXM, VXL, ZVXM, ZVXL)
      QM (I) = ZVXM
 5    QL (I) = ZVXL
      IF (KO) 9, 9, 10
 9    TU = 58. 132441
      GO TO 11
 10   TU = . 93394389E-2
 11   CALL FSBS (T(1), O. , T (2), DTM, DTL)
      CALL FDYS (DTM, DTL, TU, TAUM1, TAUL1)
      CALL FSBS (T (3), O. , T(2), DTM, DTL)
      CALL FDYS (DTM, DTL, TU, TAUM3, TAUL3)
      CALL FSB (TAUM3, TAUL3, TAUM1, TAUL1, DTAUM, DTAUL)
      CALL FSB (YOUM(1), YOUL(1), YOUM(2), YOUL(2), UNM1, UNL1)
```

```
      CALL FDY (UNM1, UNL1, TAUM1, TAUL1, UM1, UL1)
      CALL FSB (YOUM (3), YOUL(3), YOUM(2), YOUL(2), UNM3, UNL3)
      CALL FDY (UNM3, UNL3, TAUM3, TAUL3, UM3, UL3)
      CALL FSB (VEEM (1), VEEL(1), VEEM (2), VEEL(2), VNM1, VNL1)
      CALL FDY (VNM1, VNL1, TAUM1, TAUL1, VM1, VL1)
      CALL FSB (VEEM(3), VEEL(3), VEEM (2), VEEL(2), VNM3, VNL3)
      CALL FDY (VNM3, VNL3, TAUM3, TAUL3, VM3, VL3)
      CALL FSB (PIM (1), PIL(1), PIM(2), PIL(2), PNM1, PNL1)
      CALL FDY (PNM1, PNL1, TAUM1, TAUL1, PM1, PL1)
      CALL FSB (PIM(3), PIL(3), PIM(2), PIL(2), PNM3, PNL3)
      CALL FDY (PNM3, PNL3, TAUM3, TAUL3, PM3, PL3)
      CALL FSB (QM(1), QL(1), QM(2), QL(2), QNM1, QNL1)
      CALL FDY (QNM1, QNL1, TAUM1, TAUL1, QM1, QL1)
      CALL FSB (QM(3), QL(3), QM(2), QL(2), QNM3, QNL3)
      CALL FDY (QNM3, QNL3, TAUM3, TAUL3, QM3, QL3)
12    NO = 1
      PAM = UM1
      PAL = UL1
      PBM = UM3
      PBL - UL3
13    CALL FMP (TAUM3, TAUL3, PAM, PAL, TMVM, TMVL)
      CALL FMP (TAUM1, TAUL1, PBM, PBL, TSVM, TSVL)
      CALL FSB (TMVM, TMVL, TSVM, TSVL, TNM, TNL)
      CALL FDY (TNM, TNL, DTAUM, DTAUL, DOM, DOL)
      CALL  FSB (PBM, PBL, PAM, PAL, TNM, TNL)
      CALL FDY (TNM, TNL, DTAUM, DTAUL, D2OM, D2OL)
      GO TO (20, 21, 22, 23), NO
20    DUM = DOM
      DUL = DOL
      D2UM = D2OM
      D2UL = D2OL
```

```
      PAM = VM1
      PAL = VL1
      PBM = VM3
      PBL = VL3
      NO = NO + 1
      GO TO 13
21    DVM = DOM
      DVL = DOL
      D2VM - D2OM
      D2VL = D2OL
      PAM = PM1
      PAL = PL1
      PBM = PM3
      PBL = PL 3
      NO = NO + 1
      GO TO 13
22    DPM = DOM
      DPL = DOL
      D2PM = D2OM
      D2PL = D2OL
      PAM = QM1
      PAL = QL1
      PBM = QM3
      PBL = QL3
      NO = NO + 1
      GO TO 13
23    DQM = DOM
      DQL = DOL
      D2QM = D2OM
      D2QL = D2OL
      CALLFMP(D2UM, D2UL, DVM, DVL, UVMM, UVML)
```

```
      CALL FMP(D2VM, D2VL, DUM, DUL, UVSM, UVSL)
      CALL FSB (UVSM, UVSL, UVMM, UVML, DM, DL)
      CALL FMP (D2PM, D2PL, DVM, DVL, PVMM, PVML)
      CALL FMP (D2QM, D2QL, DUM, DUL, QUMM, QUML)
      CALL FSB (PVMM, PVML, QUMM, QUML, AM, AL)
      CALL FMP (PIM(2), PIL (2), DVM, DVL, PVMM, PVML)
      CALL FMP (QM(2), QL(2), DUM, DUL, QUMM, QUML)
      CALL FSB (PVMM, PVML, QUMM, QUML, BM, BL)
      CALL SQ( VEEM(2), VEEL(2), VSQM, VSQL)
      CALL SQ(YOUM(2), YOUL(2), USQM, USQL)
      CALL FAD (VSQM, VSQL, USQM, USQL, UVSQM, UVSQL)
      CALL FADS(UVSQM, UVSQL, 1., CM, CL)
      CALL FMP (YOUM(2), YOUL(2), PIM(2), PIL(2), UPM, UPL)
      CALL FMP (VEEM (2), VEEL (2), QM (2), QL (2L) VQM, VQL)
      CALL FAD (UPM, UPL, VQM, VQL, EM, EL)
      CALL SQ (PIM (2), PIL (2), PSQM, PSQL)
      CALL SQ (QM (2), QL(2), QSQM, QSQL)
      CALL FAD (PSQM, PSQL, QSQM, QSQL, FM, FL)
      CALL FMP (D2QM, D2QL, D2UM, D2UL, D2QUM, D2QUL)
      CALL FMP (D2PM, D2PL, D2VM, D2VL, D2PVM, D2PVL)
      CALL FSB (D2QUM, D2QUL, D2PVM, D2PVL, GM, GL)
      CALL FMP (D2UM, D2UL, QM(2), QL(2), D2UQM, D2UQL)
      CALL FMP (D2VM, D2VL, PIM (2), PIL (2), D2VPM, D2VPL)
      CALL FSB (D2UQM, D2UQL, D2VPM, D2VPL, HM, HL)
      RM = 1.02
      RL = 0.
30    CALL CUBE (RM, RL, R3M, R3L)
      CALL FAD (R3M, R3L, R3M, R3L, R3M, R3L)
      CALL FDY (BM, BL, R3M, R3L, BRM, BRL)
      CALL FAD (AM, AL, BRM, BRL, BARM, BARL)
      CALL FDY (BARM, BARL, DM, DL, XM, XL)
```

```
      CALL SQ (XM, XL, X2M, X2L)
      CALL FMP (X2M, X2L, CM, CL, CXM, CXL)
      CALL FMP (XM, XL, EM, EL, EXM, EXL)
      CALL FAD (EXM, EXL, EXM, EXL, EXM, EXL)
      CALL FAD (CXM, CXL, EXM, EXL, ECXM, ECXL)
      CALL FAD (ECXM, ECXL, FM, FL, RNM, RNL)
      CALL DPSQRT (RNM, RNL, RNM, RNL)
      WOT 10, 3, XM, RNM
3     FORMAT (1HO10X, 4HX = 1 PE16. 7, 10X, 4HR = E16. 7)
      KNT = KNT + 1
      CALL FSB (RNM, RNL, RM, RL, DRM, DRL)
      IF (ABSF(DRM) - 1E - 8) 33, 33, 34
34    IF (KNT - 100) 35, 35, 36
35    RM = RNM
      RL = RNL
      GO TO 30
36    WOT 10, 4
4     FORMAT (1HO10X, 9HGIVING UP)
      GO TO 6
33    CALL CUBE (RNM, RNL, RNM, RNL)
      CALL FAD (RNM, RNL, RNM, RNL, RNM, RNL)
      CALL FDY(BM, BL, RNM, RNL, BRM, BRL)
      CALL FAD (AM, AL, BRM, BRL, BARM, BARL)
      CALL FDY (BARM, BARL, DM, DL, PM(1), PL(1))
      CALL FDY (HM, HL, RNM, RNL, HRM, HRL)
      CALL FAD (HRM, HRL, GM, GL, HRM, HRL)
      CALL FDY (HRM, HRL, DM, DL, PM(4), PL(4))
      CALL FMP (YOUM(2), YOUL(2), PM(1), PL(1), XUM, XUL)
      CALL FAD (XUM, XUL, PIM(2), PIL(2), PM(2), PL(2))
      CALL FMP (DUM, DUL, PM(1), PL(1), DXUM, DXUL)
      CALL FMP (YOUM(2), YOUL(2), PM(4), PL(4). DUXM, DUXL)
      CALL FAD (DXUM, DXUL, DUXM, DUXL, DYM, DYL)
```

```
      CALL FAD(DYM, DYL, DPM, DPL, PM(5), PL(5) )
      CALL FMP(VEEM(2), VEEL(2), PM(1), PL(1), VXM, VXL)
      CALL FAD(VXM, VXL, QM(2), QL(2), PM(3), PL(3) )
      CALL FMP (DVM, DVL, PM(1), PL(1), DVXM, DVXL)
      CALL FMP(VEEM(2), VEEL(2), PM(4), PL(4), DXVM, DXVL)
      CALL FAD(DVXM, DVXL, DXVM, DXVL, DXM, DXL)
      CALL FAD(DXM, DXL, DQM, DQL, PM(6), PL(6) )
      IF (KO) 17, 17, 18
17    WOT 10, 19, T (2), (PM(I), I = 1, 6)
      GO TO 63
18    WOT 10, 15, T(2), (PM(I), I - 1, 6)
15    FORMAT (1H13X, 6HTIME = F9. 2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/
     1 1HO3X, 8HPOSITION 9X, 1PE40. 7, 9X, E16. 7, 11X, E14. 7/1H
     2 6X, 20HUNITS OF EARTH RADII/1HO3X, 8HVELOCITY35X,
     3 E14. 7, 11X, E14. 7, 11X, E14. 7/1H 6X, 33HUNITS OF EARTH RADII/
     4 806. 928 SEC. ///)
19    FORMAT(1H13X, 6HTIME = F9. 2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/
     1 1HO3X, 8HPOSITION 9X, 1PE40. 7, 9X, E16. 7, 11X, E14. 7/ 1H 6X,
     2 20 HUNITS OF EARTH RADII/1HO3X, 8HVELOCITY35X, E14. 7,
     3 11X, E14. 7, 11X, E14. 7/1H 6X, 33HUNITS OF EARTH RADII/
     4 58. 13244 DA. ///)
63    DO 50 I = 1, 3
      PS = PM(I)
      PSL = PL(I)
      CALL FMPS(PS, PSL, 6378. 388, PAM, PAL)
      PZM(I) = PAM
      PZL(I) - PAL
      CALL FDYS(PAM, PAL, 149. 5042132E + 6, PAM, PAL)
      PM (I) = PAM
50    PL(I) = PAL
      DO 51 I = 4, 6
```

```
      PS = PM (I)
      PSL = PL (I)
      IF (KO) 60, 60, 61
60    CALL FMPS (PS, PSL, . 12992648E-2, PAM, PAL)
      GO TO 62
61    CALL FMPS (PS, PSL, 7. 90453641, PAM, PAL)
62    PZM (I) = PAM
      PZL (I) = PAL
      CALL FMPS (PAM. PAL. . 24079588E-4, PAM, PAL)
      PM (I) = PAM
51    PL (I) = PAL
      WOT 10, 24 T(2), (PZM (I), I = 1, 6)
      WOT 10, 27, T(2), (PM (I), I = 1, 6)
24    FORMAT (1HO3X, 6HTIME = F9. 2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/
   1  1HO3X, 8HPOSITION 9X, 1PE40. 7, 5H KM  E20. 7, 3H KM8X,
   2  E14. 7, 3H KM/1HO3X, 8HVELOCITY 35X, E14. 7, 7H KM/SEC4X,
   3  E14. 7, 7H KM/SEC4X, E14. 7H KM/SEC///)
27    FORMAT (1HO3X, 6HTIME = F9. 2, 39X, 1HX, 24X, 1HY, 24X, 1HZ/
   1  1HO3X, 8HPOSITION 9X, 1PE40. 7, 7H A. U.  E18. 7, 5H A. U.
   2  6X, E14. 7, 5H A. U. /1HO3X, 8HVELOCITY35X, E14. 7, 8H A. U. /
   3  HR3X, E14. 7, 8H A. U. /HR3X, E14. 7, 8H A. U. /HR)
40    GO TO 6
      END (1, 1, 0, 0, 0, 1)
```

c. Three Angular Position Program

Check Problem

<u>Output:</u>

Time 33. 91

Position-Units of Earth Radii

X = 2.2272140E 00 Y = -6.4261167E-01 Z = -2.4354754E-01

Velocity-Units of Earth Radii/58.13244 DAY

X = 2.5257830E-01 Y = 6.6070658E-01 Z = 8.8773156E-02

Time = 33.91

Position

X = 1.4206035E 04 KM Y = -4.0988266E O3KM Z=1.5534407E 03KM

Velocity

X = 3.2816609E-04 KM/SEC Y = 8.5843279E-04 KM/SEC Z = 1.1533984E-04 KM/SEC

Time = 33.91

Position

X = 9.5020968E-05 A. U. Y = -2.7416128E-05 A. U. Z = -1.0390615E-05 A. U.

Velocity

X = 7.9021043E-09 A. U. /HR Y = 2.0670708E-08 A. U/HR Z = 2.7773358E-09 A. U. /HR

Input:

Card 1: XO*

Card 2: F 30.006, 346.5265, -3.690944, .9217386, -.3782763, -.16402741*

Card 3: F 33.9067, 345.92583, -4.510222, .9460249, -.3214131, -.1393582*

Card 4: F 37.9351, 345.28958, -5.365694, .9667071, -.2612860, -.1132835*

C. CONVERSION OF COORDINATES FROM EQUINOX TO EQUINOX

a. Operational Directory

Purpose

To rotate a given vector $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ from the equinox of 1950.0 to other equinoxes and vice versa.

Usage

Input - All decimal input is read by a modified DBC FORTRAN subroutine which accepts variable length fields. All fields are floating point numbers except for starred fields which are integers.

Following is the list of input quantities in the order in which they are read into the program:

Card 1

*Field 1 - N5, Control Parameter

If N5 $\geq$ 1, X will be rotated from equinox of date to equinox of 1950.0.

If N5 $\leq$ 0, X will be rotated from equinox of 1950.0 to equinox of date.

*Field 2 - NP, Control Parameter

NP must be equal to the number of vectors which are to be rotated as specified by N5. 1 $\leq$ NP $\leq$ 10.

Cards 2 - NP + 1

Field 1 - Whole number component of Julian Date. The Julian Date must correspond to the beginning of the Besselian Year.

Field 2 - Decimal Component of Julian Date.

Field 3 - X component of vector to be rotated.

Field 4 - Y component of vector to be rotated.

Field 5 - Z component of vector to be rotated.

Sample Input

Card 1 - Columns 1-5

X1, 2*

Card 2 - Columns 1-32

Card 2 - Columns 1-32

F 2437203., .5, .2985, .3740, -.6739*

Card 3 - Columns 1-32

F 2437008., .5, .0046. -.5968, -.3702*

<u>Output</u> - The vector components are printed out referenced to the new equinox.

b. Conversion of Coordinates from Equinox to Equinox

```
      DAC AM(3,3), AL(3,3), PZM(10,3), PZL(10,3), DJM(10), DJL(10),
    1 X(10), Z(10), Y(10), XL(10), YL(10), ZL(10)
C     N5 = 1, REDUCTION OF X, Y, Z OF DATE TO EQUINOX OF 1950.0
C     N5 = 0, REDUCTION OF X, Y, Z OF 1950.0 TO MEAN EQUINOX OF
    1 DATE
 1    RIT 2, 3, N5, NP
 3    FORMAT (5I5)
 11   RIT 2, 3, (DJM(I), DJL(I), X(I), Y(I), Z(I), I = 1, NP)
      ERASE XL, YL, ZL
      DO 500 J = 1, NP
      DJIM = DJM(J)
      DJIL = DJL(J)
      IF (N5) 61, 71, 61
 61   CALL FSB(2433282., .5, DJIM, DJIL, VM, VL)
      GO TO 62
 71   CALL FSB(DJIM, DJIL, 2433282., .5, VM, VL)
 62   CALL FDYS(VM, VL, 36525., VM, VL)
      CALL SQ(VM, VL, VSQM, VSQL)
      CALL CUBE(VM, VL, VCUM, VCUL)
      CALL FMPS(VSQM, VSQL, .00029696, CM, CL)
      CALL FMPS(VCUM, VCUL, .00000014, DM, DL)
      CALL FSB(1., 0., CM, CL, CM, CL)
```

```
CALL FSB(CM, CL, DM, DL, AM(1, 1), AL(1, 1) )
CALL FMPS (VM, VL, -.02234941, CM, CL)
CALL FMPS(VSQM, VSQL, .00000676, DM, DL)
CALL FMPS(VCUM, VCUL, .00000221, EM, EL)
CALL FSB(CM, CL, DM, DL, CM, CL)
CALL FAD(CM, CM, EL, AM(1,2), AL(1,2) )
CALL FMPS(VM, VL, -.00971691, CM, CL)
CALL FMPS(VSQM, VSQL, .00000206, DM, DL)
CALL FMPS(VCUM, VCUL, .00000098, EM, EL)
CALL FAD(CM, CL, DM, DL, CM, DL)
CALL FAD(CM, CL, EM, EL, AM(1, 3), AL(1, 3) )
AM(2, 1) = -AM(1, 2)
AL(2, 1) =-AL(1, 2)
CALL FMPS(VSQM, VSQL, .00024975, CM, CL)
CALL FMPS(VCUM, VCUL, .00000015, DM, DL)
CALL FSB(1.,0., CM, CL, CM, CL)
CALL FSB(CM, CL, DM, DL, AM(2, 2), AL(2, 2) )
CALL FMPS(VSQM, VSQL, -.00010858, CM, CL)
CALL FMPS(VCUM, VCUL, .00000003, DM, DL)
CALL FSB(CM, CL, DM, DL, AM(2, 3), AL(2, 3) )
AM(3, 1) = AM(1, 3)
AL(3, 1) = -AL(1, 3)
AM(3, 2) = AM(2, 3)
AL(3, 2) = AL(2, 3)
CALL FMPS(VSQM, VSQL, .00004721, CM, CL)
CALL FMPS(VCUM, VCUL, .00000002, DM, DL)
CALL FSB(1.0., CM, CL, CM, CL)
CALL FAD(CM, CL, DM, DL, AM(3, 3), AL(3, 3) )
DO 7 I=1,3
CALL FMP(AM(I, 1), AL(I, 1), X(J), XL(J), CM, CL)
CALL FMP(AM(I, 2), AL(I, 2), Y(J), YL(J), DM, DL)
```

```
      CALL FMP(AM(I, 3), AL(I, 3), Z(J), ZL(J), EM, EL)
      CALL FAD(CM, CL, DM, DL, CM, CL)
7     CALL FAD(CM, CL, EM, EL, PZM(J, I), PZL(J, I) )
500   CONTINUE
      DO 9 J = 1, NP
      X (J) = PZM(J, 1)
      Y (J) = PZM(J, 2)
      Z (J) = PZM(J, 3)
      XL(J) = PZL(J, 1)
      YL(J) = PZL(J, 2)
9     ZL(J) = PZL(J, 3)
      IF (N5) 20, 20, 21
20    WOT 10, 24, (X(I), Y(I), Z(I), I = 1, NP)
24    FORMAT(1H16X, 62H   COORDINATES REFERENCED TO MEAN
  1   EQUINOX OF DATE/(1H06X, 2HX = 1PE14. 7, 10X, 2HY = E14. 7, 10X
  2   2HZ = E14. 7) )
      GO TO 1
21    WOT 10, 30
30    FORMAT (1HO)
      WOT 10, 25, (X(I), Y(I), Z(I), I = 1, NP)
25    FORMAT(1H06X, 62H   COORDINATES REFERENCED TO MEAN
  1   EQUINOX OF 1950/(1H06X, 2HX = 1PE14. 7, 10X, 2HY = E14. 7, 10X,
  2   2HZ = E14. 7) )
      GO TO 1
      END (1, 1, 0, 0, 0, 1)
```

c. Conversion of Coordinates from Equinox to Equinox

Check Problem

<u>Output:</u>

Coordinates Referenced to Mean Equinox of 1950

X = 1. 0008215E 00 Y = 5. 0628301E-02 Z = 2. 1963697E-02

Coordinates Referenced to Mean Equinox of Date

X = 1.0010150E 00   Y = 4.7274507E-02   Z = 2.0501011E-02

Input:

Card 1: X 1, 1*

Card 2: 2428072., .5, 1.0010150, .04727450, .02050101*

Card 3: X 0, 1*

Card 4: 2428072., .5, 1.0008215, .05062830, .02196369*

D. CONVERSION OF RIGHT ASCENSION AND DECLINATION FROM EQUINOX TO EQUINOX

a. Operational Directory

Purpose

To reduce right ascension and declination referenced to mean equinox of date to right ascension and delination referenced to mean equinox of 1950.0 and vice versa.

Usage

Input - All decimal input is read by a modified DBC FORTRAN subroutine which accepts variable length fields. All fields are floating point numbers except for starred fields which are integers.

The list of input quantities in the order in which they are read into the program follows:

Card 1

*Field 1 - N3, Control Parameter

If N3 $\geq$ 1, α and δ will be reduced from equinox of 1950.0 to equinox of date.

If N3 $\leq$ 0, α and δ will be reduced from equinox of date to equinox of 1950.0.

*Field 2 - NP, Control Parameter

NP must be equal to the number of reductions which are to be made as specified by N3, 1 $\leq$ NP $\leq$ 10.

Cards 2 - NP + 1

Field 1 - Whole number component of Julian Date.

Field 2 - Decimal component of Julian Date.

Field 3 - Right ascension in degrees and decimals of a degree,

Field 4 - Declination in degrees and decimals of a degree.

Sample Input

Card 1: Columns 1-5

XO, 1*

Card 2: Columns 1 - 37

F 2415020., .5, 332.2054167, -80.9374167*

Output - The right ascension and declination are printed out in degrees and decimals of a degree referenced to desired equinox.

b. Conversion of Right Ascension and Declination from Equinox to Equinox

```
      DAC DJM(10), ALPHAM(10), APL(20), APM(20), DDM(20), DDL(20),
     1 DJL(10), ALPHAL(10), DELTAM(10), DELTAL(10)
 1    RIT 2, 3, N3, NP
 3    FORMAT (5I5)
 2    RIT 2, 3, (DJM(I), DJL(I), ALPHAM(I), DELTAM(I), I = 1, NP
C     CONVERSION OF ALPHA + DELTA FROM DATE TO EQUINOX OF
     1 1950.0, N3 = 3
C     CONVERSION OF ALPHA + DELTA FROM EQUINOX TO DATE,
     1 N3 = +1
      DO 400I = 1, NP
440   FORMAT(1H1)
      ERASE KNT, ALPHL, DELTL
      DJIM = DJM(I)
```

```
      DJIL = DJL(I)
      CALL FSB(DJIM, DJIL, 2433281., .5, AMM, AML)
      CALL FDYS(AMM, AML, 365.25, RDM, RDL)
      CALL FSB(2433281., .5, DJIM, DJIL, REM, REL)
      CALL FDYS(REM, REL, 365.25, REM, REL)
C     COMPUTE M AND N, AMM = M, ANM = N
      CALL FMPS(RDM, RDL, .25, AQM, AQL)
      CALL FMPS(AQM, AQL, .000279, AMM, AML)
      CALL FADS(AMM, AML, 46.09905, AMM, AML)
      CALL FDYS(AMM, AML, 3600., AMM, AML)
      CALL FMPS(AQM, AQL, .000085, ANM, ANL)
      CALL FSB(20.0426, 0., ANM, ANL, ANM, ANL)
      CALL FDYS(ANM, ANL, 3600., ANM, ANL)
      ALPHM = ALPHAM(I)
      DELTM = DELTAM(I)
   60 CALL DPSC(ALPHM, ALPHL, SALPHM, SALPHL, CALPHM,
     1 CALPHL, 0)
      CALL DPSC(DELTM, DELTL, SDELTM, SDELTL, CDELTM,
     1 CDLTL, 0)
C     COMPUTE DALPHA/DT AND DDELTA/DT
      CALL FDY(SDELTM, SDELTL, CDELTM, CDELTL, TDELTM,
     1 TDELTL)
      CALL FMP(TDELTM, TDELTL, SALPHM, SALPHL, DADTM,
     1 DADTL
      CALL FMP(DADTM, DADTL, ANM, ANL, DADTM, DADTL)
      CALL FAD(DADTM, DADTL, AMM, AML, DADTM, DADTL)
      CALL FMP(ANM, ANL, CALPHM, CALPHL, DDDTM, DDDTL)
      KNT = KNT +1
      IF (N3) 50,50,51
   50 RDM = REM
      RDL = REL
```

```
51      CALL FMP(RDM, RDL, DADTM, DADTL, APIM, APIL)
        CALL FAD(APIM, APIL, ALPHAM(I), ALPHAL(I), APM(KNT)
    1   APL(KNT) )
        CALL FMP(RDM, RDL, DDDTM, DDDTL, DDIM, DDIL)
        CALL FAD(DDIM, DDIL, DELTAM(I), DELTAL(I), DDM(KNT),
    1   DDL(KNT) )
        IF (KNT - 2) 53, 54, 54
54      CALL FSB(APM(KNT), APL(KNT), APM(KNT-1), APL(KNT-1),
    1   DAP, DAL)
        CALL FSB(DDM(KNT), DDL(KNT), DDM(KNT-1) DDL(KNT-1),
    1   DDD, DAL)
        IF (ABSF(DAP) - 1E-8)55, 53, 53
55      IF (ABSF(DDD) - 1E-8)62, 53, 53
53      CALL FAD(APM(KNT), APL(KNT), ALPHAM(I), ALPHAL(I), ALPHM,
    1   ALPHL)
        CALL FMPS(ALPHM, ALPHL, .5, ALPHM, ALPHL)
        CALL FAD(DDM(KNT), DDL(KNT), DELTAM(I), DELTAL(I), DELTM,
    1   DELTL)
        CALL FMPS(DELTM, DELTL, .5, DELTM, DELTL)
        IF (KNT-20) 60, 80, 80
80      WOT 10, 8
 8      FORMAT(1HO6X, 9HGIVING  UP)
        GO TO 400
62      ALPHAM(I) = APM(KNT)
        ALPHAL(I) = APL(KNT)
        DELTAM(I) = DDM(KNT)
        DELTAL(I) = DDL(KNT)
400     CONTINUE
        IF (N3) 70, 70, 71
70      WOT 10, 5, (ALPHAM(I), DELTAM(I), I = 1, NP)
 5      FORMAT(1H115X, 62HASCENSION AND DECLINATION
```

```
    1     REFERENCED TO MEAN EQUINOX OF 1950.0/(1HO20X,
    2     6HALPHA = 1PE14.7, 16H DEGREES DELTA = E14.7, 8H DEGREES) )
          GO TO 1
71        WOT 10, 6(ALPHAM(I), DELTAM(I), I = 1, NP)
 6        FORMAT (1H115X, 62HASCENSION AND DECLINATION
    1     REFERENCED TO MEAN EQUINOX OF DATE/(1HO20X,
    2     6HALPHA - 1PE14.7, 16H DEGREES DELTA = 14.7, 8 H DEGREES) )
          GO TO 1
          END (1, 1, 0, 0, 0, 1)
```

c. Conversion of Right Ascension and Declination from Equinox to Equinox

Check Problem

<u>Output:</u>

Right Ascension and Declination Referenced to Mean Equinox of Date

Alpha = 3.4327915E 02 Degrees

Delta = -7.9466670E 00 Degrees

Alpha = 3.4111123E 02 Degrees Delta = -1.0696111E 01 Degrees

<u>Input:</u>

Card 1: X 1, 2*

Card 2: F 2428064., .5, 343.46521, -7.87047*

Card 3: F 2428080., .5, 341.29838, -10.621060*

Right Ascension and Declination Referenced to Mean Equinox of 1950.0

Alpha - 3.4346521E 02 Degrees

Delta = - 7.8704700E 00 Degrees

Alpha - 3.4129838E 02 Degrees

Delta = -1.0621060E 01 Degrees

<u>Input:</u>

Card 1: X 0, 2*

Card 2: F 2428064., .5, 343.27915, -7.946667*

Card 3: F 2428080., .5, 341.111249, -10.696111*

E. COMPUTATION OF RECTANGULAR GEOCENTRIC SITE COORDINATES AND CONVERSION OF AZIMUTH AND ELEVATION TO RIGHT ASCENSION AND DECLINATION

a. Operational Directory

Purpose

To compute the geocentric site coordinate and to convert azimuth and elevation to right ascension and declination.

Usage

Input - All decimal input is read by a modified DBC FORTRAN subroutine which accepts variable length fields. All fields are floating point numbers except for starred fields which are integers. Following is the list of input quantities in the order in which they must be read into the program and then the input for a sample problem.

*Field 1 - N4, Control parameter

If N4 $\geq$ 1, right ascension and declination will be computed from the corresponding azimuth and elevation.

If N4 $\leq$ 0, right ascension and declination will not be computed.

*Field 2 - NP, Control Parameter which must be equal to the number of cases to be run. $1 \leq NP \leq 10$

Field 3 - ϕ, geocentric latitude of site in degrees and decimals of a degree (+ if North, - if South).

Field 4 - h, height of site above sea level in units of Earth's equatorial radius. (6378.388 km. = equatorial radius of Earth)

Field 5 - λ, geographic longitude of site in degrees and decimals of a degree. (+ if East, - if West)

Field 6 - Whole number component of Julian Date.

Field 7 - Decimal component of Julian Date.

Field 8 - Decimal component of corresponding universal time.

<u>NOTE:</u> If N4 $\leq$ 0, no more input is required; however, the input must include "NP" sets of data, each set of which must be composed of Fields 3-8.

If N4 $\geq$ 1, then each set of the "NP" sets must be composed of Fields 3-10.

Field 9 - Elevation in degrees and decimals of a degree.

Field 10 - Azimuth in degrees and decimals of a degree.

Sample Inputs

Example 1

Card 1: X 1, 1*

Card 2: F 50.35, 023E-6, 4.358, 2428048.,
.5, 0,. 25.46, 209.35*

Example 2

Card 1: X 0, 2*

Card 2: F 53.2, .013E-4, 6.37, 2428072., .5, 0*

Card 3: F 55.2, .014E-5, 6.39, 2428078., .5, 0*

<u>Output</u> - If N4 $\leq$ 0, only the rectangular geocentric site coordinates will be printed out in units of Earth's equatorial radius.

If N4 $\geq$ 1, the right ascension and declination corresponding to each site vector will be printed out in degrees. Both site coordinates and angles will be referenced to mean equinox of date.

b. Computation of Rectangular Geocentric Site Coordinates and Conversion of Azimuth and Elevation to Right Ascension and Declination

```
      DAC PHIPM(10), PHIPL(10), RM(10), GMSTM(10),
    1 GMSTL(10), PHI(10), H(10), BAMDA(10), DJM(10), DJL(10), T(10).
```

```
      2   BSTM(10), BSTL(10), SPHIPM(10), SPHIPL(9), CPHIPM(10)
      3   CPHIPL(10), X(10). Y(10), Z(10)
          DAC CDELTM(10), CDELTL(10), SDELTM(10), SDELTL(10),
      1   DELTAM(10), DELTAL(10), ALPHAM(10), ALPHAL(10), E(10),
      2   A(10), SALPHM(10), CALPHM(10), SALPHL(10), CALPHL(10),
      3   HAB(10)
    4     FORMAT (15)
    1     RIT 2, 4, N4, NP
          IF (N4) 3, 3, 6
    3     RIT 2, 4, (PHI(I), H(I), BAMDA(I), DJM(I), DJL(I), T(I), I = 1,  NP)
          GO TO 7
    6     RIT 2, 4(PHI(I), H(I), BAMDA(I), DJM(I), DJL(I), T(I), E(I), A(I), I =
      1   1, NP)
    7     DO 100I = 1,  NP
          PHIM = PHI(I)
          HT = H(I)
          DJIM = DJM(I)
          DJIL = DJL(I)
          BAMDAM = BAMDA(I)
          TPHIM = 2. *PHIM
          FPHIM = 4. *PHIM
          SPHIM = 6. *PHIM
          TA = T(I)
C         COMPUTE THE GEOCENTRIC LATITUDE PHI PRIME (I-1)
          CALL DPSC(TPHIM,  0., STPHIM, STPHIL, CTPHIM, CTPHIL,  0)
          CALL DPSC(FPHIM,  0., SFPHIM, SFPHIL, CFPHIM, CFPHIL,  0)
          CALL DPSC(SPHIM,  0., SSPHIM, SSPHIL, CSPHIM, CSPHIL, 0)
          CALL FMPS(STPHIM, STPHIL, -695.6635, CM, CL)
          CALL FMPS(SFPHIM, SFPHIL, 1.1731, DM, DL)
          CALL FMPS(SSPHIM, SSPHIL, -.0026, EM, EL)
          CALL FAD(CM, CL, DM, DL, CM, CL)
```

```
      CALL FAD(CM, CL, EM. EL, CM, CL)
      CALL FDYS(CM, CL, 3600. , CM, CL)
      CALL FADS(CM, CL, PHIM, PHIPM(I), PHIPL(I) )
C     COMPUTE THE GEOCENTRIC RADIUS VECTOR OF SITE R(II-2)
      CALL FMPS(CTPHIM, CTPHIL, I.683494E-3, CM, CL)
      CALL FMPS(CFPHIM, CFPHIL, 3.549E-6, DM, DL)
      CALL FMPS(CSPHIM, CSPHIL, 8.0E-9, EM, EL)
      CALL FSB(CM, CL, DM, DL, CM, CL)
      CALL FAD(CM, CL, EM, EL, CM, CL)
      CALL FADS(CM, CL, .998320047, CM, CL)
      CALL FADS(CM, CL, HT, RM(I), RL(I) )
C     COMPUTE THE GREENWICH MEAN SIDEREAL TIME AT U.T.
     1 (II-3)
      CALL FSBS(DJIM, DJIL, 2415020.0, CM, CL)
      CALL FDYS(CM, CL, 36525. ,CM, CL)
      CALL SQ(CM, CL, CSQM, CSQL)
      CALL FMPS(CSQM, CSQL, .0929, TERM, TERL)
      CALL FMP(CM, CL, 8640184. , .542, TM, TL)
      CALL FAD(TERM, TERL, TM, TL, GMSTM(I), GMSTL(I) )
      CALL FADS(GMSTM(I), GMSTL(I), 23925.836, GMSTM(I), GMSTL(I) )
      CALL FDYS(GMSTM(I) GMSTL(I), 3600. , GMSTM(I), GMSTL(I) )
   60 CALL FSBS(GMSTM(I), GMSTL(I), 24. , GMSTM(I), GMSTL(I) )
      IF(GMSTM(I) -24.) 61, 61, 60
C     COMPUTE THE LOCAL SIDEREAL TIME (II-4)
   61 CALL FDYS(BAMDAM, 0. 15. , CM, CL)
      CALL FMPS(  TA, 0. , 24. , AM, AL)
      CALL FAD(AM, AL, CM, CL, CM, CL)
      CALL FMPS(AM, AL, .0027369, DM, DL)
      CALL FAD(DM, DL, CM, DL, STM, STL)
      CALL FAD(STM, STL, GMSTM(I), GMSTL(I), STM, STL)
      CALL FMPS(STM, STL, 15. , BSTM(I), BSTL(I) )
```

```
C      (II-5)
       CALL DPSC(BSTM(I), BSTL(I), SLSTM, SLSTL, CLSTM, CLSTL, 0)
       CALL DPSC(PHIPM(I), PHIPL(I), SPHIPM(I), SPHIPL(I)
     1 CPHIPM(I), CPHIPL(I), 0)
C      COMPUTE RECTANGULAR COMPONENTS (II-6)
       CALL FMP(CPHIPM(I), CPHIPL(I), CLSTM, CLSTL, XM, XL)
       CALL FMP(XM, XL, RM(I), RL(I), X(I), XL)
       CALL FMP(CPHIPM(I), CPHIPL(I), SLSTM, SLSTL, YM, YL)
       CALL FMP(YM, YL, RM(I), RL(I), Y(I), YL)
   100 CALL FMP(SPHIPM(1),SPHIPL(I), RM(I), RL(I), Z(I), ZL)
       WOT 10, 5, (X(I), Y(I), Z(I), BSTM(I), I = 1, NP)
     5 FORMAT (1H16X, 99HRECTANGULAR SITE COORDINATES
     1 REFERENCED TO EQUINOX OF DATE AND IN UNITS OF
     2 EARTHS EQUATORIAL RADIUS/(1HO3X, 2HX = 1PE14.7, 7X,
     3 2HY = E14. 7, 7X, 2HZ = E14. 7, 7X, 20HLOCAL SIDEREAL TIME =
     4 E14. 7, 8 H DEGREES) )
C      CONVERSION OF AZIMUTH AND ELEVATION TO RT. ASC.
     1 AND DEC (III-1)
       IF (N4) 1, 1, 8
     8 DO 200 I = 1, NP
       EM = E(I)
       AM = A(I)
       SPHIM = SPHIPM(I)
       SPHIL = SPHIPL(I)
       CPHIM = CPHIPM (I)
       CPHIL = CPHIPL(I)
       CALL DPSC(EM, 0. , SEM, SEL, CEM, CEL, 0)
       CALL DPSC(AM, 0. , SAM, SAL, CAM, CAL, 0)
       CALL FMP(SPHIM, SPHIL, SEM, SEL, CCM, CCL)
       CALL FMP(CPHIM, CPHIL, CEM, CEL, DM, DL)
       CALL FMP(DM, DL, CAM, CAL, DM, DL)
```

```
      CALL FAD(DM, DL, CCM, CCL, SDELTM(I), SDELTL(I) )
      CALL SQ(SDELTM(I), SDELTL(I), SDLTSM, SDLTSL)
      CALL FSB(1. ,0. , SDLTSM, SDLTSL, CDLTSM, CDLTSL)
      CALL DPSQRT(CDLTSM, CDLTSL, CDELTM(I), CDELTL(I) )
      CALL DPASIN(SDELTM(I), SDELTL(I), DELTAM(I), DELTAL(I),
     1 KZ)
      CALL FMPS(DELTAM(I), DELTAL(I), 57.2957795, DELTAM(I)
     1 DELTAL(I) )
C     COMPUTE RIGHT ASCENSION (III-2)
      CALL FMP(CEM, CEL, -SAM, -SAL, DM, DL)
      CALLFDY(DM, DL, CDELTM(I), CDELTL(I), SINHAM, SINHAL)
      CALL FMP(SEM, SEL, CPHIM, CPHIL, CM, CL)
      CALL FMP(CEM, CEL, SHPIM, SPHIL, DM, DL)
      CALL FMP(DM, DL, CAM, CAL, DM, DL)
      CALLFSB(CM, CL, DM, DL, CM, CL)
      CALL FDY(CM, CL, CDELTM(I), CDELTL(I), COSHAM, COSHAL)
      CALLDPASIN(SINHAM, SINHAL, HAM, HAL, KZ)
      IF (COSHAM) 20, 21, 21
   20 IF (SINHAM) 22, 23, 23
   22 CALL FSB(PIM, PIL, HAM, HAL, HAM, HAL)
      GO TO 25
   23 CALL FSB(PIM, PIL, HAM, HAL, HAM, HAL)
C     PIM, PIL, ARE TO BE READ IN ON BINARY CARDS
      GO TO 25
   21 IF (SINHAM) 24, 25, 25
   24 CALL FAD(TPIM, TPIL, HAM, HAL, HAM, HAL)
   25 CALL FMPS(HAM, HAL, 57.2957795, HAM, HAL)
      HAB(I) = HAM
  200 CALL FSB(BSTM(I), BSTL(I), HAM, HAL, ALPHAM(I), ALPHAL(I) )
      WOT 10, 103, (DELTAM(I), ALPHAM(I), HAB(I), I = 1, NP)
  103 FORMAT (1H06X, 61HRIGHT ASCENSION AND DECLINATION
```

```
 1     REFERENCED TO EQUINOX OF DATE//(1H06X, 6HDELTA =
 2     1PE14. 7, 16H DEGREES ALPHA = E14. 7, 1X20H DEGREES HOUR
 3     ANGLE = E14. 7, 8 HDEGREES) )
110    GO TO 1
       END (1, 1, 0, 0, 0, 1)
```

c. Computation of Rectangular Geocentric Site Coordinates and Conversion of Azimuth and Elevation to Right Ascension and Declination

Check Problem

Output:

Rectangular Site Coordinates Referenced to Equinox of Date

Units of Earth's Equatorial Radius

Local Sidereal Time = 9. 2112108E 00 Degrees

X = 6. 2516253E-01 Y = 1. 0137977E-01 Z = 7. 7127761E-01

Right Ascension and Declination Referenced to Equinox of Date

Delta = -9. 4294895E 00 Degrees

Alpha = -1. 7721417E 01 Degrees

Hour Angle = 2. 6932628E 01 Degrees

Input:

Card 1: X 1, 1*

Card 2: F 50. 7985278, . 7023213E - 6, 4. 3583333, 2428072. , 5, 0*

Card 3: F 25. 5638611, 209. 69025*

F. EPHERMERIS COMPUTATION PROGRAM

a. Operational Directory

Purpose

The program computes the rectangular geocentric site coordinates at Time T referenced to the mean equinox of date; these coordinates are reduced to the mean equinox of 1950. 0. The topocentric distance observed at time T is computed from the site coordinates and the rectangular geocentric coordinates of the observed body. The right ascension and declination of the observed body at time T are computed referenced to the mean equinox of 1950.0 and reduced to mean equinox of date. The azimuth and elevation of the observed body at time T are computed.

Usage

All decimal input is read by a modified DBC FORTRAN subroutine which accepts variable length fields. All fields are floating point numbers except for starred fields which are integers. Following is the list of input quantities in the order in which they are read into the program:

Field 1 - T, Whole number component of Julian Date.

Field 2 - T, Decimal Component of Julian Date.

Field 3 - X
Field 4 - Y
Field 5 - Z
Rectangular geocentric coordinates of observed body at time T in units of Earth's equatorial radius and reference to the mean equinox of 1950.0. (6378.388 km. = equatorial radius of Earth)

Field 6 - ϕ, Geocentric latitude of site in degrees and decimals of a degree (+ if North, - if South).

Field 7 - h, Height of site above sea level in units of Earth's equatorial radius.

Field 8 - λ, Geographic longitude of site in degrees and decimals of a degree (+ if East, - if West).

Field 9 - Decimal component of the universal time corresponding to the Julian Date in Fields 1 and 2.

Sample Input:

Example

Card 1: F 2428072., .5, 2.3358772, - 3709524?, -.20

Card 2: F 50.7985278, .7023213E-6, 4 3583333, 0*

Output:

The rectangular geocentric site coordinates in units of Earth's